



Mark Scheme (Final)

January 2026

International Advanced Level in Further
Pure Mathematics F1

WFM01/01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)

Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - \square or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
 - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$
 - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$
- Formula
 - Attempt to use the correct formula (with values for a , b and c).
- Completing the square
 - Solving $x^2 + bx + c = 0 : \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

- Differentiation
 - Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)
- Integration
 - Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

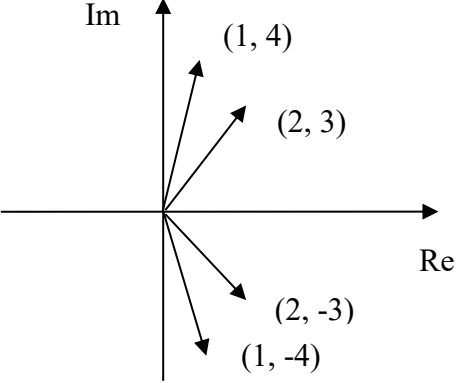
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1.	$r(r+3) = r^2 + 3r \quad \text{so} \quad \sum_{r=1}^n r(r+3) = \sum_{r=1}^n r^2 + 3 \sum_{r=1}^n r$ $= \frac{n}{6}(n+1)(2n+1) + 3 \frac{n}{2}(n+1)$	M1A1
	$= \frac{n}{6}(n+1)[2n+1+9]$	dM1
	$\left(= \frac{n}{6}(n+1)[2n+10] \right) = \frac{n}{3}(n+1)(n+5) \quad \text{or } a = 3 \text{ and } b = 5$	A1
		(4)
		Total 4

Notes

- M1:** Attempts to expand and attempts to substitute **at least one correct** standard formula into their resulting expression.
- A1:** Correct expression (or equivalent)
- dM1:** Attempt to factorise at least $n(n+1)$ **having attempted to substitute both correct** standard formulae.
May be done in stages
Dependent on the previous M mark.
- A1:** Correct completion with no errors and must be in terms of n
- Note:** Condone work in terms of r for all but the final mark.

Attempts that use induction would generally score no marks, but if there are any cases you are unsure about please send to review

Question Number	Scheme	Notes	Marks
2	$f(z) = z^4 - 6z^3 + 38z^2 - 94z + 221$ Condone consistent work in x throughout, but mixed variables which are not recovered loses final A mark		
(a)	$z = 2 - 3i \text{ (is also a root)}$		B1
	$(z - (2 + 3i))(z - (2 - 3i)) = \dots$ or $(2 + 3i) + (2 - 3i) = \dots(4) \text{ and } (2 + 3i)(2 - 3i) = \dots(13)$		M1
	$z^2 - 4z + 13$		A1
	$\frac{z^4 - 6z^3 + 38z^2 - 94z + 221}{z^2 - 4z + 13}$ $= az^2 + bz + c \text{ with } a \neq 0$ or $z^4 - 6z^3 + 38z^2 - 94z + 221 = (z^2 - 4z + 13)(az^2 + bz + c) \Rightarrow a = \dots, b = \dots, c = \dots$		M1
	$= z^2 - 2z + 17$		A1
	$z = \frac{2 \pm \sqrt{4 - 68}}{2} = \dots \text{ or } (z - 1)^2 + 16 = 0 \Rightarrow z = \dots$		M1
	$= 1 \pm 4i$		A1
	(7)		
(b)		Accept points, crosses, vectors or coordinates	B1B1ft
	(2)		
Total 9			

Notes

(a)

B1: $z = 2 - 3i$ seen identified as another root anywhere in the solution for (a) or (b). May only be seen at the end but must be written down somewhere in the solution. May only be on the argand diagram for (b) which is ok

M1: Correct attempt at quadratic factor by either finding the product of the two linear factors, or using the sum and product of the known roots.

A1: $z^2 - 4z + 13$ (sight of this quadratic would score first M1A1)

M1: Correct attempt at the other quadratic factor e.g. long division/ comparing coefficients/ synthetic Division/inspection using their quadratic factor to obtain a second quadratic factor. Must be some minimal working shown.

A1: $z^2 - 2z + 17$

M1: Attempt to solve their second $3TQ = 0$ (we must see a method here as strict calculator warning- QF or completing the square)

A1: Correct roots.

Note: attempts that solve the quartic on a calculator with no other working score 0

(b)

If no working is shown in (a) they can score both marks in part (b) for plotting their roots correctly.

B1: $2 \pm 3i$ correctly plotted and labelled. Must be reasonably symmetric about the real axis.

