Pearson

Mark Scheme (Results)

January 2026

Pearson Edexcel International Advanced Level in
Futher Pure Mathematics F3
WFM03/01

Edexcel and BTEC Qualifications

January 2026
Question Paper Log Number P78989A
Publication Code WFM03_01_2601_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - * – The answer is printed on the paper or ag- answer given

- ⎿ or d… – The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer

Question	Scheme	Marks
1 (i)+(ii)	$a^2 = 48$ or $ae = 8$ (oe)	B1
	E.g. $b^2 = (ae)^2 - a^2 = "8"^2 - "48" = \ldots$ or $(ae)^2 = 64 \Rightarrow e^2 = \frac{"64"}{a^2} = \frac{"64"}{"48"} = \ldots$ or $e = \frac{8}{a} = \cdots$	M1
	$b = 4$ or $e = \frac{2\sqrt{3}}{3}$	A1
	e.g. $b^2 = a^2\left(e^2 - 1\right) \Rightarrow e^2 = \frac{"16"}{48} + 1 \Rightarrow e = \ldots$ or $b^2 = "64"\left("\frac{64}{48}" - 1\right) = \ldots \Rightarrow b = \ldots$	M1
	$b = 4$ and $e = \frac{2\sqrt{3}}{3}$	A1
		(5)
		(5 marks)

Notes:

(i)+(ii) Mark as a whole

B1: States or implies $ae = 8$ or $a^2 = 48$ (oe)

M1: Complete method to find either b or e or their squares. E.g. uses $b^2 = a^2\left(e^2 - 1\right)$ and expands and uses their ae and a^2 (which may be incorrect) to find b^2. Alternatively, may use $(ae)^2 = 64 \Rightarrow e^2 = \frac{64}{a^2} = \frac{64}{48}$ to get to e^2 first.

A1: For either $b = 4$ or $e = \frac{2\sqrt{3}}{3}$.

M1: Complete method to find the other unknown. E.g. uses $b^2 = a^2\left(e^2 - 1\right)$ with their a and b to find e or with their a and e to find b.

A1: cao. Both values correct. Accept $\frac{2}{\sqrt{3}}$ for e. Must reject the negative values.

Question	Scheme	Marks
2(a)	$\det \mathbf{A} = k\times(2\times4-3\times3)-0-0(=-k)$	B1
		(1)
(b)	Matrix of minors: $(\mathbf{M}) = \begin{pmatrix} 8-9 & 16-3 & 12-2 \\ 0 & 4k & 3k \\ 0 & 3k & 2k \end{pmatrix}$	M1
	Adj: $(\mathbf{C}^{\mathrm{T}}) = \begin{pmatrix} -1 & 0 & 0 \\ -13 & 4k & -3k \\ 10 & -3k & 2k \end{pmatrix}$	M1
	$\mathbf{A}^{-1} = \frac{1}{-k}\begin{pmatrix} -1 & 0 & 0 \\ -13 & 4k & -3k \\ 10 & -3k & 2k \end{pmatrix} = \frac{1}{k}\begin{pmatrix} 1 & 0 & 0 \\ 13 & -4k & 3k \\ -10 & 3k & -2k \end{pmatrix} = \begin{pmatrix} \frac{1}{k} & 0 & 0 \\ \frac{13}{k} & -4 & 3 \\ \frac{-10}{k} & 3 & -2 \end{pmatrix}$	A1
		(3)
(c)	$\mathbf{AB} = \mathbf{A}^{\mathrm{T}} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}\mathbf{A}^{\mathrm{T}} = \begin{pmatrix} 1 & 0 & 0 \\ 13 & -4 & 3 \\ -10 & 3 & -2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 1 \\ 0 & 2 & 3 \\ 0 & 3 & 4 \end{pmatrix} = \ldots$	M1
	$= \begin{pmatrix} 1 & 4 & 1 \\ 13 & 53 & 13 \\ -10 & -40 & -9 \end{pmatrix}$	A1A1
		(3)
		(7 marks)

Notes:

(a)
B1: Correct expression for the determinant. It does not need to be simplified for this mark.
(b)
M1: Attempts the matrix of minors with at least 4 correct.
M1: Applies the cofactors and takes the transpose (the determinant need not be introduced yet).
A1: Correct inverse, accept with the determinant outside or inside. (Any expression in the scheme is acceptable.)
(c)
M1: Correctly multiplies across by $\mathbf{A}^{-1}$, applies the transpose and multiplies.
A1: At least 5 correct entries
A1: Fully correct answer.
Alt
M1: Attempts $\mathbf{AB} = \mathbf{A}^{\mathrm{T}}$ after applying the transpose, multiplies the LHS and equates elements
A1: At least 5 correct elements
A1: All elements correct

Question	Scheme	Marks
3a	$\sinh x + \operatorname{sech} x = \dfrac{e^x - e^{-x}}{2} + \dfrac{2}{e^x + e^{-x}}$	B1
	$= \dfrac{(e^x - e^{-x})(e^x + e^{-x}) + 4}{2(e^x + e^{-x})}$	M1
	$= \dfrac{e^{2x} - e^{-2x} + 4}{2e^x + 2e^{-x}} \times \dfrac{e^{2x}}{e^{2x}} = \cdots$	M1
	$= \dfrac{e^{4x} + 4e^{2x} - 1}{2e^{3x} + 2e^x}$ *	A1*
		(4)
3b	$\sinh x + \operatorname{sech} x = \dfrac{1}{2}e^x \Rightarrow \dfrac{e^{4x} + 4e^{2x} - 1}{2e^{3x} + 2e^x} = \dfrac{1}{2}e^x \Rightarrow \cancel{e^{4x}} + 4e^{2x} - 1 = \cancel{e^{4x}} + e^{2x}$	M1
	$3e^{2x} = 1$	A1
	$\Rightarrow x = \dfrac{1}{2}\ln\dfrac{1}{3}$	M1
	$= -\dfrac{1}{2}\ln 3$	A1
		(4)
		(8 marks)

Notes:

(a)
B1: Correct expression in terms of exponentials given.
M1: Combines the terms over a common denominator.
M1: Expands and simplifies to a point where positive indices for the exponentials are evident.
A1*: Achieves the given answer with no incorrect work seen, and a step with the brackets all expanded before the final answer. Allow e.g. $e^{2x} - e^{-2x}$ for the expanded brackets.

Note: There may be variations using other identities, e.g. $\dfrac{\sinh x \cosh x + 1}{\cosh x} = \dfrac{\sinh 2x + 2}{2\cosh x}$ before applying identities. In such case the scheme can be followed – B1 for use of correct exponential identities when they are applied, M1 for combining the fraction (scored before the B mark here), then M1 for simplifying to positive indices in the exponents, A1 if all correct.

(b)
M1: Applies the result of (a) and cross multiplies to achieve an equation in e^{2x} only.
A1: Correct equation as shown or $e^{2x} = \dfrac{1}{3}$
M1: Solves the exponential equation of form $ae^{kx} = b$ to find a value for x
A1: cao.

