Pearson

Mark Scheme (Results)

January 2026

International Advanced Level in Further Pure

Mathematics WFM02/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and completing an attempt to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   1. ft – follow through
   2. cao – correct answer only
   3. cso - correct solution only. There must be no clear errors in this part of the question to obtain this mark
   4. isw – ignore subsequent working
   5. awrt – answers which round to
   6. SC: special case
   7. oe – or equivalent
   8. dM – dependent method mark
   9. dp decimal places
   10. sf significant figures
   11. * The answer is given on the paper – apply cso
   12. d... Dependent mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1ft to indicate that previous wrong working is to be followed through.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out provided it is not cursory.
    - either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer unless the mark scheme indicates otherwise.

8. Mark question parts separately unless the mark scheme indicates otherwise.

<u>Method mark for solving a 3 term quadratic:</u>

(Note: There may be other schemes where the below does not apply)

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|$, leading to x = ...

2. Formula

Correct attempt to use the correct formula leading to x = ... (unsimplified). There is no credit just for quoting the correct formula if it is incorrectly used or if only a simplified calculator answer is seen.

3. Completing the square (where *a* = 1; if *a* ≠ 1 must divide by *a* first but allow equivalent work e.g., if a is a perfect square)

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q \pm c=0, \quad q\neq 0$, leading to x = ...

Question Number	Scheme	Notes	Marks
1(a)	$\frac{1}{(3n-1)(3n+5)} \equiv \frac{A}{3n-1}+\frac{B}{3n+5}$ $\Rightarrow A = ...\ \left\{\frac{1}{6}\right\}, B = ...\ \left\{-\frac{1}{6}\right\}$	Uses correct partial fractions and finds values for both *A* and *B*	**M1**
	$\equiv \frac{1}{6(3n-1)}-\frac{1}{6(3n+5)}$	Correct **partial fractions** in any form e.g., $\frac{1}{18n-6}-\frac{1}{18n+30}$, $\frac{\frac{1}{6}}{3n-1}-\frac{\frac{1}{6}}{3n+5}$, $\frac{1}{6}\left(\frac{1}{3n-1}+\frac{-1}{3n+5}\right)$	**A1**
			(2)
(b)	Allow the M marks if *r* is used for *n* Ignore errors with terms that are cancelled		
	$\frac{20}{6}\sum_{r=1}^{n}\left(\frac{1}{3n-1}-\frac{1}{3n+5}\right)=$ $\frac{20}{6}\left(\frac{1}{2}-\frac{1}{8}+\frac{1}{5}-\frac{1}{11}+\frac{1}{8}-\frac{1}{14}+...+\frac{1}{3n-7}-\frac{1}{3n-1}+\frac{1}{3n-4}-\frac{1}{3n+2}+\frac{1}{3n-1}-\frac{1}{3n+5}\right)$ Uses the method of differences for at least the first and last rows. Must attempt to put their rows together. Allow with their partial fractions provided there are two which form a difference and have different 2 term linear denominators in *n* and numeric numerators. Ignore errors/omission with the 20 or the "$\frac{1}{6}$". Allow this mark to be implied if their 4 terms in the correct positions are the only ones that result.		**M1**
	$=\frac{20}{6}\left(\frac{1}{2}+\frac{1}{5}-\frac{1}{3n+2}-\frac{1}{3n+5}\right)$	Any correct expression for the required sum. The "20" may appear later - allow this mark if a correct sum without the 20 was seen but is then misprocessed before the 20 is reapplied.	**A1**
	$=\frac{20}{6}\left(\frac{7(3n+2)(3n+5)-10(6n+7)}{10(3n+2)(3n+5)}\right)$	Achieves a consistent single fraction expression. Must come from adding 4 fractions - 2 numeric and 2 with different two term linear denominators and numeric numerators. Ignore omission/error with the 20 and/or the "$\frac{1}{6}$".	**dM1**
	$=\frac{1}{3}\left(\frac{63n^2+87n}{(3n+2)(3n+5)}\right)=\frac{n(21n+29)}{(3n+2)(3n+5)}$	Correct expression in the correct form. Not just values for the constants unless form is seen	**A1**
			(4)
			Total 6