B1ft: Their other conjugate pair correctly plotted and labelled. Must be reasonably symmetric about the real axis.

Note: For part (b) the axes don't need to be labelled, or could be labelled as 'x' and 'y' and the points can be shown with dots/crosses/vector lines/coordinates from the origin but must be clearly indicated. Accept axes labelled numerically as long as the points are identified. Condone (2, 3i) etc.

Question Number	Scheme	Marks
3.(a)	$3\left(\frac{4}{t}\right) - 2(4t) = 10$	M1
	$8t^2 + 10t - 12 (= 0)$	A1
	$(8t - 6)(t + 2) = 0 \Rightarrow t = \dots$ or $(4t - 3)(2t + 4) = 0 \Rightarrow t = \dots$ or $2(4t - 3)(t + 2) = 0 \Rightarrow t = \dots$ or $t = \frac{-10 \pm \sqrt{100 + 384}}{16} = \dots \left(t = \frac{3}{4}, -2 \right)$ or $4\left(t + \frac{5}{8}\right)^2 - \frac{25}{16} - 6 = 0 \Rightarrow t = \dots \left(t = \frac{3}{4}, -2 \right)$	dM1
	Points are $\left(3, \frac{16}{3}\right)$ and $(-8, -2)$	ddM1 A1
		(5)
(a) Alternative		
	$y = \frac{10 + 2x}{3}$ or $x = \frac{3y - 10}{2}$ and $xy = 16$ $\Rightarrow x^2 + 5x - 24 (= 0)$ or $3y^2 - 10y - 32 (= 0)$	M1A1
$x^2 + 5x - 24 = 0 \Rightarrow (x + 8)(x - 3) \Rightarrow x = \dots$ $x^2 + 5x - 24 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{25 + 96}}{2} \Rightarrow x = \dots$ $x^2 + 5x - 24 = 0 \Rightarrow \left(x + \frac{5}{2}\right)^2 - \frac{121}{4} = 0 \Rightarrow x = \dots$	$3y^2 - 10y - 32 = 0 \Rightarrow (3y - 16)(y + 2) = 0 \Rightarrow y = \dots$ $3y^2 - 10y - 32 = 0 \Rightarrow y = \frac{10 \pm \sqrt{100 + 384}}{6} \Rightarrow y = \dots$ $3y^2 - 10y - 32 = 0 \Rightarrow 3\left(y - \frac{5}{3}\right)^2 - \frac{121}{3} = 0 \Rightarrow y = \dots$	dM1
Points are $\left(3, \frac{16}{3}\right)$ and $(-8, -2)$		ddM1 A1
(b)	$\left(\frac{3 - 8}{2}, \frac{\frac{16}{3} - 2}{2}\right) \rightarrow \left(-\frac{5}{2}, \frac{5}{3}\right)$	M1 A1
		(2)
(7 marks)		
Notes		
<p>M1: Substitutes $x = 4t$ and $y = \frac{4}{t}$ into the printed equation to obtain an equation in t only.</p> <p>A1: Obtains a correct three term quadratic equation.</p> <p>dM1: Correct method (e.g. factorising, quadratic formula or completing the square) of solving a 3TQ to find $t = \dots$</p> <p>Dependent on the previous M mark.</p> <p>ddM1: Correct substitution for at least one of their values for t into the given parametric equations and obtains two sets of corresponding values for x and y. Could be implied by correct points provided dM1 scored.</p> <p>Dependent on both previous M marks.</p> <p>A1: Both correct points found. No need to label them as A and B if given in the order shown in the MS but</p>		

if the order is reversed, they must be labelled. So $(-8, -2)$ and $\left(3, \frac{16}{3}\right)$, scores A0 unless labelled correctly.

Must also be written as coordinates.

Alternative:

M1: Solves simultaneously $3y - 2x = 10$ with $xy = k$, $k \neq 0$ to form an equation in x or y only.

A1: Obtains a correct three term quadratic equation.

dM1: Correct method (e.g. factorising, completing the square or applying the quadratic formula) of solving a 3TQ to find $x = \dots$ or $y = \dots$

Dependent on the previous M mark.

ddM1: Correct substitution for at least one of **their** values **and** obtains *two sets* of corresponding values for x and y . Could be implied by correct points provided dM1 scored.

Dependent on both previous M marks.

A1: Both correct points found. No need to label them as A and B if given in the order shown in the MS but if

the order is reversed, they must be labelled. So $(-8, -2)$ and $\left(3, \frac{16}{3}\right)$, scores A0 unless labelled correctly.

Must be written as coordinates.

