Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

BLANK PAGE

1 Solve the inequality $2 - 5x > 2|x - 3|$. [4]

2   On a sketch of an Argand diagram, shade the region whose points represent complex numbers z
satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + \mathrm{i}| \leqslant 1$.   [4]

3   The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot\theta$.   [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

4 Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

....

....

(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

....

....

....

....

....

....

....

....

....

6 **(a)** Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

7 **(a)** Verify that $-1 + \sqrt{5}\,i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Find the other roots of this equation. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x\frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

[Turn over

9 Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2.

[5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

10

The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....