Pearson

Mark Scheme (Final)

January 2026

Pearson Edexcel International Advanced Level in Pure Mathematics

WFM01/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol √ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⎿ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
    a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1	$\mathrm{f}(x)=3\sqrt{x}-\frac{1}{x}-5$ [3, 3.8]	
(a)	$\mathrm{f}(3)=-0.1371809...$(awrt −0.14 or −0.13). [$\mathrm{f}(3.8)=0.5849187...$(awrt 0.58)] $\mathrm{f}(3.4)=0.2376090...$ (awrt 0.24 or 0.23 or just 0.2)	M1 A1
	$\mathrm{f}(3.2)=0.0540631...$ (awrt 0.05) $\mathrm{f}(3.1)=-0.0405355...$ (awrt −0.04)	**d**M1
	$\mathrm{f}(3.15)=0.007011487...$ (awrt 0.007) $\Rightarrow \alpha \approx 3.1$	A1
		(4)
(b)(i)	$\mathrm{f}'(x)=\frac{3}{2}x^{-\frac{1}{2}}+x^{-2}$ or e.g., $\frac{3}{2\sqrt{x}}+\frac{1}{x^2}$	M1 A1
		(2)
(b)(ii)	$\left\{x_0="3.1", \quad \alpha=\right\}$ $"3.1"-\frac{\mathrm{f}("3.1")}{\mathrm{f}'("3.1")}=3.1-\frac{3\sqrt{3.1}-\frac{1}{3.1}-5}{\frac{3}{2\sqrt{3.1}}+\frac{1}{3.1^2}}=...$	M1
	$\left\{=3.1-\frac{-0.0405...}{0.95600...}\right\}$ $=3.1424(01196)$	A1
		(2)
		Total 8

Notes

(a) NB Use of linear interpolation will score no marks for part (a).

M1: Attempts a value for f(3.4) **and** either f(3) or f(3.8) with one correct to 2dp (rounded or truncated)

A1: Correct value for f(3.4) to at least 1s.f. which may be rounded or truncated if full figure not seen.

dM1: **Requires previous M mark.** Proceeds to calculate values for f(3.2) and then f(3.1), or appropriate evaluations for their results to narrow to within a 0.1 range, with at least one correct evaluation.

Useful values: f(3.3) = 01464… f(3.5) = 0.3267…, f(3.6) = 0.4143… , f(3.7) = 0.5003…

A1: 3.1 following calculation of f(3.15) and all correct values. May use *x* or "root" = There is no requirement for "f(*x*) continuous" or "sign change" or any reasoning. Ignore if they proceed with further evaluations (e.g. f(3.125)).

SC: 3.1 following signs of values only (e.g., "f(3) = –ve...") is max 1010

(b)(i)

M1: For either $3x^{\frac{1}{2}} \to px^{-\frac{1}{2}}$ or $x^{-1} \to qx^{-2}$ oe

A1: Fully correct differentiation (any equivalent form)

(ii)

M1: Applies correct Newton-Raphson formula with their 3.1 or other appropriate value and their $\mathrm{f}'(x)$ and obtains a value. Accept for method seen a correct statement of the formula followed by a value (no need to see the substitution but the formula must be shown).

This mark can be implied by awrt 3.1424 as long as the answer to (a) was 3.1 and (b)(i) was correct, otherwise working must be seen.

If there was no answer to part (a) allow for a correct application with their chosen starting value, but method must be seen. Allow if they use an unrounded value from (a) etc.

A1: awrt 3.1424 following a correct derivative and 3.1 used and ignore further iterations (isw).

Note the exact root is 3.1426 [3.142593527]

Question Number	Scheme	Marks
2	$\mathrm{f}(z) = z^4 + \lambda z^3 + 14z^2 + 9\lambda z + 45$	
(a)	$\{\mathrm{f}(3i) =\}\quad (3\mathrm{i})^4 + \lambda(3\mathrm{i})^3 + 14(3\mathrm{i})^2 + 9\lambda(3\mathrm{i}) + 45$ $= 81\mathrm{i}^4 + 27\lambda\mathrm{i}^3 + 126\mathrm{i}^2 + 27\lambda\mathrm{i} + 45$ $= 81 - 27\lambda\mathrm{i} - 126 + 27\lambda\mathrm{i} + 45$ $= 0$*	M1 A1*
		(2)
(b)	$(z+3i)(z-3i) = z^2 + 9$	B1
	$\Rightarrow z^2 + \lambda z + 5$	M1 A1
	$b^2 - 4ac < 0 \Rightarrow \lambda^2 - 20 < 0$	M1
	$-2\sqrt{5} < \lambda < 2\sqrt{5}$	A1
		(5)
		Total 7