Question	Scheme	Marks
4(a)	$\mathbf{b}\times\mathbf{c}=(\mathbf{i}+2\mathbf{j}-2\mathbf{k})\times(3\mathbf{i}+\mathbf{j}+\mathbf{k})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & -2\\ 3 & 1 & 1\end{vmatrix}$ $=(2\times1-1\times-2)\mathbf{i}\pm(1\times1-3\times-2)\mathbf{j}+(1\times1-3\times2)\mathbf{k}$	M1
	$\mathbf{b}\times\mathbf{c}=4\mathbf{i}-7\mathbf{j}-5\mathbf{k}$	A1
		(2)
(b)	$\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})=(2\mathbf{i}-\mathbf{k})\bullet(4\mathbf{i}-7\mathbf{j}-5\mathbf{k})=2\times4+0\times-7+(-1)\times(-5)=...$	M1
	$=13$	A1
		(2)
(c)	Volume $OABC=\frac{1}{6}\lvert\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})\rvert=\frac{13}{6}$	B1ft
		(1)
(d) Way 1	Area triangle $OBC=\frac{1}{2}\lvert\mathbf{b}\times\mathbf{c}\rvert=\frac{1}{2}\sqrt{4^2+7^2+5^2}=...$	M1
	Vol $OABC=\frac{1}{3}h\times$ Area $OBC\Rightarrow\frac{1}{3}h\times\text{"}\frac{1}{2}\sqrt{90}\text{"}=\frac{13}{6}\Rightarrow h=...$	M1
	$=\frac{13}{3\sqrt{10}}$ or $\frac{13\sqrt{10}}{30}$	A1
		(3)
Way 2	Plane OBC is $\mathbf{r}\bullet\text{"}4\mathbf{i}-7\mathbf{j}-5\mathbf{k}\text{"}=0$ (using the origin as a point on the plane)	M1
	$d=\frac{\lvert4\times2-7\times0-5\times-1+0\rvert}{\sqrt{4^2+7^2+5^2}}=...$ oe	M1
	$=\frac{13}{3\sqrt{10}}$ or $\frac{13\sqrt{10}}{30}$	A1
		(3)
Way 3	Finding e.g. $\pm(\mathbf{c}-\mathbf{a})$ or $\pm(\mathbf{b}-\mathbf{a})$	M1
	Resolving along the direction of the normal e.g. $d=\left\lvert\frac{\pm(\mathbf{c}-\mathbf{a})\bullet\begin{pmatrix}4\\-7\\-5\end{pmatrix}}{\sqrt{4^2+7^2+5^2}}\right\rvert$ or $d=\left\lvert\frac{\pm(\mathbf{b}-\mathbf{a})\bullet\begin{pmatrix}4\\-7\\-5\end{pmatrix}}{\sqrt{4^2+7^2+5^2}}\right\rvert$	M1
	$=\frac{13}{3\sqrt{10}}$ or $\frac{13\sqrt{10}}{30}$	A1

		(3)

(8 marks)
Notes:

Accept equivalent vector notation, e.g. column vectors, throughout.

(a)

M1: Attempts the vector product with at least two components correct (simplified or unsimplified).

A1: Correct vector

(b)

M1: Takes the scalar product of **a** with their answer to (a). Obtaining a vector is M0.

A1: Correct value.

(c)

B1ft: For $\frac{1}{6}\times$|their answer to (b)|

(d)

Way 1

M1: Correct method for the area of the triangle *OBC*.

M1: Sets up and solves a correct equation for the shortest distance using the formula $V=\frac{1}{3}A_b h$ with their area and their volume from (c).

A1: Correct answer. Accept equivalents such as $\frac{13}{\sqrt{90}}$ or $\frac{13\sqrt{90}}{90}$

Way 2

M1: Identifies or implies the equation of the plane containing *OBC* (following their (a)).

M1: Uses the perpendicular distance formula with their plane and point *A*.

A1: Correct answer. Accept equivalents such as $\frac{13}{\sqrt{90}}$ or $\frac{13\sqrt{90}}{90}$

Way 3

M1: Finds the directional vector $\pm(\mathbf{c}-\mathbf{a})$ or $\pm(\mathbf{b}-\mathbf{a})$

M1: Resolves along the direction of the normal vector to the plane

A1: Correct answer. Accept equivalents such as $\frac{13}{\sqrt{90}}$ or $\frac{13\sqrt{90}}{90}$