Question Number	Scheme	Notes	Marks
2(a)	$y=(1-3x)^A+e^{Bx} \Rightarrow \frac{dy}{dx}=-3A(1-3x)^{A-1}+Be^{Bx}$	Correct first derivative. Accept unsimplified	**B1**
	$\frac{d^2y}{dx^2}=9A(A-1)(1-3x)^{A-2}+B^2e^{Bx}$ $\frac{d^3y}{dx^3}=-27A(A-1)(A-2)(1-3x)^{A-3}+B^3e^{Bx}$ or e.g., $-27(A^3-3A^2+2A)(1-3x)^{A-3}+B^3e^{Bx}$ M1: Obtains $\frac{d^3y}{dx^3}=g(A)(1-3x)^{A-3}+h(B)e^{Bx}$ A1: Fully correct third derivative. Could be unsimplified. Condone poor bracketing unless further work confirms error. If g(A) is only seen expanded then it must be correct		**M1** **A1**
			(3)
(b)	$f'(0)=-3A+B=0$	Substitutes $x=0$ into their first derivative and sets = 0 and achieves a linear equation in A and B	**M1**
	$f''(0)=9A(A-1)+B^2=0$ $\Rightarrow 9A^2-9A+(3A)^2=0$ or $\Rightarrow 9\left(\frac{B}{3}\right)\left(\frac{B}{3}-1\right)+B^2=0$	Substitutes $x=0$ into their second derivative, sets = 0 and obtains a quadratic equation in A (or B) only	**dM1**
	$18A^2-9A=0 \Rightarrow 9A(2A-1)=0 \Rightarrow A=...$ or $2B^2-3B=0 \Rightarrow B(2B-3)=0 \Rightarrow B=...$	Solves for A and B. No working is required but must get real values and neither can be 0 unless the zero solution is additional	**ddM1**
	$A=\frac{1}{2},\ B=\frac{3}{2}$	Both correct values. Accept equivalent fractions/decimals. Ignore extra zero solution	**A1**
	Allow all marks in (b) even if full marks were not obtained in (a)		**(4)**
(c)	$\frac{d^3y}{dx^3}=-27A(A-1)(A-2)(1-3x)^{A-3}+B^3e^{Bx}$ $\left(\frac{d^3y}{dx^3}\right)_{x=0}=-27\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)+\left(\frac{3}{2}\right)^3=...\quad\left(-\frac{27}{4}\right)$ Substitutes their A and their B and $x=0$ into their 3rd derivative and obtains a value. A and B must be non-zero and must have come from simultaneous equations (one linear, one quadratic). If $k=$ ... they must have divided by 6		**M1**
	$k=-\frac{27}{4}\div 6=...$ Divides their value by 6 or 3! to find a value for k. These two M marks are likely to scored at the same time. If just a value is given without a substitution attempt it must be consistent.		**dM1**
	$k=-\frac{9}{8}$	Correct value for k. This fraction or $-1\frac{1}{8}$ This mark requires a fully correct solution.	**A1**
			(3)
			Total 10
	Alternative for (b) & (c) by series expansions: $y=(1-3x)^A+e^{Bx}=1-3Ax+\frac{A(A-1)}{2}(-3x)^2+\frac{A(A-1)(A-2)}{3!}(-3x)^3+...+1+Bx+\frac{(Bx)^2}{2}+\frac{(Bx)^3}{3!}+...$ $=2+kx^3+... \Rightarrow -3A+B=0\quad$ (M1) $\frac{9A(A-1)}{2}+\frac{B^2}{2}=0 \Rightarrow 9A^2-9A+9A^2=0\quad$ (dM1) $2A^2-A=A(2A-1)=0 \Rightarrow A=\frac{1}{2},\quad B=\frac{3}{2}\quad$ (ddM1A1) $k=-\frac{9}{2}A(A-1)(A-2)+\frac{B^3}{6}\quad$ (M1) $\Rightarrow k=-\frac{9}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)+\frac{1}{6}\left(\frac{3}{2}\right)^3=...,\quad -\frac{9}{8}\quad$ (dM1, A1)		