Notes

(a)

M1: Attempts f(3i) and obtains 2 of the first 4 terms on line 3 in the box for (a) (ie with the powers of i dealt with correctly in two of the first four terms)

A1*: Obtains 0 from correct work. (No need for a concluding comment.)

(b)

B1: Obtains $z^2 + 9$, may be implied (e.g. by use of product and sum of roots to directly deduce the other factor).

M1: Uses $z^2 \pm kz \pm 9$and f(z) to attempt a further quadratic with real coefficients, finding at least two terms. Ignore remainder if long division used. Correct first and last terms achieved can imply the mark. If the quadratic is never directly given but working to find the middle coefficient is shown then allow the mark.

A1: Correct factor. Condone work in x

M1: Uses $b^2 - 4ac < 0$ oe with their 3TQ. Condone $\leqslant 0$ used for the M mark.

A1: Correct exact range. Any equivalent notation e.g., $|\lambda| < \sqrt{20}$, or if using interval notaion accept $(-2\sqrt{5}, 2\sqrt{5})$ Condone being given as separate inequalities. Must not be in terms of a variable other than λ

SC 1 If they obtain an incorrect factor $z^2 - \lambda z + 5$ they could still obtain the correct final answer and we will allow B1M1A0M1A1 in such cases.

SC 2 Some are finding a quadratic factor $z^2 + bz + 5$ without fiding b in terms of λ, going to apply $b^2 - 20 < 0$ and only changing b to λ later **without any clear justification**. We will alllow SC max B1M1A0M1A1 for such cases.

Question Number	Scheme	Marks
3	$\mathbf{BAB}=\mathbf{I} \qquad \mathbf{B}=\begin{pmatrix}3 & k\\ -4 & -2k\end{pmatrix}$	
(a)	$\mathbf{B}^{-1}\mathbf{BABB}^{-1}=\mathbf{B}^{-1}\mathbf{IB}^{-1}\Rightarrow \mathbf{A}=\mathbf{B}^{-1}\mathbf{B}^{-1}$	B1*
		(1)
(b)	$\det\mathbf{B}\ (=-6k+4k)=-2k$	B1
	$\mathrm{Adj}(\mathbf{B})=\begin{pmatrix}-2k & -k\\ 4 & 3\end{pmatrix}$	B1
	$\mathbf{A}=\mathbf{B}^{-1}\mathbf{B}^{-1}=\left(-\frac{1}{2k}\right)\begin{pmatrix}-2k & -k\\ 4 & 3\end{pmatrix}\left(-\frac{1}{2k}\right)\begin{pmatrix}-2k & -k\\ 4 & 3\end{pmatrix}=\ldots$	M1
	$\frac{1}{4k^2}\begin{pmatrix}4k^2-4k & 2k^2-3k\\ -8k+12 & -4k+9\end{pmatrix}$ or $\begin{pmatrix}1-\frac{1}{k} & \frac{1}{2}-\frac{3}{4k}\\ \frac{3}{k^2}-\frac{2}{k} & \frac{9}{4k^2}-\frac{1}{k}\end{pmatrix}$ oe	A1
		(4)
		Total 5

Notes

(a)

B1*: Correct proof. Minimum as shown. May be done in stages but the multiplications must be on consistent sides in such cases. The $\mathbf{B}^{-1}\mathbf{B}$ and $\mathbf{BB}^{-1}$ must be seen or explained.

(b)

B1: $\det\mathbf{B}=-2k$ must be simplifed but may be implied.

B1: Correct Adj($\mathbf{B}$). Ignore any incorporated determinant.

M1: Attempts $\mathbf{B}^{-1}\mathbf{B}^{-1}$ with their $\mathbf{B}^{-1}=\frac{1}{\text{their}\det\mathbf{B}}\begin{pmatrix}\text{changed}\\ \text{matrix}\end{pmatrix}$ using row by column multiplication (evidenced in at least two entries). Note they may start with this line.

