Pearson

Mark Scheme (Results)

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International Advanced Level in Further Pure Mathematics F2

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Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given
   - ⌞ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread

however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

EDEXCEL IAL MATHEMATICS
General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
  - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$
  - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$
- Formula
  - Attempt to use the correct formula (with values for *a*, *b* and *c*).
- Completing the square
  - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

- Differentiation
  - Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)
- Integration
  - Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

1. Solve the equation

$$z^5 = 32$$

Give your answers in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$

(5)

(Total for Question 1 is 5 marks)

Question Number	Scheme	Marks
1	$r = 2$	B1
	At least one of $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ oe	B1
	At least 3 of $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ oe	M1
	$r = 2$, and at least 3 of: $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$	A1
	All correct: $r = 2,\ \theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$	A1
		(5)

Notes

B1: $r = 2$ which may be implied by their answers.

B1: For at least **one** correct argument e.g. one of $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ but allow a correct value out of range e.g. at least one of $-\frac{2\pi}{5}, -\frac{4\pi}{5}, -\frac{6\pi}{5}, -\frac{8\pi}{5}$ which may be implied by their answers.
Note that $z = 2$ implies both B marks.

M1: For at least 3 correct and **<u>distinct arguments</u>** which may be out of range as above.
May be implied by their answers. Allow r to be incorrect for this mark.

A1: For the correct value of r and at least 3 correct and distinct arguments **in range**.
e.g. $r = 2$ with any 3 of $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ which may be implied by their answers.

A1: For the correct value of r and all correct arguments **in range**.

e.g. $r = 2$ with $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ which may be implied by their answers.
Ignore extra answers outside the range.
For a fully correct solution that has extra solutions in range, deduct the final A mark.
Answers in **degrees**: Penalise once the first time it occurs but do not penalise 0°
For reference: Angles in degrees are: 0, 72, 144, 216, 288
If they subsequently convert the angles to radians then allow recovery.

Note that answers in a general form can score full marks if correct.
e.g. these score full marks:

$$z = 2\left(\cos\frac{2k\pi}{5} + \mathrm{i}\sin\frac{2k\pi}{5}\right), (k = 0, 1, 2, 3, 4):\ r = 2,\ \theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$$

$$(z =)\,2\mathrm{e}^{\frac{2k\pi}{5}\mathrm{i}}, (k = 0, 1, 2, 3, 4)$$

But

$$z = 2\left(\cos\frac{2k\pi}{5} + \mathrm{i}\sin\frac{2k\pi}{5}\right) \text{ scores B1 only.}$$

Note that if $r = 2$ with $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ is found and the solutions are then formed incorrectly e.g. $z = 2\left(\cos\frac{2\pi}{5} + \sin\frac{2\pi}{5}\right)$ or e.g. $2\mathrm{e}^{\frac{2k\pi}{5}}$ then isw can be applied.

2. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use algebra to find the set of values of x for which

$$|x^2 - 9| < |1 - 2x|$$

(6)

(Total for Question 2 is 6 marks)

Question Number	Scheme	Marks
2	$\lvert x^2-9\rvert < \lvert 1-2x \rvert$ **(ignore use of any strict/non-strict inequalities instead of "=" when finding cv's)**	
	$x^2-9=1-2x \Rightarrow x^2+2x-10=0 \Rightarrow x=...$ **or** $x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	M1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **or** $x=-2,\ 4$	A1
	$x^2-9=1-2x \Rightarrow x^2+2x-10=0 \Rightarrow x=...$ **and** $x^2-9=-1+2x \Rightarrow x^2-2x-8=0 \Rightarrow x=...$	**d**M1
	$x=\dfrac{-2\pm\sqrt{44}}{2}$ **and** $x=-2,\ 4$	A1
	One of: $-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	A1
	Both $-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	A1
		(6)

Notes

M1: Attempts to solve $x^2-9=1-2x$ oe **or** $x^2-9=-1+2x$ oe to obtain 2 non-zero values of x.
Must be solving a 3TQ – see general guidance but also allow calculator solutions so this mark may be implied by correct values for their 3TQ (you may need to check).

A1: **One** correct **pair** of values. May be seen embedded in an inequality or e.g. on a sketch.
Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.3, –4.3 or truncated 2.3...,–4.3…

dM1: Attempts to solve $x^2-9=1-2x$ oe **and** $x^2-9=-1+2x$ oe to obtain 4 non-zero values of x.
Must be solving a 3TQ – see general guidance but also allow calculator solutions so this mark may be implied by correct values for their 3TQ (you may need to check).
Depends on the first method mark.

A1: **Both pairs** of values correct. May be seen embedded in an inequality or e.g. on a sketch.
Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.3, –4.3 or truncated 2.3..., –4.3…

For the final 2 marks (which must follow both previous method marks):

For $-1+\sqrt{11}$ allow $\dfrac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\dfrac{-2-\sqrt{44}}{2}$ but must be exact here.

Allow alternative notation e.g. $\left(-1+\sqrt{11},\ 4\right)$, $\left(-1-\sqrt{11},\ -2\right)$, $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$

A1: One correct inequality.