Question Number	Scheme	Notes	Marks
3(a)	$$4\frac{dy}{dx}=y-2xy^5 \qquad u=y^{-4} \qquad \left\{y=u^{-\frac{1}{4}}\right\}$$		
	e.g., $\frac{du}{dx}=-\frac{4}{y^5}\frac{dy}{dx}$, $\frac{dy}{dx}=-\frac{1}{4}u^{-\frac{5}{4}}\frac{du}{dx}$	Correct differentiation of $u=y^{-4}$ to obtain an equation in $\frac{du}{dx}$ and $\frac{dy}{dx}$ May be implied by later work	**B1**
	$4\left(-\frac{y^5}{4}\right)\frac{du}{dx}=y-2xy^5$, $-u^{-\frac{5}{4}}\frac{du}{dx}=y-2xy^5$	Use the given differential equation and substitutes. Requires a correct form from differentiation of $u=y^{-4}$	**M1**
	$4\left(-\frac{y^5}{4}\right)\frac{du}{dx}=y-2xy^5 \Rightarrow \frac{du}{dx}+\frac{1}{y^4}=2x \Rightarrow \frac{du}{dx}+u=2x$* or, e.g., $-u^{-\frac{5}{4}}\frac{du}{dx}=y-2xy^5 \Rightarrow \frac{du}{dx}=2u^{\frac{5}{4}}xy^5-u^{\frac{5}{4}}y$ or $2x-u \Rightarrow \frac{du}{dx}+u=2x$* Achieves the given answer with no errors and an intermediate step after substitution shown		**A1***
			(3)
(b)	$I=e^{\int dx}=e^x$	Correct integrating factor	**B1**
	$ue^x=\int 2xe^x\,dx$	For $Iu=\int 2xI\,dx$ Allow with y for u. I must be a function of x	**M1**
	Note that a tabular "DI" approach may be used for parts. dM1 for a correct form		
	$ue^x=2xe^x-\int 2e^x\,dx$	Applies parts to RHS and achieves correct form - allow with y for u and condone + used for - in the parts formula	**dM1**
	$ue^x=2xe^x-2e^x\,(+c)$	Correct equation with or without a constant of integration. Must have u	**A1**
	$u=2x-2+ce^{-x}$	Any correct equation with "u=..." and the constant correctly placed	**A1**
	If a 2nd order method is used, CF: $\{A\}e^{-x}$(B1), PF: $\lambda x+\mu$(M1), valid method for λ and μ(dM1), A marks as above		**(5)**
(c)	$1=c-2\Rightarrow c=3$	Uses the given conditions (y=1 or u=1 and x=0) to find the constant of integration. Condone poor algebra e.g, $\frac{1}{a}+\frac{1}{b}$ used for $\frac{1}{a+b}$	**M1**
	$y^4=\frac{1}{2x-2+3e^{-x}}$	Any correct equation in the required $y^4=\ldots$ form e.g., $y^4=\frac{1}{2(x-1)+\frac{3}{e^x}}$ Allow $y^4=\frac{1}{2x-2+ce^{-x}}$, $c=3$	**A1**
			(2)
			Total 10