A1: Correct $\mathbf{A}$ from correct work. Any equivalent and isw any incorrect simplifications. Condone missing closing bracket.

SC if $\frac{1}{\det\mathbf{B}}$ in a stated formula becomes $-2k$ in error, treat this as a correct determinant but incorrect application of the determinant for B1B1M0A0.

Alt: May find $\mathbf{B}^2$ first and then find the inverse of this. Allow B1 for correct squared matrix $\begin{pmatrix}9-4k & 3k-2k^2\\ -12+8k & -4k+4k^2\end{pmatrix}$, B1ft for correct unsimpllfied determinant of their squared matrix $(9-4k)(4k^2-4k)-(12-8k)(2k^2-3k)$, M1 for correct attempt at inverse and A1 for correct answer $\frac{1}{(9-4k)(4k^2-4k)-(12-8k)(2k^2-3k)}\begin{pmatrix}-4k+4k^2 & -3k+2k^2\\ 12-8k & 9-4k\end{pmatrix}$. May see some unsimplified discriminants this method.

Question Number	Scheme	Marks
4(a)	Focus is (2, 0)	B1
	$y-0=\frac{8}{15}(x-2)$ or $y=\frac{8}{15}x+c\Rightarrow c=-\frac{8}{15}\times 2=...$	M1
	$y=\frac{8}{15}x-\frac{16}{15},\quad y^2=8x$ $\Rightarrow y^2=15y+16\Rightarrow y^2-15y-16=0$ or $\frac{64}{225}x^2-\frac{256}{225}x+\frac{256}{225}=8x\Rightarrow 8x^2-257x+32=0$	M1
	$(y+1)(y-16)=0\Rightarrow y=-1,\ 16$ or $(8x-1)(x-32)=0\Rightarrow x=\frac{1}{8},\ 32$	M1
	$\left(\frac{1}{8},-1\right)\qquad(32, 16)$	A1 A1
		(6)
(b)	$\frac{"32"+"\frac{1}{8}"}{2}+2=...$	M1
	$\frac{289}{16}$	A1
		(2)
		Total 8

Notes

(a)

B1: Correct focus seen or used.

M1: Forms the equation of *l* correctly with their focus.

M1: Eliminates *x* or *y* to obtain a 3TQ in *y* or *x*.

Alt: you may see $y=(\pm)\sqrt{8}\sqrt{x}\Rightarrow\sqrt{8}\sqrt{x}=\frac{8}{15}x-\frac{16}{15}\Rightarrow 8t^2\pm15\sqrt{8}t-16=0$ where $t=\sqrt{x}$, which is also fine for the M. Accept if only the positive root is considered, the correct points are recovered when t is squared.

M1: Solves their 3TQ (usual rules, and accept if just one correct root is given for their 3TQ if no working – you may need to check).

A1: One correct point, allow if not paired but at least one pair of coordinates are correct implicitly.

A1: Both correct points. Accept listed rather than as coordinates as long as they are correctly paired.

(b)

M1: Adds 2 to the mean of their positive and distinct *x* coordinates.

A1: $\frac{289}{16}$ or $18\frac{1}{16}$ or 18.0625 from correct *x* coordinates.

Question Number	Scheme	Marks
5(a)	Rotation...	M1
	...of $45°$ or $\frac{\pi}{4}$ [or $315°$ or $\frac{7\pi}{4}$ **clockwise**] about (0,0)/*O*/the origin	A1
		(2)
(b)	$\mathbf{B} = \begin{pmatrix} 2\sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{C} = \begin{pmatrix} 2\sqrt{2} & 0 \\ 0 & 2\sqrt{2} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}}+1 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}+1 \end{pmatrix} = \begin{pmatrix} 2+2\sqrt{2} & -2 \\ 2 & 2+2\sqrt{2} \end{pmatrix}$ **or** $\det \mathbf{B} = (2\sqrt{2})(2\sqrt{2}) - (0)(0) = 8$ **and** $\det(\mathbf{A}+I) = \left(\frac{1}{\sqrt{2}}+1\right)\left(\frac{1}{\sqrt{2}}+1\right) - \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) = 2+\sqrt{2}$	M1
	$\det \mathbf{C} = (2+2\sqrt{2})(2+2\sqrt{2}) - 2(-2)$ **or** $8(2+\sqrt{2})$	M1
	area of $T = \frac{12(1+\sqrt{2})}{16+8\sqrt{2}}$	M1
	$\frac{3\sqrt{2}}{4}$	A1
		(4)
		Total 7

Notes

(a)

M1: Identifies the transformation as a rotation.

