Mark Scheme (Results)

January 2026

Pearson Edexcel International Advanced Level in
Pure Mathematics P3
WMA13/01

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January 2026
Question Paper Log Number P78910A
Publication Code WMA13_01_2601_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - * – The answer is printed on the paper or ag- answer given

- ⎿ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
	$\mathrm{g}(x)=3x-5+5\sin\left(x^2\right)$	
1 (a)	$\mathrm{g}(0.7)=-0.55,\quad \mathrm{g}(0.8)=0.39$	M1
	Change of sign and continuous function, hence root in $[0.7, 0.8]$	A1
		(2)
(b)	$0=3x-5+5\sin\left(x^2\right)\Rightarrow x=\left[\arcsin(1-0.6x)\right]^{\frac{1}{2}}$	B1
		(1)
(ci)	$x_2=\left[\arcsin(1-\text{"}0.6\text{"}\times 0.7)\right]^{\frac{1}{2}}=\text{awrt } 0.7866$	M1, A1
(cii)	$\alpha=0.7589$	A1
		(3)
		(6 marks)

(a)

M1 Attempts the value of g at 0.7 and 0.8 with one correct (to 1 s.f. rounded or truncated). Use of a narrower range is possible, providing the lower x value gives a negative function value and the higher one gives a positive function value. You may need to check these function values.

A1 Requires

- Both values correct to 1 s.f., rounded or truncated.
- Reason: sign change (or $\mathrm{g}(0.7)<0, \mathrm{g}(0.8)>0$ or $\mathrm{g}(0.7)\mathrm{g}(0.8)<0$, etc) and "continuous function" (or "continuous graph/curve", or just "continuity" etc.)
- Minimal conclusion, e.g. "root", "root in interval", "root in [0.7,0.8]", "α lies in interval", etc). Do not condone the incorrect statement "Change of sign hence continuous function and root in interval" etc.

(b)

B1 Correct value of p stated or implied. May be embedded in formula. If embedded, allow e.g. $-\frac{3x}{5}$ in place of $-0.6x$. Condone slips in the formula if the correct value of p is clearly seen. If an incorrect p value is seen in (b) it is B0, regardless of what the candidate uses in (c). If there is no obvious attempt at (b), then you can mark any formula used in (c). Apply isw if, for example, a correct formula is given, followed by $p=-0.6$.

(ci)

M1 Attempts to substitute $x_1=0.7$ into the correct formula with their $\pm p$ to reach a value. Do not be concerned about the labelling of this second approximation, e.g. it may be labelled x_1 or x_3. This mark is available for candidates working in degrees, in which case, if a correct formula is used, they will get $x_2=5.954$. For an incorrect formula and no working, you may have to check their answer, whether working in radians or degrees.
Can be implied by 0.786... following $p=0.6$

A1 awrt 0.7866. Condone incorrect labelling.

(cii)

A1 cao $\alpha=0.7589$. Accept $x=0.7589$ or e.g. $x_{11}=0.7589$ if it is clear this is their final answer.

Question Number	Scheme	Marks
2 (i)	$y = x^2 \ln 2x \Rightarrow \left(\dfrac{dy}{dx} =\right) 2x\ln 2x + x$ o.e.	M1 A1
(ii)	$\left(f'(x) =\right) \dfrac{2(4x+3)^2 e^{2x} - 8(4x+3)e^{2x}}{(4x+3)^4}$ or $2e^{2x}(4x+3)^{-2} - 8e^{2x}(4x+3)^{-3}$	M1 A1
	$= \dfrac{\cancel{(4x+3)}e^{2x}[2(4x+3)-8]}{(4x+3)^{\cancel{4}3}}$ or $e^{2x}(4x+3)^{-3}[2(4x+3)-8]$	dM1
	$= \dfrac{2e^{2x}(4x-1)}{(4x+3)^3}$	A1
		(6 marks)

(i)

M1 Attempts the product rule to achieve $Ax\ln 2x + Bx$ o.e. with $A, B \neq 0$ which may be unsimplified.