Question Number	Scheme/Notes	Marks
4(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=x^2y+\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=2xy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ M1: Differentiates x^2y to $Axy+Bx^2\frac{\mathrm{d}y}{\mathrm{d}x}$ A1: Any correct expression for the 3rd derivative	**M1** **A1**
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ Differentiates again to obtain an expression of the form $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=Cy+Dx\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ Terms may be uncollected	**dM1**
	$\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ Fully correct 4th derivative in correct form	**A1**
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3$ and $y=1$ at $x=1$ $\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1+3=4$ Correct value for the 2nd derivative	**B1**
	$\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)_{x=1}=2(1)(1)+1\times3+4=9,\ \left(\frac{\mathrm{d}^4y}{\mathrm{d}x^4}\right)_{x=1}=9+4+4\times3+2=27$ Finds values for their 3rd and 4th derivatives at $x=1$. Allow slips	**M1**
	$\{y=\}1+3(x-1)+\frac{\text{"4"}(x-1)^2}{2!}+\frac{\text{"9"}(x-1)^3}{3!}+\frac{\text{"27"}(x-1)^4}{4!}+\ldots$ Applies Taylor's expansion correctly with their 3 values. There is no credit for quoting a correct formula - it must be applied appropriately	**dM1**
	$\{y=\}1+3(x-1)+2(x-1)^2+\frac{3(x-1)^3}{2}+\frac{9(x-1)^4}{8}+\ldots$ Correct series with simplified coefficients	**A1**
		(4)
		Total 8

Question Number	Scheme	Notes	Marks
5	"Solutions relying entirely on calculator technology are not acceptable"		
	$\left\lvert\frac{x^2+5x-2}{x^2+1}\right\rvert < 2 \Rightarrow \lvert x^2+5x-2\rvert < 2(x^2+1)$ $x^2+5x-2=2(x^2+1) \Rightarrow x=...$ **or** $x^2+5x-2=-2(x^2+1) \Rightarrow x=...$	Multiplies through by x^2+1 and attempts to find at least one pair of cvs from an initially correct equation (no fraction). Usual rules (2 solutions) if 3TQ with solving method seen. If 2TQ allow $ax^2+bx=0 \Rightarrow \{x=0,\}\quad x=\pm\frac{b}{a}$ and condone this without factorisation seen	**M1**
	$x^2-5x+4=(x-1)(x-4) \Rightarrow x=1,\ 4$ or $3x^2+5x\{=x(3x+5)\}=0 \Rightarrow x=0,\ -\frac{5}{3}$ Identifies 1 correct pair of cvs. The "0" may appear later		**A1**
	$x^2+5x-2=2(x^2+1) \Rightarrow x=...$ **and** $x^2+5x-2=-2(x^2+1) \Rightarrow x=...$	Complete method to find all 4 cvs (rules as previous M1)	**dM1**
	$x^2-5x+4=(x-1)(x-4) \Rightarrow x=1,\ 4$ or $3x^2+5x\{=x(3x+5)\}=0 \Rightarrow x=0,\ -\frac{5}{3}$ All 4 cvs correct. The "0" may appear later. Ignore extra cvs but no further marks unless they are disregarded.		**A1**
	$x<-\frac{5}{3},\ \ 0<x<1,\ \ x>4$	M1: With 4 cvs *a*, *b*, *c*, *d* where $a<b<c<d$ identifies $x<a,\quad b<x<c,\quad x>d$ Condone non-strict inequalities. A1: Fully correct. Allow any equivalent correct notation e.g. $\left(-\infty,-\frac{5}{3}\right)\cup(0,1)\cup(4,\infty)$ Condone "and" but do not allow $\cap$ for this last mark	**M1** **A1**
	Note that if no 3TQ solving method is seen 110011 is possible but the last two marks cannot be scored if there is no algebra at all. Condone use of a calculator for the last two marks provided there has been some algebra at some point. If squaring is used this leads to: $3x^4-10x^3-13x^2+20x=x(3x^3-10x^2-13x+20)=x(3x+5)(x-1)(x-4)$ but the first 2 M marks are only scored if there is clear evidence of a non-calculator method for factorising (e.g., factor theorem, grid, etc.) You may also see the difference of two squares e.g., $2^2(x^2+1)^2-(x^2+5x-2)^2=(3x^2+5x)(x^2-5x+4)=x(3x+5)(x-1)(x-4)=0 \Rightarrow ...$ We must see the factorisation of the 3TQ as with the main scheme		**(6)**
			Total 6