A1: Both inequalities correct. Allow the inequalities to be written separately or with "," or "or" or "and" or $\cup$ between them but not $\cap$ between them.

In an otherwise fully correct solution, if any extra incorrect regions are given, deduct the final A mark.

	Q2 Alternative by squaring (ignore use of any strict/non-strict inequalities instead of "=" when finding cv's)	
	$(x^2-9)^2=(1-2x)^2 \Rightarrow x^4-18x^2+81=1-4x+4x^2$	
	$x^4-22x^2+4x+80=0 \Rightarrow x=...$	M1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **or** $x=-2, 4$	A1
	$x=\frac{-2\pm\sqrt{44}}{2}$ **and** $x=-2, 4$	**d**M1A1
	$-1+\sqrt{11}<x<4$ **or** $-1-\sqrt{11}<x<-2$	A1
	$-1+\sqrt{11}<x<4$ **and** $-1-\sqrt{11}<x<-2$	A1

Notes

M1: Attempts to square both sides, collect terms, rearrange and solve a quartic equation to obtain at least 2 values of x that are non-zero. Allow calculator solutions so this mark may be implied by correct values for their quartic (you may need to check).

A1: **One** correct pair of values. It is for $-2, 4$ or $-1\pm\sqrt{11}$. May be seen embedded in an inequality or e.g. on a sketch.
Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.3, −4.3 or truncated 2.3...,−4.3…

dM1: Obtains 4 non-zero values of x. Allow calculator solutions so this mark may be implied by correct values for their quartic (you may need to check).
Depends on the first method mark.

A1: **Both** pairs of values correct. May be seen embedded in an inequality or e.g. on a sketch.
Allow the irrational roots to be at least as given here or $-1\pm\sqrt{11}$ or awrt 2.3, −4.3 or truncated 2.3..., −4.3…

For the final 2 marks (which must follow both previous method marks):

For $-1+\sqrt{11}$ allow $\frac{-2+\sqrt{44}}{2}$, for $-1-\sqrt{11}$ allow $\frac{-2-\sqrt{44}}{2}$ but must be exact here.

Allow alternative notation e.g. $(-1+\sqrt{11}, 4)$, $(-1-\sqrt{11}, -2)$, $4>x>-1+\sqrt{11}$, $x>-1+\sqrt{11}$ **and** $x<4$, $-2>x>-1-\sqrt{11}$, $x>-1-\sqrt{11}$ **and** $x<-2$

A1: One correct inequality.

A1: Both inequalities correct. Allow the inequalities to be written separately or with "," or "or" or "and" or $\cup$ between them but not $\cap$ between them.

In an otherwise fully correct solution, if any extra incorrect regions are given, deduct the final A mark.

3.

$$(\cos x)\frac{dy}{dx} + (\sin x)y = 2\cos^3 x\sin x - 3 \qquad 0 \leqslant x < \frac{\pi}{2}$$

(a) Find the general solution of this differential equation.
Give your answer in the form $y = f(x)$.

(7)

(b) Find the particular solution of this differential equation for which $y = 3\sqrt{3}$ at $x = \frac{\pi}{3}$

Give your answer in the form $y = f(x)$.

(3)

(Total for Question 3 is 10 marks)

Question Number	Scheme	Marks
3. (a)	$\frac{\mathrm{d}y}{\mathrm{d}x}+(\tan x)y=2\cos^2 x\sin x-3\sec x$	M1
	Integrating Factor: $I=\mathrm{e}^{\int \tan x\,\mathrm{d}x}$	dM1
	$I=\sec x$	A1
	$y\sec x=\int \sec x\left(2\cos^2 x\sin x-3\sec x\right)(\mathrm{d}x)$ or $\frac{\mathrm{d}}{\mathrm{d}x}(y\sec x)=\sec x\left(2\cos^2 x\sin x-3\sec x\right)$	M1
	$\int 2\cos x\sin x\,\mathrm{d}x=\sin^2 x$ or $-\cos^2 x$ or $-\frac{1}{2}\cos 2x$ **Must follow the previous method mark**	A1
	$\int -3\sec^2 x\,\mathrm{d}x=-3\tan x$ **Must follow the previous method mark**	A1
	Follow through their integration and their integrating factor: i.e. $y=\text{their}\left(\sin^2 x-3\tan x+k\right)\div\text{their}\sec x$ Examples of a correct answer: $y=\cos x\sin^2 x-3\sin x+k\cos x$ or $y=-\frac{1}{2}\cos x\cos 2x-3\sin x+k\cos x$ or $y=-\cos^3 x-3\sin x+k\cos x$	A1ft
		(7)

Notes

M1: Attempt to divide through by cos x. If the intention is not clear must see at least 2 terms divided by cos x. May be implied by a correct integrating factor.

dM1: $I=\mathrm{e}^{\int \pm\text{their P}(x)\,(\mathrm{d}x)}$ from $\frac{\mathrm{d}y}{\mathrm{d}x}+Py=...$

Dependent on the first method mark. May be implied by use of sec x oe as the integrating factor.