A1 $\left(\dfrac{dy}{dx} =\right) 2x\ln 2x + x$. Condone $\ln 2x(2x) + x$. isw after a correct answer.

(ii)

M1 Attempts the quotient rule to achieve $\dfrac{P(4x+3)^2 e^{2x} - Q(4x+3)e^{2x}}{(4x+3)^4}$ with $P, Q > 0$. Allow $-Q(4x+3)e^{2x}$ to be expanded, i.e. if correct $-(32x+24)e^{2x}$ or $-32xe^{2x} - 24e^{2x}$.

Alternatively, attempts the product rule. Look for $Pe^{2x}(4x+3)^{-2} - Qe^{2x}(4x+3)^{-3}$ with $P, Q > 0$.

In either case, Q may be written as the product of two integers.

Condone miscopying slips on $(4x+3)$, e.g. $(3x+4)$

A1 Using quotient rule: $\left(f'(x) =\right) \dfrac{2(4x+3)^2 e^{2x} - 8(4x+3)e^{2x}}{(4x+3)^4}$ o.e.

Using product rule: $\left(f'(x) =\right) 2e^{2x}(4x+3)^{-2} - 8e^{2x}(4x+3)^{-3}$ o.e.

In either case, allow 8 to be written as (2)(4).

dM1 Dependent on previous M1. Correctly takes out a common factor of $(4x+3)e^{2x}$ from their numerator and cancels $(4x+3)$. If using product rule, must take out a common factor of $(4x+3)^{-3}e^{2x}$. They may also take out a factor of 2 at this stage, but this is not required for dM1.

A1 Achieves $\dfrac{2e^{2x}(4x-1)}{(4x+3)^3}$. Do not allow $2e^{2x}(4x-1)(4x+3)^{-3}$. Allow recovery of invisible brackets at any stage.

Question Number	Scheme	Marks
3 (i)	$\int 2(3x+5)^7 \, dx = \frac{(3x+5)^8}{12} (+c)$	M1 A1
		(2)
(ii) (a)	By division $x^2+x-2 \overline{)4x^3+x^2-7x+14}$ quotient $4x-3$	M1 A1
	Remainder $=4x+8$	M1
	$\frac{4x+8}{x^2+x-2} = \frac{4\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$	
	$\frac{4x^3+x^2-7x+14}{(x-1)(x+2)} \equiv 4x-3+\frac{4}{x-1}$	A1
		(4)
(b)	$\int_3^9 f(x)\,dx = \left[2x^2-3x+4\ln(x-1)\right]_3^9$	M1 A1ft
	$=126+4\ln 8-4\ln 2=126+8\ln 2$	dM1 A1
		(4)
		(10 marks)

(i)

M1 Integrates and achieves $k(3x+5)^8$ where k is a constant. Any attempt that involves expanding $(3x+5)^7$ is unlikely to reach an answer in its simplest form, so probably scores M0A0, but if you think there is a meaningful attempt to factorise the answer into the required form, send to review. If a substitution is used, they must revert to x and reach $k(3x+5)^8$

A1 $\frac{(3x+5)^8}{12}$ with or without + c

(ii) (a)

By division

M1 Divides the cubic by x^2+x-2 and achieves a linear quotient $4x \pm ...$. Watch out for different ways to present the division, e.g. grid method. They may divide first by $(x+2)$ and then by $(x-1)$ or vice versa. See exemplification below. Do not allow misreads for the quadratic divisor; or if dividing by the two linear factors separately, these must be correct.

A1 Achieves the linear quotient $4x-3$. This mark can be given even if slips in the division lead to an incorrect remainder.

M1 See one of the following 3 cases below for award of this mark. All 3 cases depend on a valid attempt at division and $Q(x)$ is their linear quotient after division.