SC: If they show working to arrive at a correct quadratic in x , y , or t and then use a calculator to solve their quadratic, arriving at two correct points then award SC M1A1dM0ddM0A1

(b)

M1: Correct method for the midpoint of their points. Must be adding and dividing by 2

A1: Correct answer. Cannot leave unsimplified to score this mark, so they must combine the numerical values but accept e.g. $\left(-2.5, \frac{5}{3}\right)$ or $(-2.5, 1.66\cdots)$ or $\left(-2.5, 1.\dot{6}\right)$

Question Number	Scheme	Notes	Marks
4.	$(z - 2i)(z^* - 2i) = 21 - 12i$		
	$z^* = x - iy$		B1
	$(x + iy - 2i)(x - iy - 2i) = \dots$ $= x^2 - x(y + 2)i + x(y - 2)i + y^2 - 4$ $= x^2 + y^2 - 4 - 4xi$		M1
	$x^2 + y^2 - 4 = 21$ and $4x = 12$		M1
	$4x = 12 \Rightarrow x = \dots$		M1
	$x = 3$		A1
	$y = \pm 4$		A1
Total 6			
Alternative 1			
	$(z - 2i)(z^* - 2i) = zz^* - 2i(z + z^*) - 4$		M1
	$= (x + iy)(x - iy) - 2i(x + iy + x - iy) - 4$		B1
	$= x^2 + y^2 - 4xi - 4$ $x^2 + y^2 - 4 = 21$ and $4x = 12$		M1
	$4x = 12 \Rightarrow x = \dots$		M1
	$x = 3$		A1
	$y = \pm 4$		A1
Alternative 2			
	$(z - 2i)(z^* - 2i) = zz^* - 2i(z + z^*) - 4$		M1
	$zz^* - 2i(z + z^*) - 4 = 21 - 12i$		
	$zz^* - 4 = 21, \quad 2(z + z^*) = 12$		M1
	$z^2 - 6z + 25 = 0 \left(\text{or } (z^*)^2 - 6z^* + 25 = 0 \right)$		B1
	$z^2 - 6z + 25 = 0 \left(\text{or } (z^*)^2 - 6z^* + 25 = 0 \right)$ $\Rightarrow z = \frac{6 \pm \sqrt{36 - 100}}{2} \Rightarrow z = \dots \text{ or } (z - 3)^2 + 16 = 0 \Rightarrow z = \dots$		M1
	$x = 3$ $y = \pm 4$		A1 A1

Alternative 3		
	$(z - 2i)(z^* - 2i) = zz^* - 2i(z + z^*) - 4$ $zz^* - 2i(z + z^*) - 4 = 21 - 12i$	M1
	$zz^* - 4 = 21, \quad 2(z + z^*) = 12$	M1
	$x^2 + y^2 - 4 = 21$ <p>and $4x = 12$</p>	B1
	$4x = 12 \Rightarrow x = \dots$	M1
	$x = 3$	A1
	$y = \pm 4$	A1

Notes

(a)

B1: Correct complex conjugate.

M1: Substitutes for z and their z^* and attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real **and** imaginary parts.

M1: Solves to obtain at least one value of x or y . There must be no clear use of a calculator.

A1: $x = 3$

A1: $y = \pm 4$ (accept e.g. $z = 3 \pm 4i$)

Alternative 1:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

B1: $z^* = x - iy$ (may be implied).

M1: Compares real **and** imaginary parts.

M1: Solves to obtain at least one value of x or y . There must be no clear use of a calculator.

A1: $x = 3$

A1: $y = \pm 4$ (accept e.g. $z = 3 \pm 4i$)

Alternative 2:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real and imaginary parts.

B1: Correct quadratic.

M1: Solves (must see a method as strict calculator warning-see general rules on solving a 3TQ) to obtain at least one value of z or z^*

A1: $x = 3$

A1: $y = \pm 4$ (accept e.g. $z = 3 \pm 4i$)

Alternative 3:

M1: Attempts to expand. No need to check individual terms, but look for any attempt to multiply out the brackets

M1: Compares real **and** imaginary parts.

B1: Use of $z^* = x - iy$ Implied by use of $zz^* = x^2 + y^2$ or $2(z + z^*) = 4x$

M1: Solves to obtain at least one value of x or y . There must be no clear use of a calculator.