Question	Scheme		Marks
5(a)	$\frac{dx}{dt} = 2\sinh 2t$ and $\frac{dy}{dt} = 4\cosh t$		B1
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4\sinh^2 2t + 16\cosh^2 t$		M1
	$= 4(2\sinh t\cosh t)^2 + 16\cosh^2 t$	$= 4(\cosh^2 2t - 1) + 16\cosh^2 t$ $= 4\left(2\cosh^2 t - 1\right)^2 - 4 + 16\cosh^2 t$	M1
	$= 16\cosh^2 t\left(\sinh^2 t + 1\right)$	$= 16\cosh^4 t - 16\cosh^2 t + 4 - 4 + 16\cosh^2 t$	M1
	$= 16\cosh^4 t$		A1
			(5)
(b)	$s = \int_0^{\frac{3}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\frac{3}{2}} 4\cosh^2 t\, dt = \int_0^{\frac{3}{2}} 2\left(1 + \cosh 2t\right) dt$		M1
	$= \left[2t + \sinh 2t\right]_0^{\frac{3}{2}} = \left(3 + \sinh 3\right) - \left(0 + 0\right)$		dM1
	$= 3 + \sinh 3$		A1
			(3)
(c)	$S_x = 2\pi \int_0^{\frac{3}{2}} y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 2\pi \int_0^{\frac{3}{2}} 4\sinh t \times 4\cosh^2 t\, dt$		B1
	$= 32\pi\left[\frac{1}{3}\cosh^3 t\right]_0^{\frac{3}{2}}$		M1
	$= \frac{32\pi}{3}\left[\left(\cosh^3 \frac{3}{2}\right) - \left(\cosh^3 0\right)\right]$		dM1
	$= \frac{32\pi}{3}\left(\cosh^3 \frac{3}{2} - 1\right)$		A1
			(4)
			(12 marks)

Notes:

(a)
B1: Correct derivatives.
M1: Square and adds their two derivatives (allow if coefficients not squared correctly).
M1: Applies the double argument formula for hyperbolics.
M1: Factors out the cosh *t* terms and applies Pythagorean identity for hyperbolics.
A1: Correct answer from correct work.
(b)

M1: Applies the arc length formula with the square root of their (a) and applies the double argument formula $\cosh^2 t = \pm\frac{1}{2} \pm \frac{1}{2}\cosh 2t$ to reach integrable form.

dM1: Integrates and substitutes the limits (allow bottom limit 0 implied). **Dependent on the previous method mark**.

A1: Correct answer.

Mixing up variables e.g. *x* and *t* etc loses final A mark if not recovered.

(c)

B1: States or applies the surface area formula with the square root of their (a) and *y* substituted, including the 2π but limits not needed for this mark.

M1: Integrates to the correct form (limits not needed for this mark). Alternatives may be possible here.

dM1: Substitutes the limits into their integral. **Dependent on the previous method mark.**

A1: Correct exact answer, accepting equivalents. Numerical equivalent 402.72…is A0.

Alt 1 for (c):

$$S_x = 2\pi\int_0^{\frac{3}{2}} y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^3 t\,dt = 32\pi\int_0^{\frac{3}{2}} \sinh t + \frac{1}{4}(\sinh 3t - 3\sinh t)\,dt$$

$$= 32\pi\left[\cosh t + \frac{1}{4}\left(\frac{1}{3}\cosh 3t - 3\cosh t\right)\right]_0^{\frac{3}{2}} = 32\pi\left(\frac{1}{4}\cosh\frac{3}{2} + \frac{1}{12}\cosh\frac{9}{2} - \frac{1}{4} - \frac{1}{12}\right)$$

$$= \frac{8\pi}{3}\left(3\cosh\frac{3}{2} + \cosh\frac{9}{2} - 4\right)$$

Alt 2 for (c)

$$S_x = 2\pi\int_0^{\frac{3}{2}} y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^3 t\,dt = 32\pi\int_0^{\frac{3}{2}} \sinh t + \sinh^2 \text{t sinhtd}t$$

$$= 32\pi\left[\int_0^{\frac{3}{2}} \sinh t + \int_0^{\frac{3}{2}} \cosh^2 \text{t sinhtd}t - \int_0^{\frac{3}{2}} \sinh t\right] = 32\pi\left[\frac{1}{3}\cosh^3 t\right]_0^{\frac{3}{2}}$$