A1: $\frac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$ and condone $\cos^{-1} x$ if the intention is clear.

M1: Attempts to use their integrating factor correctly e.g.

$$y\times\text{their IF}=\int Q(x)\times\text{their IF}(\mathrm{d}x) \text{ or } \frac{\mathrm{d}}{\mathrm{d}x}(y\times\text{their IF})=Q(x)\times\text{their IF}$$

from their $\frac{\mathrm{d}y}{\mathrm{d}x}+P(x)y=Q(x)$

If there is any doubt, must multiply at least one of their $Q(x)$ terms by their IF

A1: Correct integration of $2\cos x\sin x$. **Must follow the previous method mark.**

A1: Correct integration of $-3\sec^2 x$. **Must follow the previous method mark.**

A1ft: Follow through their integration (must be a changed function and not just and their $Q(x)\times\text{their IF}$) and their integrating factor but must be $y=\ldots$ with the constant dealt with correctly. **Depends on the third method mark.**

Allow any equivalent correct or correct ft expressions which may be unsimplified so allow e.g. $-3\frac{\tan x}{\sec x}$ for $-3\sin x$

(b)	e.g. $3\sqrt{3}=-\frac{1}{2}\cos\frac{\pi}{3}\cos 2\frac{\pi}{3}-3\sin\frac{\pi}{3}+k\cos\frac{\pi}{3}\Rightarrow k=...$	M1
	$y=\cos x\sin^2 x-3\sin x+k\cos x$ $\Rightarrow 3\sqrt{3}=\frac{1}{2}.\left(\frac{\sqrt{3}}{2}\right)^2-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}-\frac{3}{4}\right)$ oe $y=-\frac{1}{2}\cos x\cos 2x-3\sin x+k\cos x$ $\Rightarrow 3\sqrt{3}=-\frac{1}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}-\frac{1}{4}\right)$ oe $y=-\cos^3 x-3\sin x+k\cos x$ $\Rightarrow 3\sqrt{3}=-\left(\frac{1}{2}\right)^3-3\frac{\sqrt{3}}{2}+k\frac{1}{2}\Rightarrow k=\left(9\sqrt{3}+\frac{1}{4}\right)$ oe	A1
	$y=\cos x\sin^2 x-3\sin x+\left(9\sqrt{3}-\frac{3}{4}\right)\cos x$ oe or $y=-\frac{1}{2}\cos x\cos 2x-3\sin x+\left(9\sqrt{3}-\frac{1}{4}\right)\cos x$ oe or $y=-\cos^3 x-3\sin x+\left(9\sqrt{3}+\frac{1}{4}\right)\cos x$ oe	A1
		(3)
		Total 10

Notes

M1: Substitutes the given conditions into their $y = f(x)$ and attempts to find a value for their constant.

A1: **Correct constant** for their method from a **correct function** in part (a).

A1: Correct equation or equivalent correct equation but must be $y = \dots$

4. (a) Express $\dfrac{4r+2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence, using the method of differences, prove that

$$\sum_{r=1}^{n} \frac{4r+2}{r(r+1)(r+2)} = \frac{n(an+b)}{2(n+1)(n+2)}$$

where a and b are constants to be found.

(5)

(Total for Question 4 is 8 marks)

Question Number	Scheme	Marks
4(a)	$\dfrac{4r+2}{r(r+1)(r+2)} \equiv \dfrac{A}{r}+\dfrac{B}{r+1}+\dfrac{C}{r+2} \rightarrow A=...$ or $B=...$ or $C=...$	M1
	$\dfrac{1}{r}+\dfrac{2}{(r+1)}-\dfrac{3}{(r+2)}$	A1 A1
		(3)

Notes

M1: Correct partial fractions method e.g. substitution or compares coefficients to obtain one of A, B or C for $\dfrac{A}{r}, \dfrac{B}{(r+1)}, \dfrac{C}{(r+2)}$

A1: 2 Correct fractions (or values)

A1: Fully correct **partial fractions**.
It is not for just the correct values unless the correct fractions are seen in (b).

Correct answer with no working scores full marks in (a)

NB use of e.g. $\dfrac{4r+2}{r(r+1)(r+2)} \equiv \dfrac{A}{r(r+1)}+\dfrac{B}{(r+1)(r+2)} \rightarrow \dfrac{1}{r(r+1)}+\dfrac{3}{(r+1)(r+2)}$

Scores no marks in part (a) unless the fractions are split further.