A1: $x = 3$

A1: $y = \pm 4$ (accept e.g. $z = 3 \pm 4i$)

Question Number	Scheme	Notes	Marks
5.	$x^2 - 2x + 3 = 0$		
(a)(i)	$\alpha + \beta = 2, \alpha\beta = 3$		B1
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 6 = -2$		M1A1
(iii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 8 - 3 \times 3 \times 2 = -10$		M1A1
			(5)
(b)(i)	$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + 2(\alpha\beta)^2 + \beta^4 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$		B1*
ALT 1	$\alpha^4 + \beta^4 = \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$		B1*
ALT 2	$(\alpha + \beta)^4 = \alpha^4 + 4\alpha^3\beta + 6\alpha^2\beta^2 + 4\alpha\beta^3 + \beta^4$ $\Rightarrow \alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2$ $= (\alpha + \beta)^4 - 4\alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta] - 6\alpha^2\beta^2$ $= (\alpha + \beta)^4 - 4\alpha\beta(\alpha + \beta)^2 + 2\alpha^2\beta^2$ $= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 4\alpha^2\beta^2 + 2\alpha^2\beta^2$ $= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$		B1*
(ii)	$\text{Sum} = \alpha^3 + \beta^3 - (\alpha + \beta) = -10 - 2 = -12$		B1
	$\text{product} = (\alpha^3 - \beta)(\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta = 27 - (4 - 18) + 3 = 44$		M1A1
	$\Rightarrow x^2 + 12x + 44 = 0$		M1A1
			(6)
	Total 11		

Notes

- (a)(i)**
B1: Both $\alpha + \beta = 2, \alpha\beta = 3$
- (ii)**
M1: Use of a correct identity for $\alpha^2 + \beta^2$ (May be implied by their work) e.g. $(\text{their sum})^2 - 2 \times \text{their product}$
A1: -2 from a correct solution only.
- (iii)**
M1: Use of a correct identity for $\alpha^3 + \beta^3$ (May be implied by their work) e.g. $(\text{their sum})^3 - 3 \times \text{their product} \times \text{their sum}$
A1: -10 from a correct solution only.

(b)(i)

B1*: Correct algebraic proof starting with RHS. Must see full correct expansion of $(\alpha^2 + \beta^2)^2$ before the printed answer and no incorrect work.

ALT 1 Starts with LHS but must see $\alpha^4 + \beta^4 = \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2$ before the printed answer and no incorrect work

ALT 2 Starts with the expansion of $(\alpha + \beta)^4$ then rearranges to obtain $\alpha^4 + \beta^4$, factorises and progresses to the correct printed answer with no errors

In all attempts accept $2\alpha^2\beta^2$ as an equivalent to $2(\alpha\beta)^2$

(ii)

NOTE: there are no ft marks available in part (b)

B1: Correct working without using explicit roots leading to a correct sum.

M1: Attempts to expand. Must have at least three correct terms to score this mark.

A1: Correct product.

M1: Applies $x^2 - (\text{sum})x + \text{product}$ (may have to check for their sum and product and $= 0$ not required for this mark)

A1: $x^2 + 12x + 44 = 0$ (must have the $= 0$ for this mark)

If p, q, r , stated but quadratic not stated then M1A0

NOTE: any attempts which try to solve the equation to find the roots will generally score zero

Question Number	Scheme	Marks
6.(a)	$\det \mathbf{A} = 2p(5q) - 3p(3q)$	B1
	$\begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix} \rightarrow \begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$	M1
	$\mathbf{A}^{-1} = \frac{1}{pq} \begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$ or $\mathbf{A}^{-1} = \begin{pmatrix} \frac{5}{p} & -\frac{3}{p} \\ -\frac{3}{q} & \frac{2}{q} \end{pmatrix}$	A1
		(3)
(b)	$\mathbf{X} = \mathbf{BA}^{-1} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix} \frac{1}{pq} \begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix} = \dots$ Or $\mathbf{X} = \mathbf{BA}^{-1} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix} \begin{pmatrix} \frac{5}{p} & -\frac{3}{p} \\ -\frac{3}{q} & \frac{2}{q} \end{pmatrix} = \dots$	M1
	$= \frac{1}{pq} \begin{pmatrix} 2pq & -pq \\ -3pq & 4pq \\ pq & pq \end{pmatrix}$ or equivalent e.g. $\begin{pmatrix} 5-3 & -3+2 \\ 30-33 & -18+22 \\ 25-24 & -15+16 \end{pmatrix}$ 4 correct elements from those above – simplified or unsimplified. The elements may not be in a matrix, or a matrix of the correct dimension.	A1
	$= \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	dM1A1
		(4)