$$== \frac{32\pi}{3}\left(\cosh^3\frac{3}{2} - 1\right)$$

Question	Scheme	Marks
6(a)	$\begin{pmatrix}-2 & 2 & 2\\-3 & -7 & -3\\1 & 1 & -3\end{pmatrix}\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}-2-2\\-3+3\\1+3\end{pmatrix}=\ldots$	M1
	$=\begin{pmatrix}-4\\0\\4\end{pmatrix}=-4\times\begin{pmatrix}1\\0\\-1\end{pmatrix}$ has an eigenvector	A1
	Eigenvalue is -4	B1
		(3)
(b)	$\begin{vmatrix}-2-\lambda & 2 & 2\\-3 & -7-\lambda & -3\\1 & 1 & -3-\lambda\end{vmatrix}=(-2-\lambda)\begin{vmatrix}-7-\lambda & -3\\1 & -3-\lambda\end{vmatrix}-2\begin{vmatrix}-3 & -3\\1 & -3-\lambda\end{vmatrix}+2\begin{vmatrix}-3 & -7-\lambda\\1 & 1\end{vmatrix}=$ $=(-2-\lambda)((-7-\lambda)(-3-\lambda)+3)-2(-3(-3-\lambda)+3)+2(-3-(-7-\lambda))$	M1A1
	$0=(-2-\lambda)(24+10\lambda+\lambda^2)-2(2\lambda+8)$ $=-\lambda^3-12\lambda^2-48\lambda-64$	M1
	$\Rightarrow-(\lambda+4)(\lambda^2+8\lambda+16)=-(\lambda+4)^3=0\Rightarrow\lambda=-4$ only	A1
		(4)
(c)	$\begin{pmatrix}-2 & 2 & 2\\-3 & -7 & -3\\1 & 1 & -3\end{pmatrix}\begin{pmatrix}1+t\\2t-1\\-3t\end{pmatrix}=\ldots$	M1
	$=\begin{pmatrix}-2-2t+4t-2-6t\\-3-3t-14t+7+9t\\1+t+2t-1+9t\end{pmatrix}=\begin{pmatrix}-4-4t\\4-8t\\12t\end{pmatrix}$	A1
	Image line is $\dfrac{x+4}{-4}=\dfrac{y-4}{-8}=\dfrac{z}{12}$	M1A1
		(4)
		(11 marks)

Notes:

(a)

M1: Multiplies the matrix by the given vector, at least two correct entries.

A1: Correct resulting vector and shows it is a multiple of the original vector.

B1: Correct eigenvalue identified. Allow simply $\lambda=-4$

(b)

Note: allow the marks for (b) to score if seen in part (a)

M1: Expands $\det(\mathbf{M}-\lambda\mathbf{I})$ fully to a cubic (need not be expanded).

A1: Fully correct expansion (need not be simplified).

M1: Sets $\det(\mathbf{M}-\lambda\mathbf{I}) = 0$ and simplifies to the cubic. May identify a factor of $\lambda+4$ in each term – allow the M for identifying this. $0=(-2-\lambda)(\lambda+4)(\lambda+6)-4(\lambda+4)=(\lambda+4)(-\lambda^2-8\lambda-16)$

A1: Reaches the cubed form and deduces only one eigenvalue. Must show the result by either fully factorising the correct cubic, or factorising the correct quadratic after the correct cubic is seen.

(c)

M1: Attempts the parametric form of the line and multiplies by the matrix **M**.

A1: Correct result of the multiplication.

M1: Forms the Cartesian equation for their resulting vector.

A1: Correct answer (oe - e.g. $6x+24=3y-12=-2z$).

