Cambridge Assessment International Education

# Cambridge International AS & A Level

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*1472641323*

**MATHEMATICS** **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.


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**1** Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

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**2** **(a)** Sketch the graph of $y = |2x - 3|$. [1]

**(b)** Solve the inequality $|2x - 3| < 3x + 2$. [3]

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**3** Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

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**4** Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, \mathrm{d}x$. [5]

**5** Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

**6** **(a)** By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places. [5]

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**(b)** Hence find the value of $x$ in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

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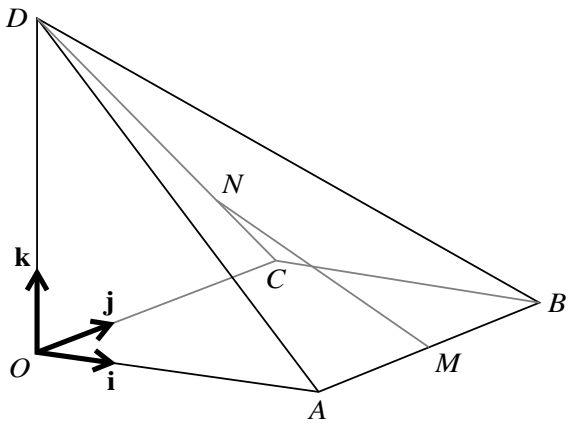
**7** The equation of a curve is $\ln(x + y) = x - 2y$.

**(a)** Show that $\frac{dy}{dx} = \frac{x + y - 1}{2(x + y) + 1}$. [4]

**(b)** Find the coordinates of the point on the curve where the tangent is parallel to the $x$-axis. [3]

**8**




In the diagram, $OABCD$ is a pyramid with vertex $D$. The horizontal base $OABC$ is a square of side 4 units. The edge $OD$ is vertical and $OD = 4$ units. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $OA$, $OC$ and $OD$ respectively.

The midpoint of $AB$ is $M$ and the point $N$ on $CD$ is such that $DN = 3NC$.

**(a)** Find a vector equation for the line through $M$ and $N$. [5]

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**(b)** Show that the length of the perpendicular from $O$ to $MN$ is $\frac{1}{3}\sqrt{82}$. [4]

**9** Let $f(x) = \dfrac{1}{(9-x)\sqrt{x}}$.

**(a)** Find the $x$-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

**(b)** Using the substitution $u = \sqrt{x}$, show that $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = \frac{1}{3}\ln 5$. [6]

**10** A large plantation of area $20\,\text{km}^2$ is becoming infected with a plant disease. At time $t$ years the area infected is $x\,\text{km}^2$ and the rate of increase of $x$ is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 1$.

**(a)** Show that $x$ and $t$ satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{19x}{20 - x}.$$ [2]

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**(b)** Solve the differential equation and show that when $t = 1$ the value of $x$ satisfies the equation $x = \mathrm{e}^{0.9+0.05x}$. [5]

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**(c)** Use an iterative formula based on the equation in part **(b)**, with an initial value of 2, to determine $x$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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**(d)** Calculate the value of $t$ at which the entire plantation becomes infected. [1]

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**11** The complex number $-\sqrt{3} + \mathrm{i}$ is denoted by $u$.

**(a)** Express $u$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of $r$ and $\theta$. [2]

**(b)** Hence show that $u^6$ is real and state its value. [2]

**(c)** **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers $z$ satisfying the inequalities $0 \leqslant \arg(z - u) \leqslant \frac{1}{4}\pi$ and $\mathrm{Re}\, z \leqslant 2$. [4]

**(ii)** Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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**Additional Page**

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