Question Number	Scheme	Notes	Marks
6	$(x+1)^2+(y-1)^2=1 \Rightarrow \lvert z-(-1+\mathrm{i})\rvert=1$	Correct locus e.g., $\lvert z+1-\mathrm{i}\rvert=1$	**B1**
	$w=\dfrac{z+2}{3z+4} \Rightarrow z=\dfrac{2-4w}{3w-1}$	M1: Attempts to make z the subject A1: Any correct rearrangement	**M1** **A1**
	$\lvert z-(-1+\mathrm{i})\rvert=1 \Rightarrow \left\lvert \dfrac{2-4w}{3w-1}+1-\mathrm{i}\right\rvert=1$	Uses a locus of the form $\lvert z\pm1\pm\mathrm{i}\rvert=1$ and their expression for z in terms of w and sets = 1	**dM1**
	$\Rightarrow \left\lvert \dfrac{2-4w+3w-3w\mathrm{i}-1+\mathrm{i}}{3w-1}\right\rvert=1 \Rightarrow \lvert 1-w-3w\mathrm{i}+\mathrm{i}\rvert=\lvert 3w-1\rvert$ $\Rightarrow \lvert 1-u-\mathrm{i}v-3u\mathrm{i}+\mathrm{i}+3v\rvert=\lvert 3u+3\mathrm{i}v-1\rvert$ M1: Finds common denominator, multiplies up and introduces $w=u+\mathrm{i}v$		**ddM1**
	$(1-u+3v)^2+(1-v-3u)^2=(3u-1)^2+9v^2$	Applies Pythagoras correctly to obtain an equation in u and v	**dddM1**
	$u^2+v^2-2u+4v+1=0$	Correct equation in this form	**A1**
			(7)

There may be various alternatives.
If using the main scheme above benefits the learners for any different approaches then please use that.

Approaches which attempt to map points can access the first mark as above (or by choosing three correct points (or complex numbers) for the circle).
The remaining marks would score as follows:
M1 for transforming 3 points using the given transformation
A1: 3 correct points
dM1ddM1dddM1: For a complete valid method to establish the equation of circle D
Could find the intersection point of two perpendicular bisectors and then find the radius and produce a circle equation, or use the transformed points in a general circle equation and solve the 3 simultaneous equations involving 3 unknowns and produce a circle equation
A1: Correct equation

Another possibility is:

$$u+\mathrm{i}v=\frac{x+\mathrm{i}y+2}{3(x+\mathrm{i}y)+4} \text{ or } x+\mathrm{i}y=\frac{2-4(u+\mathrm{i}v)}{3(u+\mathrm{i}v)-1} \text{ (the latter could get M1A1 already)}$$

$$\Rightarrow x=\frac{-12u^2-12v^2+10u-2}{(3u-1)^2+9v^2} \quad y=\frac{-2v}{(3u-1)^2+9v^2} \quad (\text{M1A1})$$

Could write this as $x+\mathrm{i}y$ and note that $(3u-1)^2+9v^2=9u^2-6u+1+9v^2$

$$\left(\frac{-12u^2-12v^2+10u-2}{(3u-1)^2+9v^2}+1\right)^2+\left(\frac{-2v}{(3u-1)^2+9v^2}-1\right)^2=1 \quad (\text{dM1ddM1dddM1, B1 if all correct})$$