A1: Fully correct description including angle and centre of rotation. Assume anticlockwise unless otherwise stated. Condone phrases such as "through the origin".

(b)

B1: Correct **B**

(c)

M1: Attempts **C** = **B**(**A**+**I**) – may use **BA** + **B** – **or** attempts both det **B** and det **A+I**. Must see evidence of the correct identity matrix. **See special case for use of C = (A + I)B**

M1: Attempts the determinant of their **C** or via the product of their det **B** and det **A+I**. May be implied by dividing by the two determinants in turn.

M1: Correct method for the area, $12(1+\sqrt{2}) \div |\text{their det}\mathbf{C}|$ - must be a positive value, but allow recovery if the make the answer positive in the final line.

A1: Any correct exact value in simplest form. Allow $\frac{3}{2\sqrt{2}}$

SC: Allow M0M1M1A1 if **C** = (**A** + **I**)**B** is clearly used, which gives the same **C** but from incorrect order of composition. But allow full marks bod in cases where it is unclear or approaches via finding the determinants first then multiplying them in any order.

Question Number	Scheme	Marks
6	$x^2+qx+r=0 \quad \frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2} \quad \alpha^2+\beta^2=5$	
(a)	$\alpha+\beta=-q \quad \alpha\beta=r$	B1
		(1)
(b)	$\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}\left\{=\frac{"-q"}{"r"}\right\}$	M1
	$(\alpha+\beta)^2-2\alpha\beta=5\ \{=("-q")^2-2"r"=5\}$	M1
	$\alpha+\beta=\frac{1}{2}\alpha\beta\Rightarrow\frac{1}{4}(\alpha\beta)^2-2\alpha\beta=5\Rightarrow(\alpha\beta)^2-8\alpha\beta-20=0$ **or** $-q=\frac{1}{2}r\Rightarrow\left(\frac{1}{2}r\right)^2-2r=5$ $\Rightarrow r^2-8r-20=0$	A1
		(3)
(c)	$(r+2)(r-10)=0$ or $(\alpha\beta+2)(\alpha\beta-10)=0$ $\Rightarrow r$ or $\alpha\beta=-2,\ 10$	M1
	($r=-2,\ 10\Rightarrow q=1,\ -5$ or $\alpha\beta=-2,\ 10\Rightarrow\alpha+\beta=-1,\ 5$)	
	$\Rightarrow\{f(x)=\}\quad x^2+x-2,\quad x^2-5x+10$	M1 A1
		(3)
		Total 7

Notes

(a)
B1: Both correct, may be indicated by (i) and (ii) if not explicitly labelled.

(b)
M1: States or uses $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$

M1: States or uses $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ or equivalent work

A1: Obtains $r^2-8r-20=0$

(c)
M1: Solves their 3TQ (usual rules). Allow for solving a 3TQ in q if they eliminate r instead of q in (b).

M1: Finds a value of q or $\alpha+\beta$ for one of their non-zero r or $\alpha\beta$ values (or vice versa) **and** substitutes correctly into x^2+qx+r or $x^2-(\alpha+\beta)x+\alpha\beta$

A1: Correct expressions (and no others).