(b)	**Must have partial fractions of the form** $\frac{A}{r}, \frac{B}{(r+1)}, \frac{C}{(r+2)}$ ***A*, *B*, *C* ≠ 0 to score the <u>first</u> M mark in (b)**	
	$\sum_{r=1}^{n} = \left(\frac{1}{1}+\frac{2}{2}-\frac{3}{3}\right)+\left(\frac{1}{2}+\frac{2}{3}-\frac{3}{4}\right)+......+\left(\frac{1}{n-1}+\frac{2}{n}-\frac{3}{n+1}\right)+\left(\frac{1}{n}+\frac{2}{n+1}-\frac{3}{n+2}\right)$	M1
	$=\frac{1}{1}+\frac{2}{2}+\frac{1}{2}-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$	A1 A1
	$=\frac{5(n+1)(n+2)-2(n+2)-6(n+1)}{2(n+1)(n+2)}$	**d**M1
	$\frac{n(5n+7)}{2(n+1)(n+2)}$	A1
		(5)
		Total 8

Notes

M1: Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their fractions identified below provided all the necessary terms (algebraic and constant) are seen**
Allow other letters for *n* (most likely to be *r*) except for the final mark – see below
If terms are found beyond the limits of the summation e.g. $r = 0$, $r = n + 1$, these can be ignored for this mark as long as at least the terms for $r = 1, 2, n - 1$ and n are seen.
No cancelling needs to be seen at this stage.

A1: $\frac{1}{1}+\frac{2}{2}+\frac{1}{2}\left(=\frac{5}{2}\right)$ identified as the only constant terms.

A1: $-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$ oe e.g. $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ identified as the only algebraic terms.

dM1: Attempt common denominator from terms of the form $A, \frac{B}{n+1}, \frac{C}{n+2}$ only.
Must see $(n + 1)(n + 2)$ in the denominator and an unsimplified or simplified polynomial of order 2 in the numerator.
Depends on the first method mark.

A1: Correct expression. Must be in terms of *n* for this mark.

Allow recovery in (b) from e.g.

$$\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)}+\frac{B}{(r+1)(r+2)} \rightarrow \frac{1}{r(r+1)}+\frac{3}{(r+1)(r+2)}$$

$$\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{1}{r(r+1)}+\frac{3}{(r+1)(r+2)} = \frac{1}{r}-\frac{1}{r+1}+\frac{3}{r+1}-\frac{3}{r+2}$$ **etc.**

Note that there will be other correct approaches in (b):

<u>Alternative 1:</u> using change of summation limits:

$$\sum_{r=1}^{n}\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=1}^{n}\frac{2}{(r+1)}-\sum_{r=1}^{n}\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=2}^{n+1}\frac{2}{r}-\sum_{r=3}^{n+2}\frac{3}{r}$$

$$=-2+\frac{2}{n+1}+3+\frac{3}{2}-\frac{3}{n+1}-\frac{3}{n+2}+\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=1}^{n}\frac{2}{r}-\sum_{r=1}^{n}\frac{3}{r}=\frac{5}{2}-\frac{1}{n+1}-\frac{3}{n+2}\text{ etc.}$$

Score as follows:

M1: for an attempt to change all summations to $\sum\frac{\dots}{r}$ and add/subtract terms to compensate.

A1: for $-2+3+\frac{3}{2}$ oe **A1**: for $\frac{2}{n+1}-\frac{3}{n+1}-\frac{3}{n+2}$ oe
M1A1: As main scheme.

If you are unsure if an attempt deserves credit use Review.

<u>Alternative 2:</u> grouping terms:

$$\sum_{r=1}^{n}\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}-\frac{1}{(r+2)}+\sum_{r=1}^{n}\frac{2}{r+1}-\frac{2}{(r+2)}$$

$$=\left(1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\ldots+\frac{1}{n-1}-\frac{1}{n+1}+\frac{1}{n}-\frac{1}{n+2}\right)+2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n}-\frac{1}{n+1}+\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$=\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)+2\left(\frac{1}{2}-\frac{1}{n+2}\right)$$

Score as follows:

M1: for an attempt to group terms and consider at least 2 terms at the start and at least 2 terms at the end for each group.

A1: for $1+\frac{1}{2}+\frac{2}{2}$ oe **A1**: for $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ oe
M1A1: As main scheme.

5. Given that

$$(2 - x^2)\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 = 3y$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{1}{(2 - x^2)}\left(2x\frac{d^2y}{dx^2}\left(1 - 5\frac{dy}{dx}\right) - 5\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx}\right)$$

(5)

Given also that $y = 3$ and $\frac{dy}{dx} = \frac{1}{4}$ at $x = 0$

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

(4)

(Total for Question 5 is 9 marks)

Note that part (a) is now being marked as MdMAddMA not MMABA

Question Number	Scheme	Marks
5(a)	$\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \to -2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ **or** $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \to 5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1
	$\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \to -2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ **and** $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 \to 5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**d**M1
	$-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\frac{\mathrm{d}y}{\mathrm{d}x}$	A1
	$\Rightarrow\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-5x\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	**dd**M1A1*
		(5)

Notes (Now marked as MdMAddMA not MMABA)

M1: Differentiates $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using the product rule to obtain $\ldots x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

May be implied by e.g. $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $-x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\to -2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

i.e. $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\to\ldots x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\ldots x^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

Or Differentiates $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using the product rule to obtain $\ldots\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\ldots x\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

It must be clear that the expressions are not simply a result of expanding the given answer.

dM1: Differentiates $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using the product rule to obtain $\ldots x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

May be implied by e.g. $\left(2-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $-x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\to -2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

i.e. $x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\to\ldots x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\ldots x^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$

And Differentiates $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using the product rule to obtain $\ldots\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\ldots x\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

Depends on the first method mark.