Question	Scheme	Marks
7(a)	$I_n = \left[\frac{1}{4}e^{4x}\tan^n x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{n}{4}e^{4x}\tan^{n-1} x\sec^2 x\, dx$	B1
	$= \left[\frac{1}{4}e^{4x}\tan^n x\right]_0^{\frac{\pi}{4}} - \frac{n}{4}\int_0^{\frac{\pi}{4}} e^{4x}\tan^{n-1} x\left(1+\tan^2 x\right) dx$	M1
	$= \left(\frac{1}{4}e^{\pi}\right) - (0) - \frac{n}{4}\left(I_{n-2} + I_{n+1}\right)$	M1
	$\Rightarrow \frac{n}{4}I_{n+1} = \frac{1}{4}e^{\pi} - \frac{n}{4}I_{n-1} - I_n \Rightarrow I_{n+1} = \frac{e^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	A1
		(4)
Alt 1:	$I_{n+1} = \int_0^{\frac{\pi}{4}} e^{4x}\tan^{n-1} x\tan^2 x\, dx = \int_0^{\frac{\pi}{4}} e^{4x}\tan^{n-1} x\left(\sec^2 x - 1\right) dx$ **M1** $= \left[e^{4x}\frac{\tan^n x}{n}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 4e^{4x}\frac{\tan^n x}{n} dx - I_{n-1}$ **B1**	B1 M1
	$= \left(\frac{e^{\pi}}{n}\right) - (0) - \frac{4}{n}I_n - I_{n-1}$	M1
	$\Rightarrow I_{n+1} = \frac{e^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	A1
		(4)
Alt 2:	$I_{n+1} = \int_0^{\frac{\pi}{4}} e^{4x}\tan^{n+1} x\, dx$ $= \left[\frac{1}{4}e^{4x}\tan^{n+1} dx\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{4}e^{4x}(n+1)\tan^n x\sec^2 x\, dx$	B1
	$= \left[\frac{1}{4}e^{4x}\tan^{n+1} x\right]_0^{\frac{\pi}{4}} - \frac{n+1}{4}\int_0^{\frac{\pi}{4}} e^{4x}\tan^n x\left(1+\tan^2 x\right) dx$	M1
	$= \left(\frac{1}{4}e^{\pi}\right) - (0) - \frac{n+1}{4}\left(I_n + I_{n+2}\right)$	M1
	$\Rightarrow I_{n+2} = \frac{e^{\pi}}{n+1} - \frac{4}{n+1}I_{n+1} - I_n$ Replacing e.g. $'n = n-1' \Rightarrow I_{n+1} = \frac{e^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	A1

Alt 3	$I_n = \int_0^{\frac{\pi}{4}} e^{4x}\tan^{n-2}x\tan^2 x\,dx = \int_0^{\frac{\pi}{4}} e^{4x}\tan^{n-2}x\left(\sec^2 x - 1\right)dx$ **M1** $= \underline{\left[e^{4x}\frac{\tan^{n-1}x}{n-1}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 4e^{4x}\frac{\tan^{n-1}x}{n-1}\,dx} - I_{n-2}$ **<u>B1</u>**	**B1** **M1**
	$I_n = \left(\frac{e^{\pi}}{n-1}\right) - (0) - \frac{4}{n-1}I_{n-1} - I_{n-2}$	**M1**
	$\Rightarrow I_{n+1} = \frac{e^{\pi}}{n} - \frac{4}{n}I_n - I_{n-1}$	**A1**
(b)	$I_0 = \int_0^{\frac{\pi}{4}} e^{4x}\,dx = \left[\frac{1}{4}e^{4x}\right]_0^{\frac{\pi}{4}} = ...$	**M1**
	$= \frac{1}{4}e^{\pi} - \frac{1}{4}$	**A1**
		(2)
(c)	$I_2 = \frac{e^{\pi}}{1} - \frac{"4"}{1}\times 3.7002 - \frac{1}{4}\left[e^{\pi} - 1\right] \quad (= 2.8047...)$	**M1**
	$I_3 = \frac{e^{\pi}}{2} - \frac{"4"}{2}\times "2.8047" - 3.7002 \quad (= 2.2607...)$ $I_4 = \frac{e^{\pi}}{3} - \frac{"4"}{3}\times "2.2607" - "2.8047" = ...$	**M1**
	$= 1.8945... = 1.89$	**A1**
		(3)
		(9 marks)

Notes:

(a)

B1: Correct application of integration by parts on the given integral (need not be simplified).

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the integral.

M1: Evaluates the definite part of the integral and replaces appropriately by I_{n-1} and I_{n+1} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted. Condone missing arguments etc if recovered, but if not recovered then final A0.