(b) Alternative

	$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$ So $2a+3b=1$ and $3a+5b=1$; $a=2, b=-1$ or $2c+3d=6$ and $3c+5d=11$; $c=-3, d=4$ or $2e+3f=5$ and $3e+5f=8$; $e=1, f=1$	M1A1
	$a=2, b=-1 \quad c=-3, d=4 \quad e=1, f=1$	dM1
	$= \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	A1

Notes

- (a)
- B1:** Correct determinant. Simplified or unsimplified.
- M1:** Interchanges elements in major diagonal and changes signs in other diagonal.
- A1:** Completely correct answer in any form, simplified or unsimplified. If the matrix is correct and they then go on to simplify incorrectly then ISW
- (b)
- M1:** Attempts \mathbf{BA}^{-1} and finds at least one element using their inverse from (a). The matrix dimension may be incorrect or not shown but one correct element for their \mathbf{BA}^{-1} after an attempt at \mathbf{BA}^{-1} is sufficient. The element may be unsimplified.
- A1:** At least 4 elements correct. The elements may not yet be simplified.
- dM1:** Finds a 3×2 matrix of 6 elements.
- A1:** Correct simplified matrix.

Alternative:

- M1:** Attempts simultaneous equations from $\mathbf{XA} = \mathbf{B}$ and solves for at least one constant.
- A1:** At least 4 correct values.
- dM1:** Finds all 6 elements for the 3×2 matrix.
- A1:** Correct simplified matrix.

Question Number	Scheme	Marks
7.(a)	$f(2) = 2.9497\dots$ or $f(2.1) = -6.0105\dots$ and $f(2.05) = -1.3160\dots$	M1A1
	$f(2.025) = 0.86846\dots$	dM1
	Interval is (2.025, 2.05) oe	A1
		(4)
(b)	$f'(x) = -\frac{7}{2}x^{-\frac{3}{2}} - 5x^4$	M1A1
		(2)
(c)	$x_1 = 2 - \frac{f(2)}{f'(2)} = \left(2 - \frac{2.9497}{-81.237}\right) = 2.04$	M1 A1
		(2)
		Total 8

Notes

(a)

M1: Evaluates (must see numerical values) *at least one* of $f(2)$ or $f(2.1)$ **and evaluates** $f(2.05)$

A1: $f(2)$ **or** $f(2.1)$ correct awrt (or truncated) to 1 sf **and** $f(2.05)$ correct awrt (or truncated) to 1 sf

dM1: Evaluates $f(2.025)$

Dependent on the previous M mark

A1: Needs accuracy for $f(2.025)$ correct to 1 figure truncated or rounded and correct interval. Accept soft or hard brackets here or e.g. $\text{root}/\alpha/x \in (2.025, 2.05)$ or $\text{root}/\alpha/x \in [2.025, 2.05]$ or 'root belongs to (2.025, 2.05)' etc

Note:

Condone $\text{root}/\alpha/x$ is between 2.05 and 2.025 but $\text{root}/\alpha/x \in (2.05, 2.025)$ or $\text{root}/\alpha/x \in [2.05, 2.025]$ is **A0**

(b)

M1: For attempt at differentiation $\frac{7}{\sqrt{x}} \rightarrow ax^{-\frac{3}{2}}$ or $-x^5 \rightarrow bx^4$

A1: Correct derivative.

(c)

M1: For correct application of Newton - Raphson (using the correct formula) with $f(2)$ and $f'(2)$ which could be implied with their values.

$$x_0 = 2 \text{ followed by } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \dots \text{ is M1}$$

$$x = 2 - \frac{\text{their } f(2)}{\dots} = \dots \text{ is M1}$$

A1: cso and needs to be given to 2dp. This mark can only be scored if full marks in (b) are scored (Newton Raphson used more than once – isw)

