## Pearson Edexcel International Advanced Level

**Tuesday 11 June 2024**

Morning (Time: 1 hour 30 minutes)

**Paper reference** **WFM03/01**

# Mathematics

**International Advanced Subsidiary/ Advanced Level**
**Further Pure Mathematics F3**

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators
must not have the facility for symbolic algebra manipulation, differentiation and
integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are
  clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without
  working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new
  answer and any working underneath.

*Turn over*

1. The hyperbola $H$ has

- foci with coordinates $\left( \pm \dfrac{13}{2}, 0 \right)$

- directrices with equations $x = \pm \dfrac{72}{13}$

- eccentricity $e$

Determine

(a) the value of $e$

(3)

(b) an equation for $H$, giving your answer in the form $px^2 - qy^2 = r$, where $p$, $q$ and $r$ are integers.

(3)

**(Total for Question 1 is 6 marks)**

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**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & 0 \end{pmatrix}$$

Given that $\mathbf{M}$ has exactly two distinct eigenvalues $\lambda_1$ and $\lambda_2$ where $\lambda_1 < \lambda_2$

(a) determine a normalised eigenvector corresponding to the eigenvalue $\lambda_1$

**(6)**

The line $l_1$ has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, where $\mu$ is a scalar parameter.

The transformation $T$ is represented by $\mathbf{M}$.

The line $l_1$ is transformed by $T$ to the line $l_2$

(b) Determine a vector equation for $l_2$, giving your answer in the form $\mathbf{r} \times \mathbf{b} = \mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are constant vectors.

**(3)**

**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

**(Total for Question 2 is 9 marks)**

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**3.**
$$y = \text{arsinh}\left(\sqrt{x^2 - 1}\right) \qquad x > 1$$

(a) Prove that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 - 1}}$

**(3)**

$$f(x) = \frac{1}{3} \text{arsinh}\left(\sqrt{x^2 - 1}\right) - \arctan x \qquad x > 1$$

(b) Determine the exact values of $x$ for which $f'(x) = 0$

**(4)**

**Question 3 continued**

**4.** (a) Use the definitions of hyperbolic functions in terms of exponentials to show that

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B$$

**(3)**

(b) Hence express $10 \sinh x + 8 \cosh x$ in the form $R \sinh(x + \alpha)$ where $R > 0$, giving $\alpha$ in the form $\ln p$ where $p$ is an integer.

**(4)**

(c) Hence solve the equation

$$10 \sinh x + 8 \cosh x = 18 \sqrt{7}$$

giving your answer in the form $\ln\left(\sqrt{7} + q\right)$ where $q$ is a rational number to be determined.

**(2)**

P 7 5 7 1 7 A 0 1 2 3 2

**Question 4 continued**

**(Total for Question 4 is 9 marks)**

P 7 5 7 1 7 A 0 1 3 3 2

**5.**
$$4x^2 + 4x + 17 \equiv (2x + p)^2 + q$$

where $p$ and $q$ are integers.

(a) Determine the value of $p$ and the value of $q$

<div align="right">(2)</div>

Given that

$$\frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} \equiv \frac{1}{\sqrt{4x^2 + 4x + 17}} + \frac{Ax + B}{\sqrt{4x^2 + 4x + 17}}$$

where $A$ and $B$ are integers,

(b) write down the value of $A$ and the value of $B$

<div align="right">(1)</div>

(c) Hence use algebraic integration to show that

$$\int_{\frac{1}{3}}^{1} \frac{8x + 5}{\sqrt{4x^2 + 4x + 17}} \, dx = k + \frac{1}{2}\ln k$$

where $k$ is a rational number to be determined.

<div align="right">(5)</div>

**Question 5 continued**

P 7 5 7 1 7 A 0 1 6 3 2

**(Total for Question 5 is 8 marks)**

P 7 5 7 1 7 A 0 1 7 3 2

**6.** The ellipse $E$ has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The line $l$ is the normal to $E$ at the point $P(5\cos\theta, 3\sin\theta)$ where $0 < \theta < \dfrac{\pi}{2}$

(a) Using calculus, show that an equation for $l$ is

$$5x\sin\theta - 3y\cos\theta = 16\sin\theta\cos\theta$$

(4)

Given that

- $l$ intersects the $y$-axis at the point $Q$

- the midpoint of the line segment $PQ$ is $M$

(b) determine the exact maximum area of triangle $OMP$ as $\theta$ varies, where $O$ is the origin.

You must justify your answer.

(5)

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P 7 5 7 1 7 A 0 2 0 3 2

**(Total for Question 6 is 9 marks)**

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7.  **In this question you must show all stages of your working.**

    **Solutions relying on calculator technology are not acceptable.**



**Figure 1**

Figure 1 shows the curve with equation

$$y = \ln\left(\tanh\frac{x}{2}\right) \qquad 1 \leqslant x \leqslant 2$$

(a) Show that the length, $s$, of the curve is given by

$$s = \int_1^2 \coth x \, \mathrm{d}x$$

**(4)**

(b) Hence show that

$$s = \ln\left(e + \frac{1}{e}\right)$$

**(4)**

*P 7 5 7 1 7 A 0 2 2 3 2*

P 7 5 7 1 7 A 0 2 3 3 2

**Question 7 continued**

P 7 5 7 1 7 A 0 2 4 3 2

**(Total for Question 7 is 8 marks)**

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**8.**

$$I_n = \int_0^k x^n (k-x)^{\frac{1}{2}} \, dx \qquad n \geqslant 0$$

where $k$ is a positive constant.

(a) Show that

$$I_n = \frac{2kn}{3+2n} I_{n-1} \qquad n \geqslant 1$$

(5)

Given that

$$\int_0^k x^2 (k-x)^{\frac{1}{2}} \, dx = \frac{9\sqrt{3}}{280}$$

(b) use the result in part (a) to determine the exact value of $k$.

(4)

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**(Total for Question 8 is 9 marks)**

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**9.** The plane $\Pi_1$ has vector equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where $s$ and $t$ are scalar parameters.

(a) Determine a Cartesian equation for $\Pi_1$

**(3)**

The plane $\Pi_2$ has vector equation $\mathbf{r}.\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 1$

(b) Determine a vector equation for the line of intersection of $\Pi_1$ and $\Pi_2$

Give your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and $\lambda$ is a scalar parameter.

**(4)**

The plane $\Pi_3$ has Cartesian equation $4x - 3y - z = 0$

(c) Use the answer to part (b) to determine the coordinates of the point of intersection of $\Pi_1$, $\Pi_2$ and $\Pi_3$

**(3)**

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**(Total for Question 9 is 10 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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