A1: Fully correct differentiation e.g. $-2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\frac{\mathrm{d}y}{\mathrm{d}x}$

ddM1: Rearranges to make $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ the subject. **Depends on both previous method marks.**

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 1: Makes $\frac{d^2y}{dx^2}$ the subject first and then quotient rule.

	$\frac{d^2y}{dx^2}=\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{\left(2-x^2\right)}$	
	$\frac{d^3y}{dx^3}=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$	M1**d**M1A1
	$\frac{d^3y}{dx^3}=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$ $=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times 2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$ $=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-10x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\frac{d^2y}{dx^2}}{\left(2-x^2\right)^2}$ $\frac{d^3y}{dx^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the quotient rule on $\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{\left(2-x^2\right)}$ Award for the form

$\frac{\left(2-x^2\right)\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'\pm\left(2-x^2\right)'\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)}{\left(2-x^2\right)^2}$ where $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'$ and $\left(2-x^2\right)'$ are attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{dy}{dx}\right)^2$ using the product rule to obtain $...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}$

dM1: Complete attempt at the quotient rule to obtain:

$$\frac{d^3y}{dx^3}=\frac{\left[...\frac{dy}{dx}+...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)\pm...x\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]}{\left(2-x^2\right)^2}$$

A1: Fully correct differentiation in any form.

ddM1: Replaces $3y$ with $\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2$ and cancels through by $\left(2-x^2\right)$

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 2: Makes $\frac{d^2y}{dx^2}$ the subject first and then product rule.

	$\frac{d^2y}{dx^2}=\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)}=\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)(2-x^2)^{-1}$	
	$\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)^{-1}+\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]\times-(2-x^2)^{-2}\times-2x$	M1**d**M1A1
	$\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)^{-1}+2x(2-x^2)^{-2}\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]$ $\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)^{-1}+2x(2-x^2)^{-2}\left[(2-x^2)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-5x\left(\frac{dy}{dx}\right)^2\right]$ $=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-10x\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)+2x(2-x^2)\frac{d^2y}{dx^2}}{(2-x^2)^2}$ $\frac{d^3y}{dx^3}=\frac{1}{(2-x^2)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the product rule on $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)(2-x^2)^{-1}$ Award for the form

$(2-x^2)^{-1}\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'+\left((2-x^2)^{-1}\right)'\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)$ where $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'$ and $\left((2-x^2)^{-1}\right)'$ are attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{dy}{dx}\right)^2$ using the product rule to obtain $...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}$

dM1: Complete attempt at the product rule to obtain:
$\frac{d^3y}{dx^3}=\left[...\frac{dy}{dx}+...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)^{-1}+...x(2-x^2)^{-2}\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]$

A1: Fully correct differentiation in any form.

ddM1: Replaces $3y$ with $(2-x^2)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2$ and cancels through by $(2-x^2)$ or equivalent work.

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 3: Makes $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ the subject first, splits and then uses the quotient rule on both terms.		
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{3y}{(2-x^2)}-\frac{5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{(2-x^2)}$	
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{3\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)(2-x^2)-3y(-2x)}{(2-x^2)^2}-\frac{\left[5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\right](2-x^2)-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2(-2x)}{(2-x^2)^2}$	M1**d**M1A1
	$=\frac{3\frac{\mathrm{d}y}{\mathrm{d}x}(2-x^2)-\left((2-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)(-2x)}{(2-x^2)^2}$ $-\frac{\left[5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right](2-x^2)-5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2(-2x)}{(2-x^2)^2}$ $=\frac{3\frac{\mathrm{d}y}{\mathrm{d}x}(2-x^2)+2x(2-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left[5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right](2-x^2)}{(2-x^2)^2}$ $=\frac{3\frac{\mathrm{d}y}{\mathrm{d}x}+2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\left[5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times2\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right]}{(2-x^2)}$ $=\frac{1}{(2-x^2)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the quotient rule on $\frac{3y}{(2-x^2)}$ **and** $\frac{5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{(2-x^2)}$

Award for the forms $\frac{(2-x^2)(3y)'\pm3y(2-x^2)'}{(2-x^2)^2}$ **and** $\frac{(2-x^2)\left(5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)'\pm(2-x^2)'\left(5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}{(2-x^2)^2}$

where $(3y)'$, $\left(5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)'$ and $(2-x^2)'$ are attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using the product rule to obtain $...\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+...x\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

dM1: Complete attempt at both quotient rules to obtain:

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{...\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)(2-x^2)\pm...xy}{(2-x^2)^2}-\frac{\left[...\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+...x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\right](2-x^2)\pm...x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{(2-x^2)^2}$$

A1: Fully correct differentiation in any form.

ddM1: Combines both fractions with a common denominator of $(2-x^2)^2$, replaces $3y$ with $(2-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ and cancels through by $(2-x^2)$

A1*: Obtains the printed answer from **fully correct** working.

Other methods e.g. rearranges, splits and uses 2 products can be marked in a similar way.
If you are unsure if a particular approach deserves credit use Review.