$$u^2+v^2-2u+4v+1=0 \quad (\text{A1})$$

Question Number	Scheme	Notes	Marks
7(a)	$\sin\theta = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)$ $\Rightarrow \sin^4\theta = \left\{\frac{1}{(2\mathrm{i})^4}\right\}\left(\left(\mathrm{e}^{\mathrm{i}\theta}\right)^4 + 4\left(\mathrm{e}^{\mathrm{i}\theta}\right)^3\left(-\mathrm{e}^{-\mathrm{i}\theta}\right) + 6\left(\mathrm{e}^{\mathrm{i}\theta}\right)^2\left(-\mathrm{e}^{-\mathrm{i}\theta}\right)^2 + 4\mathrm{e}^{\mathrm{i}\theta}\left(-\mathrm{e}^{-\mathrm{i}\theta}\right)^3 + \left(\mathrm{e}^{-\mathrm{i}\theta}\right)^4\right)$ Expands $\left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)^4$ or squares twice. Allow slips but must have at least 4 terms and include an attempt at binomial coefficients which may be uncalculated e.g., $\binom{4}{3}$... Allow via $\left\{\frac{1}{(2\mathrm{i})^4}\right\}\left(z - \frac{1}{z}\right)^4 = \left\{\frac{1}{16}\right\}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right)$		M1
	$= \frac{1}{16}\left(\mathrm{e}^{4\mathrm{i}\theta} - 4\mathrm{e}^{2\mathrm{i}\theta} + 6 - 4\mathrm{e}^{-2\mathrm{i}\theta} + \mathrm{e}^{-4\mathrm{i}\theta}\right)$ or $\frac{1}{16}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right)$ Correct simplified expansion. The $\frac{1}{16}$ may be applied later		A1
	$= \frac{1}{16}\left(\cos 4\theta + \mathrm{i}\sin 4\theta + \cos 4\theta - \mathrm{i}\sin 4\theta - 4\left(\cos 2\theta + \mathrm{i}\sin 2\theta + \cos 2\theta - \mathrm{i}\sin 2\theta\right) + 6\right)$ or $= \frac{1}{16}\left(\mathrm{e}^{4\mathrm{i}\theta} + \mathrm{e}^{-4\mathrm{i}\theta} - 4\left(\mathrm{e}^{2\mathrm{i}\theta} + \mathrm{e}^{-2\mathrm{i}\theta}\right) + 6\right) = \frac{1}{16}\left(2\cos 4\theta - 4\left(2\cos 2\theta\right) + 6\right)$ Completes an attempt to use $\mathrm{e}^{n\theta\mathrm{i}} = \cos n\theta + \mathrm{i}\sin n\theta$ or $\cos n\theta = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}n\theta} + \mathrm{e}^{-\mathrm{i}n\theta}\right)$ to express the expansion in terms of trig functions. The $\frac{1}{16}$ may be incorrect or missing		dM1
	$\frac{1}{16}\left(2\cos 4\theta - 8\cos 2\theta + 6\right) \Rightarrow \frac{1}{8}\left(\cos 4\theta - 4\cos 2\theta + 3\right)$ Correct expression in the required form		A1
			(4)
(b)	$\Rightarrow \sin^4\left(\frac{1}{2}\pi - \theta\right) = \frac{1}{8}\left(\cos 4\left(\frac{1}{2}\pi - \theta\right) - 4\cos 2\left(\frac{1}{2}\pi - \theta\right) + 3\right)$ $\cos^4\theta = \frac{1}{8}\left(\cos 4\left(\frac{1}{2}\pi - \theta\right) - 4\cos 2\left(\frac{1}{2}\pi - \theta\right) + 3\right)$ $= \frac{1}{8}\left(\cos(2\pi - 4\theta) - 4\cos(\pi - 2\theta) + 3\right)$ $= \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)$ oe e.g., $\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$ M1: $\sin^4\theta = A\cos 4\theta + B\cos 2\theta + C \Rightarrow \cos^4\theta = D\cos 4\theta + E\cos 2\theta + F$ Do not allow if something other than $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ is clearly being used A1: Any correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ (not fortuitous)		M1 A1
			(2)