Question Number	Scheme	Marks
8.(a)	$y^2 = 4ax$, at $P(ap^2, 2ap)$	
	$y = 2\sqrt{ax} \Rightarrow \frac{dy}{dx} = \sqrt{a}x^{-\frac{1}{2}}$ or (implicitly) $2y\frac{dy}{dx} = 4a$ or (chain rule) $\frac{dy}{dx} = 2a \times \frac{1}{2ap}$	M1
	When $x = ap^2$, $m_T = \frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{ap^2}} = \frac{\sqrt{a}}{\sqrt{a}p} = \frac{1}{p}$ or $m_T = \frac{dy}{dx} = \frac{4a}{2(2ap)} = \frac{1}{p}$	A1
	$y - 2ap = \frac{1}{p}(x - ap^2)$ Or $y = mx + c \Rightarrow 2ap = \frac{1}{p}(ap^2) + c \Rightarrow c = \dots(ap)$	M1
	$py - 2ap^2 = x - ap^2 \Rightarrow py = x + ap^2$ Or $\Rightarrow y = \frac{1}{p}x + ap \Rightarrow py = x + ap^2$	A1 *
		(4)
(b)	$B(-a, \frac{5}{6}a) \Rightarrow p(\frac{5}{6}a) = -a + ap^2$ or $p(\frac{5}{6}a) = x + ap^2$ or $py = -a + ap^2$	M1
	$\Rightarrow 5ap = -6a + 6ap^2 (\Rightarrow 6p^2 - 5p - 6 = 0)$	A1
	$6p^2 - 5p - 6 = 0 \Rightarrow p = \dots$ leading to $p = \dots$	M1
	$\Rightarrow \left\{ p = -\frac{2}{3} \text{ (reject)} \right\} p = \frac{3}{2}$	A1
	$0 = x + a\left(\frac{3}{2}\right)^2$	M1
	$x = -\frac{9a}{4}$	A1
	(6)	
(c)	When $p = \frac{3}{2}$, $y_P = 2a\left(\frac{3}{2}\right) = 3a$ Area(OPD) = $\frac{1}{2}\left(\frac{9a}{4}\right)(3a) = \frac{27a^2}{8}$ Or Area(OPD) = $\frac{1}{2} \begin{vmatrix} 0 & \frac{9a}{4} & -\frac{9a}{4} & 0 \\ 0 & 3a & 0 & 0 \end{vmatrix} = \frac{1}{2} \times 3a \times \frac{9a}{4}$	M1A1
		(2)
		Total 12

Notes

(a)

M1: $\frac{dy}{dx} = \pm kx^{-\frac{1}{2}}$ or $ky \frac{dy}{dx} = c$ or $\frac{\text{their } \frac{dy}{dx}}{\text{their } \frac{dx}{dt}}$

A1: $\frac{dy}{dx} = \frac{1}{p}$

M1: Applies $y - 2ap = (\text{their } m_T)(x - ap^2)$ Where (their m_T) is a function of p and has come from calculus or applies $y = mx + c$, substitutes $P(ap^2, 2ap)$ and makes progress to find the c

A1*: Obtains the printed result with at least one line of intermediate working and no incorrect work seen.

E.g. Cannot go from $y - 2ap = \frac{1}{p}(x - ap^2)$ directly to the printed answer

(b)

M1: Substitutes $x = -a$ or $y = \frac{5}{6}a$ or both into **their** equation (or their rearranged equation) or the printed answer

A1: Correct equation **in any form** with $x = -a$ and $y = \frac{5}{6}a$

M1: Attempts to solve their 3TQ in p having substituted $x = -a$ and $y = \frac{5}{6}a$ (Note that this question has the less strict calculator warning, so that calculators may be used for general processing, but not to formulate a complete solution to the question, so we can allow a calculator to be used here to solve the 3TQ)

A1: $p = \frac{3}{2}$ (Can just be stated from a correct quadratic)

M1: Substitutes their " $p = \frac{3}{2}$ " and $y = 0$ into **their** equation (or their rearranged equation) or the printed answer

A1: $x = -\frac{9a}{4}$

Note: If two values of p are substituted and two values for x are found, and the incorrect value of x is rejected then full marks can be scored in (b). If they don't reject the incorrect value, then the last two A marks are lost.