(b)	$$\frac{d^2y}{dx^2}=\frac{9}{2}$$	B1
	$$\frac{d^3y}{dx^3}=\frac{1}{2}\left(-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)=\frac{1}{2}\left(-5\times\frac{1}{16}+\frac{3}{4}\right)=\frac{7}{32}$$	M1
	$$(y=)3+\frac{1}{4}x+\frac{9}{2}\frac{x^2}{2!}+\frac{7}{32}\frac{x^3}{3!}$$	M1
	$$(y=)3+\frac{1}{4}x+\frac{9}{4}x^2+\frac{7}{192}x^3$$	A1
		(4)
		Total 9

Notes

B1: Correct value for $\frac{d^2y}{dx^2}$

M1: Attempts to find a value for $\frac{d^3y}{dx^3}$.

Working need not be seen/checked as long as a value is obtained and a value may just be written down.

M1: Correct application of Taylor's series using their values for the derivatives (accept 2! or 2 and 3! or 6)

A1: Correct simplified series. The "y =" is not required.

6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which

$y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

(Total for Question 6 is 13 marks)

Question Number	Scheme	Marks
6(a)	$m^2-6m+8=0\Rightarrow(m-2)(m-4)=0,\ m=2,4$	M1
	$(\text{CF}=)\quad A\text{e}^{2x}+B\text{e}^{4x}$	A1
	$\text{PI}:(\ y=)\alpha x^2+\beta x+\gamma$	B1
	$y'=2\alpha x+\beta \quad y''=2\alpha$ $2\alpha-6(2\alpha x+\beta)+8(\alpha x^2+\beta x+\gamma)=2x^2+x$	M1
	$8\alpha=2,\ -12\alpha+8\beta=1,\ 2\alpha-6\beta+8\gamma=0\Rightarrow\alpha=...,\ \beta=...,\ \gamma=...$	**d**M1
	$\alpha=\frac{1}{4},\ \beta=\frac{1}{2},\ \gamma=\frac{5}{16}$	A1A1
	$y=A\text{e}^{2x}+B\text{e}^{4x}+\frac{1}{4}x^2+\frac{1}{2}x+\frac{5}{16}$	A1ft
		(8)

Notes

M1: Attempts to solve $m^2-6m+8=0$ by any valid method including a calculator so may be implied by correct values or the correct CF

A1: Correct CF ("$y=$" not required)

B1: Correct form for the PI ("$y=$" not required)

M1: Differentiates their PI twice and attempts to substitute into the given differential equation. There must be evidence of $x^n\to x^{n-1}$ at least once.
The PI must be a quadratic expression with at least 2 terms and at least 2 unknowns.

dM1: Compares coefficients and solves to find values for their $\alpha,\ \beta$ and γ from a PI of $y=\alpha x^2+\beta x+\gamma,\ \alpha,\beta,\gamma\neq 0$.
You do not need to check their working and they may just write down values.
Depends on the previous M mark only.

A1: Any 2 values correct **following the award of both previous method marks.**

A1: All correct **following the award of both previous method marks.**

A1ft: A complete solution, follow through their CF and PI.
Must be $y=$... or $y=$... must be seen at some point for their GS e.g. they may start $y=$ PI + CF
All 3 M marks must have been earned.

6(b)	$y = Ae^{2x} + Be^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$	
	$1 = A + B + \frac{5}{16}$	M1
	$\frac{dy}{dx} = 2Ae^{2x} + 4Be^{4x} + \frac{1}{2}x + \frac{1}{2} \qquad 0 = 2A + 4B + \frac{1}{2}$	M1
	$A = \frac{13}{8} \quad B = -\frac{15}{16}$ oe	**dd**M1A1
	$y = \frac{13}{8}e^{2x} - \frac{15}{16}e^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ oe	A1ft
		(5)
		Total 13

Notes

M1: Substitutes $y = 1$ and $x = 0$ in their GS from (a) to obtain an equation in 2 unknowns.

M1: Differentiates their GS and substitutes $\frac{dy}{dx} = 0$, $x = 0$ to obtain another equation in 2 unknowns.
You do not need to be concerned about the detail of their differentiation provided there is an attempt and their derivative is not their original function.

ddM1: Solves their 2 equations in 2 unknowns to obtain a value for both constants. You do not need to check their working and they may just write down values.
Depends on both previous M marks.

A1: Both values correct from a correct GS and fully correct work.

A1ft: Correct particular solution. Follow through their A and B and their PI from (a).
Must be y =... or y =... must be seen at some point for their GS but do not penalise its omission if already penalised in part (a).
Depends on all 3 previous M marks.