(c)	$\int\left(\sin^4\theta + \cos^4\theta\right)\mathrm{d}\theta = \int\left(\frac{1}{8}\left(\cos 4\theta - 4\cos 2\theta + 3\right) + \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)\right)\mathrm{d}\theta$ $\left\{= \int\left(\frac{1}{4}\cos 4\theta + \frac{3}{4}\right)\mathrm{d}\theta \text{ or e.g., } \frac{1}{8}\int\left(2\cos 4\theta + 6\right)\mathrm{d}\theta\right\}$ Uses their expressions from (a) and (b) both of correct form and different and substitutes correctly within an integral. M0 if expression would reduce to just k. Must be consistent if only seen with terms collected		M1
	$= \frac{1}{16}\sin 4\theta + \frac{3}{4}\theta(+c)$	Correct 2 term result with or without + c	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
8(a)	e.g., $u = xy \Rightarrow \frac{du}{dx} = x\frac{dy}{dx} + y$ or $y = ux^{-1} \Rightarrow \frac{dy}{dx} = -ux^{-2} + x^{-1}\frac{du}{dx}$ or $\frac{dy}{dx} = \frac{x\frac{du}{dx} - u}{x^2}$ Differentiates $u = xy$ oe to obtain any correct equation in $\frac{du}{dx}$ and $\frac{dy}{dx}$		**B1**
	$\frac{du}{dx} = x\frac{dy}{dx} + y \Rightarrow \frac{d^2u}{dx^2} = x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx}$ or e.g., $\frac{d^2y}{dx^2} = -x^{-2}\frac{du}{dx} + 2x^{-3}u - x^{-2}\frac{du}{dx} + x^{-1}\frac{d^2u}{dx^2} \Rightarrow -2x^{-2}\left(x\frac{dy}{dx} + y\right) + 2x^{-2}y + x^{-1}\frac{d^2u}{dx^2}$ Differentiates again and obtains an equation of the correct form before given answer i.e., $A\frac{d^2u}{dx^2} = Bx\frac{d^2y}{dx^2} + C\frac{dy}{dx}$ oe Allow if there are clearly cancelling terms involving other variables/derivatives or the work makes completely clear how the given answer is reached		**M1**
	$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{d^2u}{dx^2} - 2\frac{dy}{dx}\right)$ *	Obtains the given answer with no errors. No intermediate step is required	**A1***
			(3)
(b)	$x\frac{d^2y}{dx^2} + 2(2x+1)\frac{dy}{dx} + 13xy = 17e^{3x} - 4y$ $\Rightarrow x\frac{1}{x}\left(\frac{d^2u}{dx^2} - 2\frac{dy}{dx}\right) + 4x\frac{1}{x}\left(\frac{du}{dx} - y\right) + 2\frac{dy}{dx} + 13xy = 17e^{3x} - 4y$ Substitutes the given second derivative and their first derivative into DE (I)		**M1**
	$\Rightarrow \frac{d^2u}{dx^2} + 4\frac{du}{dx} - 4y + 13u = 17e^{3x} - 4y$ $\Rightarrow \frac{d^2u}{dx^2} + 4\frac{du}{dx} + 13u = 17e^{3x}$ *	Obtains the given answer with an intermediate step after substitution and no errors	**A1***
			(2)
(c)	$m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3i$	Solves $m^2 + 4m + 13 = 0$ (allow miscopy and apply usual rules if necessary but may use calculator but if so must get a correct root for their 3TQ)	**M1**
	$\{u =\} e^{-2x}(A\cos 3x + B\sin 3x)$	Correct CF	**A1**
	Note other CFs are possible: $\{u =\} Ae^{(-2+3i)x} + Be^{(-2-3i)x}$, $\quad Ae^{-2x}\cos(3x + B)$		
	$u = \lambda e^{3x} \Rightarrow \frac{du}{dx} = 3\lambda e^{3x} \Rightarrow \frac{d^2u}{dx^2} = 9\lambda e^{3x}$ $\Rightarrow \frac{d^2u}{dx^2} + 4\frac{du}{dx} + 13u = (9\lambda + 12\lambda + 13\lambda)e^{3x}$	Starts with the correct PI form, differentiates twice (obtaining correct (changed) forms) and substitutes into LHS	**M1**
	$\Rightarrow 9\lambda + 12\lambda + 13\lambda = 17 \Rightarrow \lambda = ...$	Solves to find the unknown in the PI	**dM1**
	$u = e^{-2x}(A\cos 3x + B\sin 3x) + \frac{1}{2}e^{3x}$	Correct GS of equation (II). Must be $u = ...$	**A1**
			(5)