1) If the candidate has divided the cubic by x^2+x-2, achieving a remainder $px+q$, this mark can be awarded for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

2) If they have divided by $(x+2)$ and then $(x-1)$, they should reach an expression of the form $Q(x)+\frac{c}{x-1}$, where c is an integer. If there are slips in the division by $(x+2)$ leading to a non-zero remainder, award for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

3) If they have divided by $(x-1)$ and then $(x+2)$, this mark can be awarded for reaching $Q(x)+\frac{px+q}{x^2+x-2}$ or $Q(x)+\frac{px+q}{(x+2)(x-1)}$

In Cases (2) and (3) there must be a correct method for combining the two remainders into a single fraction.

A1 $4x-3+\frac{4}{x-1}$ which may be seen in (ii) (b)

Division by the two linear factors separately – exemplification

$\frac{4x^3+x^2-7x+14}{x+2}=4x^2-7x+7$; followed by $\frac{4x^2-7x+7}{x-1}=4x-3+\frac{4}{x-1}$ (M1A1M1A1)

Or $\frac{4x^3+x^2-7x+14}{x-1}=4x^2+5x-2+\frac{12}{x-1}$; followed by $\frac{4x^2+5x-2}{x+2}=4x-3+\frac{4}{x+2}$ (M1A1 here for correct quotient). Then $\frac{12}{(x-1)(x+2)}+\frac{4}{x+2}=\frac{4}{x-1}\Rightarrow \mathrm{f}(x)=4x-3+\frac{4}{x-1}$ (M1A1)

Alt: By comparing terms / using an identity. This must be of the correct form but condone slips (say in signs).
$4x^3+x^2-7x+14\equiv(ax+b)(x+2)(x-1)+c(x+2)$

M1 Correct method to find two constants, e.g. $x=1\Rightarrow 12=3c\Rightarrow c=4$, compare terms in $x^3\Rightarrow 4=a$

A1 Two correct constants

M1 Correct method to find all three constants

A1 $4x-3+\frac{4}{x-1}$ which may be seen in (ii) (b)

(ii) (b)

M1 Achieves a correct $c\ln(x-1)$ AND one of the other two terms: $\frac{1}{2}ax^2$ or bx for their a, b and c. Condone invisible brackets in the log term as long as there is a recovery when substituting limits.

A1ft $\int ax+b+\frac{c}{x-1}\,\mathrm{d}x=\frac{1}{2}ax^2+bx+c\ln(x-1)$. Follow through on their a,b and c. Condone a spurious integration symbol and dx after integration.

dM1 Dependent on the first M1. Substitutes in both limits and subtracts. Condone missing brackets around the expression involving the lower limit.

A1 $126+4\ln 4$ or $126+8\ln 2$ or any equivalent involving a single log term. Withhold this mark if a spurious integration symbol or dx features in their final answer. Isw after a correct expression.

Question Number	Scheme	Marks
4	$\log_{10} h = 2.4 - 0.25\log_{10} m$	
(a)	$\log_{10} h = 2.4 - 0.25\log_{10} 3 \Rightarrow h = 10^{2.4-0.25\log_{10} 3}$	M1
	$\Rightarrow h = 191$	A1
		(2)
(b)	Example: One correct law $\log_{10} h = 2.4 + \log_{10} m^{-0.25}$	M1
	Full method in forming an equation linking *h* and *m*, e.g. $h = 10^{2.4} \times m^{-0.25}$	dM1
	$\Rightarrow h = \frac{251}{m^{0.25}}$	A1
		(3)
(c)	The resting heart rate (in beats per minute) for (a mammal of mass) 1 kg	B1
		(1)
		(6 marks)

(a)

M1 Substitutes $m = 3$ into the given equation and uses the correct order of operations to find *h*. Can be implied by a correct value for *h*.