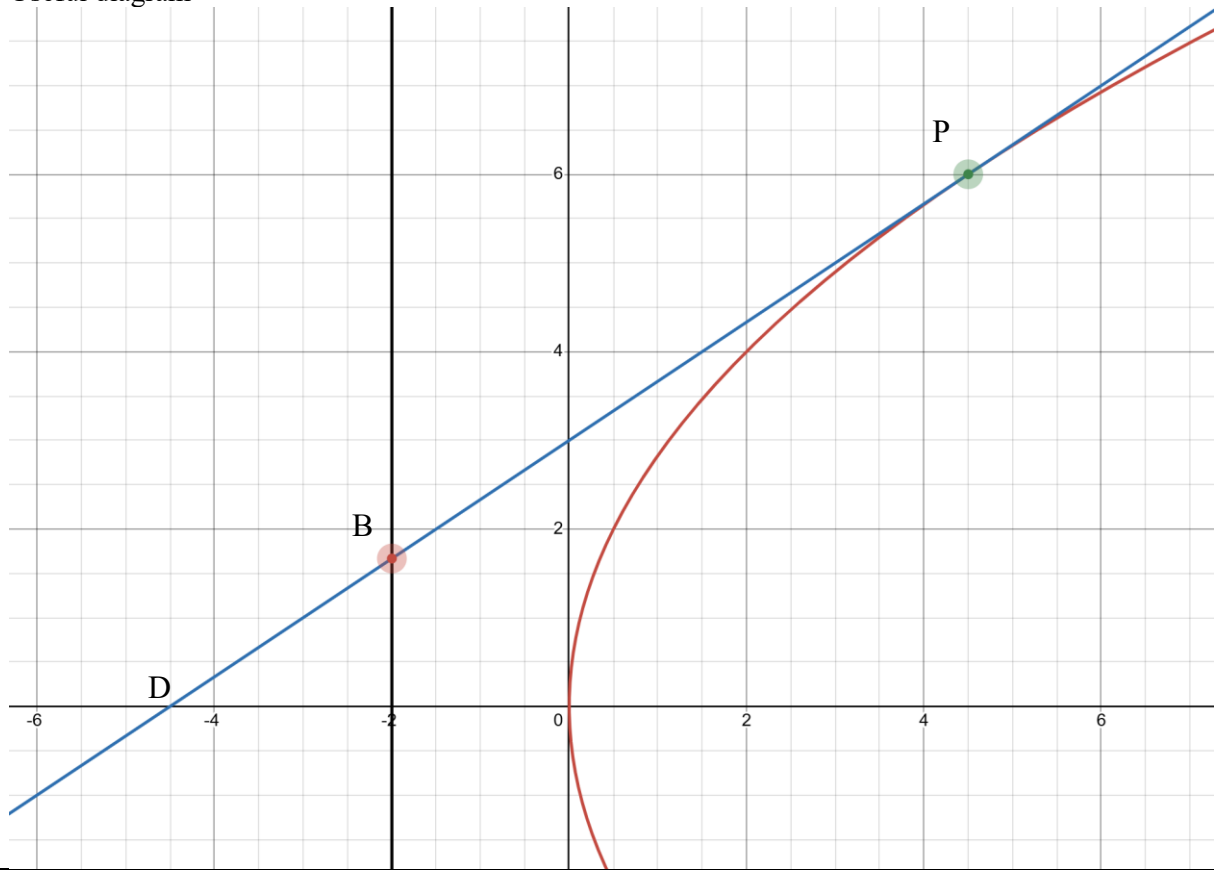
(c)

M1: Applies $\frac{1}{2}(\text{their } |OD|)(\text{their } y_p \text{ where their } y_p \neq \frac{5}{6}a)$ Allow if $OD < 0$ and a correct method in terms of a

and p e.g. $\frac{1}{2} \times -ap^2 \times 2ap$

A1: $\frac{27a^2}{8}$

Useful diagram



Question Number	Scheme	Marks
9.(a)	$\mathbf{A} = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}$, Area(R) = 10, $\mathbf{B} = \mathbf{A}^4$	
	$\det(\mathbf{A}) = 6(1) - 4(1)$	M1
	$\det(\mathbf{A}) \neq 0$ (so \mathbf{A} is non-singular)	A1
		(2)
(b)	Area(S) = 2(10) = 20	M1A1
		(2)
(c)	Area(T) = 2 ⁴ (10); = 160	M1A1
		(2)
		Total 6

Notes

(a)
M1: Correct attempt at the determinant. The working for the determinant must be seen in part (a)
A1: $\det(\mathbf{A}) = 2$ or $(6 - 4)$ **and** some reference to not being zero e.g. $2 \neq 0$ is sufficient
 Stating e.g. $\det(\mathbf{A}) > 0$ etc is insufficient for this mark

(b)
M1: their $\det(\mathbf{A}) \times 10$ ($10 \div$ their $\det(\mathbf{A})$ is M0)
A1: 20

(c)
M1: (their $\det(\mathbf{A})$)⁴ $\times 10$ ($10 \div$ (their $\det(\mathbf{A})$)⁴ is M0)
A1: 160

Accept use of \mathbf{A}^2

e.g. $\mathbf{A}^2 = \begin{pmatrix} 40 & 28 \\ 7 & 5 \end{pmatrix} \Rightarrow |\mathbf{A}^2| = 4 \Rightarrow \text{Area}(T) = 4^2(10); = 160$ Is acceptable

M1: (their $\det(\mathbf{A}^2)$)² $\times 10$
A1: 160

BUT there must be no attempt to evaluate \mathbf{A}^4 to give $\det \mathbf{A}^4 = 16$

If they think $\det(\mathbf{A}) = \frac{1}{\det(\mathbf{A})}$ then no marks in (a) but allow M's in (b) and (c).