Question Number	Scheme	Marks
7(a)	$z=\dfrac{3+\left(k\sqrt{3}\right)\text{i}}{2+\left(\sqrt{3}\right)\text{i}}\times\dfrac{2-\left(\sqrt{3}\right)\text{i}}{2-\left(\sqrt{3}\right)\text{i}}$	M1
	$=\dfrac{6+\left(2k\sqrt{3}\right)\text{i}-\left(3\sqrt{3}\right)\text{i}+3k}{7}$	dM1
	$=\dfrac{6+3k}{7}+\dfrac{\sqrt{3}(2k-3)}{7}\text{i}$	A1
		(3)
(b)		
	$\dfrac{\sqrt{3}(2k-3)}{6+3k}=\dfrac{\sqrt{3}}{3}$	M1
	$6k-9=6+3k\Rightarrow k=5$	M1A1
	$\Rightarrow z=3+\left(\sqrt{3}\right)\text{i}\ \Rightarrow \lvert z\rvert=\sqrt{3^2+\left(\sqrt{3}\right)^2}=\ldots$	M1
	$\sqrt{12}$ or $2\sqrt{3}$	A1
		(5)
Alt	$z=r\left(\cos\dfrac{\pi}{6}+\text{i}\sin\dfrac{\pi}{6}\right)=r\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}r\text{ i}$	M1
	$\Rightarrow 6+3k=\dfrac{7r}{2}\sqrt{3},\quad \sqrt{3}(2k-3)=\dfrac{7r}{2}$	M1A1
	$\Rightarrow k=\dfrac{1}{3}\left(\dfrac{7r}{2}\sqrt{3}-6\right)\Rightarrow r=\dfrac{2}{7}\left(\sqrt{3}\left(\dfrac{2}{3}\left(\dfrac{7r}{2}\sqrt{3}-6\right)-3\right)\right)\Rightarrow r=\ldots$	M1
	$r=2\sqrt{3}$	A1
		(5)
		Total 8

Notes

(a)

M1: A correct rationalising multiplier seen or implied

dM1: Obtains denominator of 7 (need not be fully simplified) **and** numerator of the correct form with the i^2 processed to give two real terms and two imaginary terms.

A1: Any correct expression in $a + b\text{i}$ form (with single i term) or expressions for a and b. Terms in i must be factored out.

(b)

M1: Uses their $\dfrac{b}{a}=\dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$ to form an equation in k. Condone if by error only one component of z contained a k. M0 if they leave i in the equation and never recover but allow if the later realise it should not be there and drop the i.

M1: Solves for k from an equation that had at least two k's in (both components of z must have been in terms of k). Must achieve a real value, but accept any real value following a suitable equation having been set up..

A1: Correct k Allow from an incorrect denominator in (a) if all else was correct.

M1: Correct method to find a value for $|z|$ using their real k. E.g. uses k to find z and applies modulus, or via use of the numerator and denominator of original z and taking ratio of moduli, or may find $|z|$ in terms of k first and then substitute for k. Must be correct application of the modulus in whichever method.

A1: Either correct exact value

Alt:

M1: Attempts to evaluate the polar form using arg $z = \frac{\pi}{6}$ with evaluated trig terms.

M1: Equates coefficients to produce two equations in r and k.

A1: Correct equations.

M1: Solves the equations to find the value for r (if k is found first they must proceed to find r to score this mark.

A1: Correct answer, either form as per main scheme.

Question Number	Scheme	Marks
8(a)	$\sum_{r=1}^{n}(2r-1)^2 = 4\sum_{r=1}^{n}r^2 - 4\sum_{r=1}^{n}r + \sum_{r=1}^{n}1$	M1
	$= 4\times\frac{n}{6}(n+1)(2n+1) - 4\times\frac{n}{2}(n+1) + n$	M1 A1
	$= n\left(\frac{2}{3}(n+1)(2n+1) - 2(n+1) + 1\right)$ or $\frac{n}{3}\left(4n^2+6n+2-6n-6+3\right)$	M1
	$= \frac{n}{3}\left(4n^2-1\right)$	A1
		(5)
(b)	smallest = 25^2 = 13th term or largest = 95^2 = 48th term	B1
	$\frac{48}{3}\left(4\times48^2-1\right) - \frac{12}{3}\left(4\times12^2-1\right)$	M1
	145 140	A1
		(3)
		Total 8

Notes

(a)

M1: Expands and splits summation to obtain two correct terms. (Limits need not be shown.)

M1: Uses any two of $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$ or $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ or $\sum_{r=1}^{n} 1 = n$

A1: Fully correct expression.

M1: Extracts a common factor of $\frac{n}{3}$ or n from **all three terms** in their expression (n must be a factor), allow one slip on one term. Do not accept attempts to solve the cubic via a calculator and "build up" the solution.

A1: Achieve the correct expression following the award of all the previous marks.