7. (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}\,(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,\mathrm{d}\theta$$

(4)

(Total for Question 7 is 14 marks)

Question Number	Scheme	Marks
7(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	A1
		(3)
(a) Alternative		
	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3},\ \left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$	M1A1
	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^3+3z+\frac{3}{z}+\frac{1}{z^3}\right)\left(z^3-3z+\frac{3}{z}-\frac{1}{z^3}\right)=...$ $=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	A1

(a) Notes

M1: Writes $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$ as $\left(z^2-\frac{1}{z^2}\right)^3$ and expands to obtain at least 3 terms of the correct form e.g. 3 from $...z^6,\ ...z^2,\ ...z^{-2},\ ...z^{-6}$

A1: For $\left(z^2-\frac{1}{z^2}\right)^3=z^6-3z^2+\frac{3}{z^2}-z^{-6}$

A1: $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$ Condone $z^6-z^{-6}-3\left(z^2-z^{-2}\right)$

Correct answer with no errors seen. The "$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=$" does not need to be seen.

Alternative:

M1: Attempt to expand both cubic brackets to obtain at least 3 terms of the correct form in both cases e.g. 3 from $...z^3,\ ...z,\ ...z^{-1},\ ...z^{-3}$

A1: Both expansions correct.

A1: Expands, collects terms and obtains the correct answer with no errors seen.

Condone $z^6-z^{-6}-3\left(z^2-z^{-2}\right)$ for $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$

The "$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=$" does not need to be seen.

Question Number	Scheme	Marks
(b)(i)(ii)	$z^n=\cos n\theta+\mathrm{i}\sin n\theta$	B1
	$z^{-n}=\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$ or $=\cos n\theta-\mathrm{i}\sin n\theta$	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta$*, $z^n-\frac{1}{z^n}=2\mathrm{i}\sin n\theta$*	A1*
		(3)

Notes

B1: $z^n=\cos n\theta+\mathrm{i}\sin n\theta$ seen anywhere.

M1: States or uses $z^{-n}=\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$ (with or without the brackets) or $z^{-n}=\cos n\theta-\mathrm{i}\sin n\theta$

A1*: Both correct results obtained following sight of $z^{-n}=\cos n\theta-\mathrm{i}\sin n\theta$

(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2\mathrm{i}\sin\theta)^3$ oe e.g. $-2^6\mathrm{i}\cos^3\theta\sin^3\theta$	B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ oe	B1ft
	$(2\cos\theta)^3(2\mathrm{i}\sin\theta)^3=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ $\Rightarrow 8\cos^3\theta\times(-8\mathrm{i}\sin^3\theta)=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ $-64\mathrm{i}\sin^3\theta\cos^3\theta=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$	**d**M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *	A1*
		(4)

(c) Notes

B1: Uses $z+\frac{1}{z}=2\cos\theta$ and $z-\frac{1}{z}=2\mathrm{i}\sin\theta$ to express $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$ correctly in terms of $\sin\theta$ and $\cos\theta$. May be implied by e.g. lhs = $\left(\frac{1}{2}\left(z+\frac{1}{z}\right)\right)^3\left(\frac{1}{2\mathrm{i}}\left(z-\frac{1}{z}\right)\right)^3$.

May be seen in a rearranged form e.g. $\frac{-1}{64\mathrm{i}}\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\cos^3\theta\sin^3\theta$

B1ft: Uses $z^6-\frac{1}{z^6}=2\mathrm{i}\sin 6\theta$ and $z^2-\frac{1}{z^2}=2\mathrm{i}\sin 2\theta$ to express their $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$ correctly in terms of $\sin\theta$ and $\cos\theta$. Follow through their *k* from part (a).

dM1: Equates both expressions and attempts to simplify $2^3\times(2\mathrm{i})^3$ to obtain $\alpha\mathrm{i}$ which may be implied. **Depends on both B marks.**

A1*: Reaches the printed answer with no errors.

(d)	$$\int \cos^3\theta\sin^3\theta\,d\theta = \int \frac{1}{32}(3\sin 2\theta - \sin 6\theta)\,d\theta$$ $$= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]$$	M1A1
	$$= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$$ $$= \frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{3}{2} + \frac{1}{6}\right)\right] = \frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$$	**d**M1A1
		(4)
		Total 14

(d) Notes

M1: Substitutes and integrates to obtain $p\cos 2\theta + q\cos 6\theta$

A1: Fully correct integration including the $\frac{1}{32}$

dM1: Substitutes **both** the given limits and subtracts the correct way round.
Do not allow if working in decimals.

There must be some evidence of both trigonometric terms evaluated with $\theta = \frac{\pi}{8}$ to give an expression containing $\sqrt{2}$ and both trigonometric terms evaluated with $\theta = 0$ to give a rational number.

A1: Correct exact value with terms collected e.g. $\frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$ or any exact equivalent e.g. $\frac{1}{24} - \frac{5\sqrt{2}}{192}$

8.