<u>Alt 1:</u>

B1: Correct application of integration by parts on a suitably adapted integral (need not be simplified). Awarded after the M1 in this approach but score as the first mark.

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the original integral. (First mark awarded in this approach but score as the second mark.)

M1: Evaluates the definite part of the integral and replaces appropriately by I_{n-1} and I_{n+1} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted.

Variations on these methods may be seen but can be scored via this scheme, but the final A will need indices correctly adjusted if they work from different index at the start of the appropriate method.

<u>Alt 2:</u>

B1: Correct application of integration by parts on the given integral (need not be simplified).

M1: Applies $\sec^2 x = \pm 1 \pm \tan^2 x$ to the integral.

M1: Evaluates the definite part of the integral and replaces appropriately by I_n and I_{n+2} in the integral.

A1: Achieves the correct answer from completely correct work with no errors seen and the I_{n-1} and I_{n+1} correctly established and substituted. If I_{n+2} is correctly found, then accept a change of variable to obtain the correct reduction formula e.g. letting $'n = n-1'$

(b)

M1: Integrates to ke^{4x}, $k = \pm 4$ or $\pm\frac{1}{4}$ and substitutes the limits.

A1: Correct answer.

(c)

M1: Applies the reduction formula at least once with their I_0 and the given value for I_1 and $n = 1$.

M1: Proceeds to apply twice more to reach a value for I_4. Evidence of the use of (a) must be clear.

A1: Correct answer.

Question	Scheme	Marks
8(a)	$\frac{2x}{16}+\frac{2y}{9}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{3\cos\theta}{-4\sin\theta}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{9}{2}\left(1-\frac{x^2}{16}\right)^{-\frac{1}{2}}\times-\frac{2x}{16}$	B1
	$m_N=-\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{4\sin\theta}{3\cos\theta}\left(=\left(\frac{-2(4\cos\theta)\times 9}{16\times 2(3\sin\theta)}\right)^{-1}\right)$	M1
	$y-3\sin\theta=\text{"}\frac{4\sin\theta}{3\cos\theta}\text{"}(x-4\cos\theta)$	M1
	$3y\cos\theta-9\sin\theta\cos\theta=4x\sin\theta-16\sin\theta\cos\theta$ $\Rightarrow 4x\sin\theta-3y\cos\theta=7\sin\theta\cos\theta$	A1*
		(4)
(b)	$x_A=\frac{7\cos\theta}{4}$	B1
	Area $OAP=\frac{1}{2}x_A\times y_P=\frac{1}{2}\times\text{"}\frac{7}{4}\cos\theta\text{"}\times 3\sin\theta$	M1
	$=\frac{21}{8}\sin\theta\cos\theta=\frac{21}{8}\times\frac{1}{2}\sin 2\theta$	M1
	$=\frac{21}{16}\sin 2\theta$	A1
		(4)
(c)	$\frac{21}{16}$	B1ft
		(1)
		(9 marks)

Notes:

(a)

B1: Any correct differential statement for the ellipse given.

M1: Substitutes P and applies the negative reciprocal to find the gradient of the normal in terms of θ

M1: Attempts the equation of the normal using their attempt at the normal gradient – any adapted $\frac{\mathrm{d}y}{\mathrm{d}x}$ or equivalent method. If using the '$y=mx+c$' method then they must make progress to finding the 'c'.

A1*: Multiplies by the "$4\cos\theta$" to get rid of the fractions in the equation and correctly reaches the given equation with derivative having been seen and no errors in the working.

(b)

B1: States or uses the correct x coordinate at point A.

M1: Correct method for the area of triangle OAP.

M1: Applies the double angle identity for sin to reach the required form.

A1: Correct answer.

(c)

B1ft: Identifies their value of k as the maximum value for the area.