(b)

B1: Identifies either 25^2 as u_{13} or 95^2 as u_{48}

M1: Attempts f(a) – f(b) with their f and a = 48 or 47 & b = 12 or 13. Substitution must be seen

A1: 145 140 only

Question Number	Scheme	Marks
9(a)	$y=\frac{c^2}{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{c^2}{x^2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{-ct^{-2}}{c}$	B1
	$y-\frac{c}{t}=\text{"}-\frac{1}{t^2}\text{"}(x-ct)$ or $y=\text{"}-\frac{1}{t^2}\text{"}x+b\Rightarrow\frac{c}{t}=\text{"}-\frac{1}{t^2}\text{"}(ct)+b\Rightarrow b=...$	M1
	$t^2y-ct=-x+ct$ or $y=-\frac{1}{t^2}x+\frac{2c}{t}\Rightarrow t^2y=-x+2ct$ $\Rightarrow x+t^2y=2ct$ *	A1*
		(3)
(b)	$y-\frac{c}{t}=\text{"}t^2\text{"}(x-ct)$ or $y=\text{"}t^2\text{"}x+b\Rightarrow\frac{c}{t}=ct^3+b\Rightarrow b=\ldots\Rightarrow y=t^2x+\text{"}\frac{c}{t}-ct^3\text{"}$	B1ft
		(1)
(c)	At $X, y=0\Rightarrow x=2ct$; At $Y, x=0\Rightarrow y=\frac{2c}{t}$	B1
	At Z, $x=0\Rightarrow y=\frac{c}{t}-ct^3$	M1
	E.g. Area $\Delta XYZ=\frac{1}{2}\left(\text{"}\frac{2c}{t}\text{"}-\left(\text{"}\frac{c}{t}-ct^3\text{"}\right)\right)\times\text{"}2ct\text{"}$	M1
	$=ct\left(\frac{c}{t}+ct^3\right)=c^2\left(1+t^4\right)$*	A1*
		(4)
(d)	$c^2\left(1+3^4\right)=410\Rightarrow c=\sqrt{5}\Rightarrow P\left(3\sqrt{5},\frac{\sqrt{5}}{3}\right)$	B1
	$PQ=2\times\sqrt{\left(3\sqrt{5}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}$	M1
	$\frac{2\sqrt{410}}{3}$	A1
		(3)
		Total 11

Notes

(a)

B1: Any correct equation in $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Correct straight line method with their tangent gradient in terms of t and/or c

A1: Achieves given answer with an intermediate step and no errors. Accept $t^2y+x=2ct$

No calculus can score 011 max

(b)

B1ft: Correct equation with the negative reciprocal of their tangent gradient in terms of t and/or c. If $y = mx + b$ is used b must be correct for their t^2 and they must proceed to give the equation of the line.

(c)

B1: Correct x coordinate of X and correct y coordinate of Y. Allow if they mix up X and Y but the coordinates are correct.

M1: Uses $x = 0$ in their normal to find an expression for y at Z

M1: Correct full method for area of triangle XYZ with their coordinates. Condone for the M if they subtract y coordinates the wrong way and work with a "negative length" (but must treat the coordinate of Z correctly if combining separate triangles). Note that other methods are possible here, any correct full method is sufficient.

A1: Fully correct proof with intermediate step. Accept if the y coordinates were subtracted the wrong way but are later made positive.

Some alternative methods for the triangle area:

Using XY as base and PZ as height,

$$\text{Area } \Delta XYZ = \frac{1}{2}XY \times PZ = \frac{1}{2}\sqrt{(2ct)^2 + \left(\frac{2c}{t}\right)^2}\sqrt{(ct)^2 + \left(\frac{c}{t} - \frac{c}{t} + ct^3\right)^2}$$

$$= \frac{1}{2}2c\sqrt{t^2 + \frac{1}{t^2}}.c\sqrt{t^2 + t^6} = c^2\sqrt{\frac{t^4+1}{t^2}}\sqrt{t^2(t^4+1)} = c^2(t^4+1)$$

Shoelace approach,

$$\text{Area } \Delta XYZ = \frac{1}{2}\begin{vmatrix} 2ct & 0 & 0 \\ 0 & \frac{2c}{t} & \frac{c}{t} - ct^3 \end{vmatrix} = \frac{1}{2}\left(2ct \times \frac{2c}{t} - 2ct\left(\frac{c}{t} - ct^3\right)\right) = \frac{1}{2}(4c^2 - 2c^2 + 2ct^4) = c^2(1+t^4)$$

Separate triangles,

$$\text{Area } \Delta XYZ = \frac{1}{2}y_Y \times x_X - \frac{1}{2}y_Z \times x_X$$ follows main scheme once factorised.