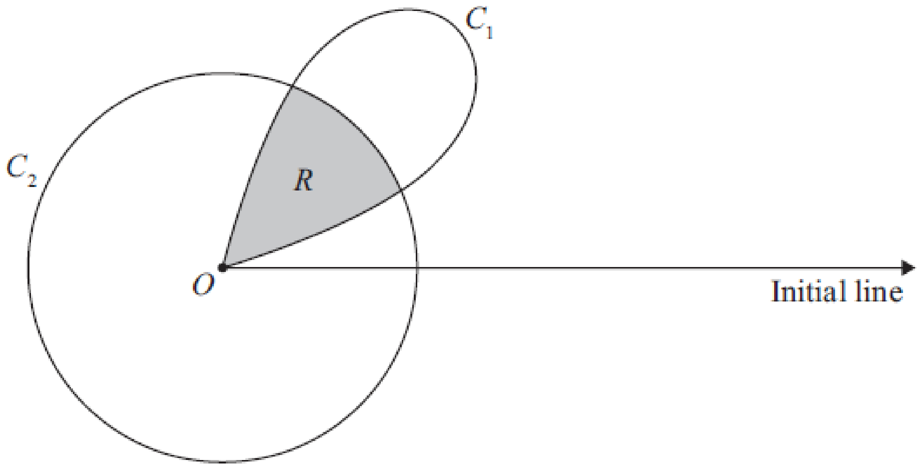

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows the curve C_1 with polar equation $r = 2a\sin 2\theta,\ 0 \leqslant \theta \leqslant \frac{\pi}{2}$, and the circle C_2 with polar equation $r = a,\ 0 \leqslant \theta \leqslant 2\pi$, where a is a positive constant.

(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2

(3)

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region R is shaded in Figure 1.

(b) Use algebraic integration to find the area of the shaded region R, giving your answer in the form $\frac{1}{12}a^2\left(p\pi + q\sqrt{3}\right)$, where p and q are integers.

(7)

(Total for Question 8 is 10 marks)

Question Number	Scheme	Marks
8(a)	$a = 2a\sin 2\theta \Rightarrow \sin 2\theta = \dfrac{1}{2} \Rightarrow 2\theta = ...$	M1
	$\sin 2\theta = \dfrac{1}{2} \Rightarrow 2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$	A1
	$\left(a, \dfrac{\pi}{12}\right), \left(a, \dfrac{5\pi}{12}\right)$	A1
		(3)

(a) Notes

M1: Sets $a = 2a\sin 2\theta$ and attempts to solve for 2θ

A1: Obtains $2\theta = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ which may be implied.

A1: Both points correct. Can be written as e.g. $r = a$, $\theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

Apply isw once the correct r and θ are seen for both points and condone coordinates written the wrong way round.

(b)	$\frac{1}{2}\times a^2\times\frac{\pi}{3}$ oe	B1
	$\left(\frac{1}{2}\right)\int r^2\,(\mathrm{d}\theta)=\left(\frac{1}{2}\right)\int(2a\sin 2\theta)^2\,(\mathrm{d}\theta)$	M1
	$\cos 4\theta=1-2\sin^2 2\theta\Rightarrow\sin^2 2\theta=\frac{1}{2}(1-\cos 4\theta)$	M1
	$\int(1-\cos 4\theta)(\mathrm{d}\theta)=\theta-\frac{1}{4}\sin 4\theta$ **or** $2\int(1-\cos 4\theta)(\mathrm{d}\theta)=2\theta-\frac{1}{2}\sin 4\theta$	A1
	$I=a^2\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{12}}=a^2\left\{\left(\frac{\pi}{12}-\frac{1}{4}\sin 4.\frac{\pi}{12}\right)-(0)\right\}$	**dd**M1
	$R=2I+\frac{a^2\pi}{6}=2a^2\left(\frac{\pi}{12}-\frac{\sqrt{3}}{8}\right)+\frac{a^2\pi}{6}$	**d**M1
	$R=\frac{1}{12}a^2\left(4\pi-3\sqrt{3}\right)$	A1
		(7)
		Total 10

(b) Notes

B1: Any correct expression for the sector area in any form.

May be seen from an **evaluated** integral e.g. $\frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} a^2\mathrm{d}\theta=\frac{1}{2}\left[a^2\theta\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}=\underline{\frac{1}{2}a^2\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)}$

M1: Attempts to use $\int r^2\mathrm{d}\theta$ with $r=2a\sin 2\theta$. Condone the omission of "dθ"

Limits not needed (ignore any shown) and the $\frac{1}{2}$ may be missing.

M1: Uses $\sin^2 2\theta=\frac{\pm1\pm\cos 4\theta}{2}$ in an attempt to reach an integrable form.

Allow equivalent work e.g. uses $\sin^2 2\theta=1-\cos^2 2\theta$ and then $\cos^2 2\theta=\frac{\pm1\pm\cos 4\theta}{2}$.

A1: Fully correct integration. Condone the omission of "dθ" earlier and condone the omission of the a^2. Limits not needed (ignore any shown) and the $\frac{1}{2}$ may be missing.

ddM1: An attempt to find one or both of the regions either side of the sector.
ie uses limits 0 and their $\frac{\pi}{12}$ and/or their $\frac{5\pi}{12}$ and $\frac{\pi}{2}$ both limits to be substituted and subtracted (if non-zero after substitution).
Limits to be used the correct way round. If two integrals seen award mark if either correct.
Depends on both previous method marks.

dM1: Fully correct strategy for the complete area (their sector + 2*I* or their sector + *I* if the $\frac{1}{2}$ was missing throughout.) All areas must be positive.
Depends on all the previous method marks.

A1: Correct expression in the required form.