(d)	$y = \frac{1}{x}\left(e^{-2x}(A\cos 3x + B\sin 3x) + \frac{1}{2}e^{3x}\right)$	Correct follow through equation. Allow with their CF + PI provided both are functions of *x* (allow e.g., CF with no constant, PI with constant) Must be $y = ...$	**B1ft**
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
9	"Solutions relying entirely on calculator technology are not acceptable"		
(a)	$1+\sin\theta = 1+\cos 2\theta \Rightarrow \sin\theta = 1-2\sin^2\theta \Rightarrow 2\sin^2\theta+\sin\theta-1=0$ Sets $1+\sin\theta = 1+\cos 2\theta$ and obtains a 3TQ in $\sin\theta$ (terms may not all be on one side) by using $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or equivalent work condoning sign errors only in identities		**M1**
	$2\sin^2\theta+\sin\theta-1=0 \Rightarrow (2\sin\theta-1)(\sin\theta+1)=0$ $\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = ...$	Solves 3TQ in $\sin\theta$ for θ Must see factorisation or equivalent work	**dM1**
	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$ Allow $r=\frac{3}{2}$, $\theta=\frac{\pi}{6}$ even if followed by $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ Correct polar coordinates. Ignore extra coordinate pairs unless correct *P* is rejected		**A1**
			(3)
(b)	Note that the $\frac{1}{2}$s may have already been applied		
	$\int (1+\sin\theta)^2\,d\theta = \int (1+2\sin\theta+\sin^2\theta)\,d\theta = \int \left(1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)d\theta$ $\int (1+\cos 2\theta)^2\,d\theta = \int (1+2\cos 2\theta+\cos^2 2\theta)\,d\theta = \int \left(1+2\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)d\theta$		
	$\int (1+\sin\theta)^2\,d\theta = \int (1+2\sin\theta+\sin^2\theta)\,d\theta$ and uses $\sin^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ $\int (1+\cos 2\theta)^2\,d\theta = \int (1+2\cos 2\theta+\cos^2 2\theta)\,d\theta$ and uses $\cos^2 2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 4\theta$		**M1(one)** **dM1(both)**
	Allows the Ms for $r_1^2 \pm r_2^2$ but $(r_1 \pm r_2)^2$ is M0dM0		
	$\int \left(1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)d\theta = \frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$ $\int \left(1+2\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)d\theta = \frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta (+c)$		**A1(one)** **Requires 1 previous M mark** **A1(both)**
	Area of *R*: $\frac{1}{2}\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}} + \frac{1}{2}\left[\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2}\left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} - \left(-\frac{3\pi}{4}\right)\right) + \frac{1}{2}\left(\frac{3\pi}{4} - \left(\frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16}\right)\right)$ or $\frac{1}{2}\left(\pi - \frac{9\sqrt{3}}{8}\right) + \frac{1}{2}\left(\frac{\pi}{2} - \frac{9\sqrt{3}}{16}\right)$ A completely valid method using the $\frac{1}{2}$ twice and the correct limits, obtaining a numerical expression with trig terms evaluated. May not be consistent - if the limits are seen correctly on the square brackets accept the **sum** of two substitutions. If just $a\pi + b\sqrt{3}$ results it must be $\frac{3\pi}{4} - \frac{27}{32}\sqrt{3}$		**ddM1**
	$= \frac{3\pi}{4} - \frac{27}{32}\sqrt{3}$ Correct expression in this form. Requires all previous marks.		**A1**
			(6)
			Total 9