Other combinations are possible.

(d)

B1: Correct coordinates of P

M1: Correct attempt at PQ for their P or any other full method. May find Q and then distance PQ directly for example. They may identify Q by symmetry or via a longer approach finding the equation of the line through O and P and then finding where this intersects the hyperbola again.

A1: Correct exact value

Question Number	Scheme	Marks
10	$u_1 = 8 \quad u_2 = 10 \quad u_{n+2} = u_{n+1} + 2u_n \quad n \in \mathbb{Z}^+ \quad u_n = 3\times 2^n - 2\times(-1)^n$	
	$n = 1 \Rightarrow u_1 = 3\times 2^1 - 2\times(-1)^1 = 8$ $n = 2 \Rightarrow u_2 = 3\times 2^2 - 2\times(-1)^2 = 10$	B1
	$u_k = 3\times 2^k - 2\times(-1)^k \qquad u_{k+1} = 3\times 2^{k+1} - 2\times(-1)^{k+1}$ $u_{k+2} = u_{k+1} + 2u_k = 3\times 2^{k+1} - 2\times(-1)^{k+1} + 2\left(3\times 2^k - 2\times(-1)^k\right)$	M1 A1
	$= 3\times 2^{k+1} - 2\times(-1)^{k+1} + 6\times 2^k - 4\times(-1)^k$ $= 3\times 2^{k+1} + 2\times(-1)^{k+2} + 3\times 2^{k+1} - 4\times(-1)^{k+2}$ $= 6\times 2^{k+1} - 2\times(-1)^{k+2}$	M1
	$= 3\times 2^{k+2} - 2\times(-1)^{k+2}$	A1
	True for $n = 1$ and $n = 2$, true for $n = k+2$ when assumed true for $n = k$ and $n = k+1$, true for all $n \in \mathbb{Z}^+$	A1
		(6)
		Total 6

Notes

B1: Uses general formula to obtain $u_1 = 8$ and $u_2 = 10$ with substitution seen, as a minimum accept $u_1 = 3\times 2 - 2\times -1 = 8$ and $u_2 = 3\times 4 - 2\times 1 = 10$. No need to state true for n = 1 and n = 2 here, but if it is stated it can count as part of the conclusion. Ignore attempts at further terms.

M1: Begins the inductive step by attempting the recurrence relation with expressions from the formula for u_k and u_{k+1}. No required to state the assumption. They may work from k to k – 2 rather than k + 2 to k, or even k + 1 to k −1 and accept if they are working in n instead of k. Allow slips as long at they are trying to substitute in the correct places.

A1: Correct expression for u_{k+2} or their appropriate term in terms of k (or n) only, condoning for this mark if they have e.g. (-1^k) instead of $(-1)^k$ as slip.

M1: Combines terms to $p\times 2^{\cdots} + q\times(-1)^{\cdots}$ where ... are functions of k (or n) by *matching the powers* before the target expression. The indices need not be the required ones at this stage e.g. reaching $6\times 2^{k+1} - 2\times(-1)^k$ is fine. Not dependent on the previous M but some attempt at the using the recurrence relation must have been made.

A1: Correct expression from fully correct work. Must have the correct powers, k + 2 or appropriate for their step. Bracketing around the (−1)'s must have been correct.

A1: Requires all previous marks. Correct conclusion given, which may be in stages. E.g. "True for n = 1 and n = 2" may have been given at the start, but the "if true for n = k +1 and n = k then shown true for n = k + 2" and "true for all n" must be given in the final conclusion. Must have both preceding cases including, though strong induction may be used (ie "if true for all $k < n$ then true for n"). The logic in the conclusion must be correct, so stating "true for n = k, k + 1 and k + 2" or similar without the "if.. then" connotation is A0.