Question	Scheme	Marks
9(a)	$\dfrac{dy}{dx}=\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}\times\ldots$	M1
	$\dfrac{dy}{dx}=\ldots\times\dfrac{1\times\sqrt{1+x^2}-x\times\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}\times2x}{\left(\sqrt{1+x^2}\right)^2}\left(=\dfrac{1}{\left(1+x^2\right)^{\frac{3}{2}}}\right)$	M1
	$\dfrac{dy}{dx}=\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}\times\dfrac{1\times\sqrt{1+x^2}-x\times\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}\times2x}{\left(\sqrt{1+x^2}\right)^2}$	A1
	$=\dfrac{1}{1-\dfrac{x^2}{1+x^2}}\times\dfrac{\sqrt{1+x^2}-x^2\left(1+x^2\right)^{-\frac{1}{2}}}{1+x^2}=\dfrac{1+x^2-x^2}{\left(1+x^2-x^2\right)\sqrt{1+x^2}}=\dfrac{1}{\sqrt{1+x^2}}$ *	A1*
		(4)
(b)	$\dfrac{dy}{dx}=\dfrac{1}{\sqrt{1+x^2}}\Rightarrow y=\displaystyle\int\dfrac{1}{\sqrt{1+x^2}}\,dx=\text{arsinh } x+c$ $\Rightarrow \text{artanh}\left(\dfrac{x}{\sqrt{1+x^2}}\right)=\text{arsinh } x+c$	M1
	$x=0\Rightarrow \text{artanh}(0)=\text{arsinh}(0)+c\Rightarrow c=0$ Therefore $\text{artanh}\left(\dfrac{x}{\sqrt{1+x^2}}\right)=\text{arsinh } x$	A1
		(2)
		(6 marks)

Notes:

(a)

M1: Attempts $\dfrac{dy}{dx}$, with correct form for the artanh, ie the $\dfrac{1}{1-\left(\dfrac{x}{\sqrt{1+x^2}}\right)^2}$ (and this may be all)

Need not be simplified and allow if the squaring is not fully correct (e.g. numerator or denominator squared is sufficient to imply the attempt at squaring).

M1: Attempts $\dfrac{dy}{dx}$ with a correct attempt at the chain rule (or product rule) used (the artanh derivative need not be correct for this mark).

A1: A correct unsimplified expression for the derivative.

A1*: Simplifies to the give answer with suitable intermediate step and no errors seen (e.g. either of those shown in the scheme).

(b)

M1: Integrates the answer to part (a) to achieve the arsinh answer with constant included and equates to the initial function. **A1:** Shows the constant of integration is zero to complete the proof with conclusion, no errors.		
9(a) Alt	$\tanh y = \dfrac{x}{\sqrt{1+x^2}} \Rightarrow \text{sech}^2 y \dfrac{\text{d}y}{\text{d}x} = \dfrac{A\sqrt{1+x^2} - Bx\left(1+x^2\right)^{-\frac{1}{2}} \times x}{\left(\sqrt{1+x^2}\right)^2}$	**M1**
	$\Rightarrow \left(1-\tanh^2 y\right)\dfrac{\text{d}y}{\text{d}x} = \left(1-\dfrac{x^2}{1+x^2}\right)\dfrac{\text{d}y}{\text{d}x} = ...$	**M1**
	$\left(1-\dfrac{x^2}{1+x^2}\right)\dfrac{\text{d}y}{\text{d}x} = \dfrac{\sqrt{1+x^2} - x^2\left(1+x^2\right)^{-\frac{1}{2}}}{1+x^2}$	**A1**
	$\Rightarrow \left(\dfrac{1+x^2-x^2}{1+x^2}\right)\dfrac{\text{d}y}{\text{d}x} = \dfrac{1+x^2-x^2}{\left(1+x^2\right)\sqrt{1+x^2}} \Rightarrow \dfrac{\text{d}y}{\text{d}x} = \dfrac{1}{\sqrt{1+x^2}}$ *	**A1***
		(4)
Notes:		
(a) **M1:** Takes tanh of both sides and differentiates using the product or quotient rule on the RHS, achieving the form shown. **M1:** Applies $\text{sech}^2 y = 1 \pm \tanh^2 y$ and replaces tanh y to get a derivative statement in terms of x only. **A1:** A correct unsimplified expression in terms of x in the derivative. **A1*:** Simplifies to the give answer with suitable intermediate step and no errors seen.		