So e.g. if they have $\det(\mathbf{A}) = \frac{1}{6-4} = \frac{1}{2}$ in (a) then M0A0 in (a)

But if they then have $10 \times \frac{1}{2}$ in (b) this can score M1A0 in (b)

If they have $10 \times \left(\frac{1}{2}\right)^4$ in (c) then this can score M1A0 in (c)

$$\text{NB } \mathbf{A}^4 = \begin{pmatrix} 6 & 4 \\ 1 & 1 \end{pmatrix}^4 = \begin{pmatrix} 1796 & 1260 \\ 315 & 221 \end{pmatrix}$$

Question Number	Scheme	Marks
	Condone working in other letters such as n etc	
10	If $n = 1$, $\sum_{r=1}^n r^2(2r-1) = 1$ and $\frac{1}{6}n(n+1)(3n^2+n-1) = 1$, (LHS=RHS) (so true for $n = 1$).	B1
	$\sum_{r=1}^{k+1} r^2(2r-1) = \frac{1}{6}k(k+1)(3k^2+k-1) + (k+1)^2(2(k+1)-1)$	M1
	$= \frac{1}{6}(k+1)(3k^3+13k^2+17k+6) \text{ or } = \frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$	A1
	$= \frac{1}{6}(k+1)(k+2)(3k^2+7k+3) = \frac{1}{6}(k+1)((k+1)+1)(3(k+1)^2+(k+1)-1)$	A1
	True for $n = k + 1$ if true for $n = k$, and as true for $n = 1$ true (by induction) for all n. (true for $n = 1$ may be stated at the start and not at the end which is fine)	A1cso
		(5)
		(Total 5)

Notes

The conclusion may be given as a narrative throughout the question, or it may be stated at the end, but the statements in bold are required somewhere in the solution

In all working condone the occasional missing bracket etc if recovered

- B1:** Shows true for $n = 1$. Don't need to see the substitution explicitly, but need to see that both sides have been evaluated when $n = 1$ and arrive at 1 for both sides- getting 1 for both sides is sufficient. Don't need to explicitly say LHS = RHS but must show clearly that both sides are equivalent by evidence of evaluating both.
- M1:** Adds the $(k + 1)^{\text{th}}$ term to the sum of the first k terms.
- A1:** Obtains $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ **or** $= \frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$ from correct work (i.e. extracts 'first' linear factor or expresses as the product of two quadratics, and allow one term to be unsimplified after the $1/6$ has been extracted).
- A1:** Achieves this result with no errors and $3k^2+7k+3$ seen (i.e. extracts 'second' linear factor and progresses to showing the correct form required with $n = k + 1$). Condone ' $k + 2$ ' in place of ' $(k + 1) + 1$ '
Allow work that shows equivalence between
e.g. $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ and $\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$
- A1:** Full conclusion and all previous marks scored. As long as the conclusion is conveyed throughout the solution then this mark can be scored if all previous marks are scored.

For the final mark

True for $n = k \Rightarrow$ true for $n = k + 1$, and as/if true for $n = 1$, true for all $n \in \mathbb{N}$ (or $n \in \mathbb{Z}^+$ or integers)

Or

True for $n = k + 1$ when true for $n = k$ and as/if true for $n = 1$, true for all $n \in \mathbb{N}$ (or $n \in \mathbb{Z}^+$ or integers)

Condone 'true for all $n \in \mathbb{Z}$ ' but do not allow 'true for $n \in \mathbb{R}$ ' or just 'true'

Accept 'correct/it works for' in place of 'true'

Note that some candidates may resort to 'assume true for $n = k + 1$ and show true for $n = k + 2$ etc which marks the same way as above

$$\mathbf{M1:} \sum_{r=1}^{k+2} r^2(2r-1) = \frac{1}{6}(k+1)(k+2)(3(k+1)^2 + (k+1) - 1) + (k+2)^2(2(k+2) - 1)$$

$$\mathbf{A1:} = \frac{1}{6}(k+2)(3k^3 + 22k^2 + 52k + 39)$$

$$\mathbf{A1:} = \frac{1}{6}(k+2)(k+3)(3k^2 + 13k + 13) = \frac{1}{6}(k+2)(k+3)(3(k+2)^2 + (k+2) - 1)$$

A1: Full conclusion and all previous marks scored.

If there are any attempts you are not sure about, please send to review

Any attempts which do not use proof by induction score 0 marks