A1 awrt 191

(b)

M1 Makes a first step towards achieving an answer. Sight of a correct rule or law used. Will usually be awarded for one of:

- Application of a power rule e.g. $-0.25\log_{10} m \to \log_{10} m^{-0.25}$
- An attempt to make *h* the subject. E.g. $\log_{10} h = 2.4 - 0.25\log_{10} m \to h = 10^{2.4-0.25\log_{10} m}$
- Both of the above applied at once, e.g. $\log_{10} h = 2.4 - 0.25\log_{10} m \to h = \frac{10^{2.4}}{m^{0.25}}$ or $10^{2.4} \times m^{-0.25}$

In this case, M1 dM1 are scored together.

dM1 Dependent on the first M1. Full and complete method to make *h* the subject, with logs being correctly removed, e.g. $\log_{10} h = 2.4 + \log_{10} m^{-0.25} \Rightarrow h = 10^{2.4} \times m^{-0.25}$

A1 Achieves $h = \frac{251}{m^{0.25}}$ with at least one intermediate step (see M1) between the given equation and the final answer. Accept ¼ or 0.250 in place of 0.25 and allow awrt 251. Do not accept $p = 251, q = 0.25$ for this mark without seeing the full equation.

Note: It is possible to score 000 in (b) despite a numerator of 251 if an incorrect method is used.

Note: Allow full marks for part (b) if full working is shown correctly in (a).

SC: Award SC110 for use of $m = 3$ in part (a), and making *h* the subject, reaching $h = \frac{10^{2.4}}{3^{1/4}}$ and then writing down $p = 251, q = 0.25$ in part (b).

(c)

B1 "The (resting) heart rate (in beats per minute) for (a mammal of mass) 1 kg". Also accept "The beats per minute for (a mammal of mass) 1 kg."

Question Number	Scheme	Marks
5 (a)	$\text{ff}(x)=\dfrac{2\times\left(\dfrac{2x+16}{x-4}\right)+16}{\left(\dfrac{2x+16}{x-4}\right)-4}, \dfrac{2\times(2x+16)+16(x-4)}{(2x+16)-4(x-4)}, = \dfrac{10x-16}{-x+16}$	M1, dM1, A1
		(3)
(b)	$\dfrac{2x+16}{x-4}=12 \Rightarrow 10x=64 \Rightarrow x=6.4$	M1 A1
		(2)
(c)	$\dfrac{2\ln a+16}{\ln a-4}=\ln a \Rightarrow (\ln a)^2-6\ln a-16=0$	M1
	$\Rightarrow(\ln a-8)(\ln a+2)=0 \Rightarrow \ln a=8,-2$	dM1
	$\Rightarrow a=e^{-2}, e^{8}$	A1
		(3) **8 marks**

(a)

M1 Attempts to fully substitute f(*x*) for *x* in $\dfrac{2x+16}{x-4}$. Condone **one** slip as long as the intention is clear and there are two substitutions of f(*x*) for *x*.

dM1 Dependent on the first M1. Correct attempt to form a single fraction (see scheme) by multiplying two terms in the numerator and two terms in the denominator by $(x-4)$

A1 $\dfrac{10x-16}{-x+16}$ or exact simplified equivalent

(b)

M1 Attempts to solve $\text{f}(x)=12$ using correct order of operations (multiplying by $x-4$ and grouping terms in *x*).

Alternatively, reaches $\text{f}^{-1}(x)=\dfrac{ax+b}{cx+d}$ using correct order of operations and attempts to substitute $x=12$. Note: $\text{f}^{-1}(x)=\dfrac{4x+16}{x-2}$

A1 6.4 o.e.

(c) **Note:** If correct answers appear without a 3TQ, send to review.

M1 Sets up a 3TQ in $\ln a$. Must have three terms, but not necessarily on one side of the equation. Condone a four term quadratic with all terms on one side if it is followed by a correct attempt to solve. Condone missing "=0".

dM1 Dependent on the first M1. Full attempt to solve a 3TQ in $\ln a$, using any valid method (including calculator), reaching two values, which do not necessarily have to be labelled $\ln a$. Allow a substitution, e.g. $x=\ln a$ or even $a=\ln a$.

A1 $a=e^{8}, e^{-2}$ (or $\dfrac{1}{e^2}$) and no others. Accept answers labelled as *x* or unlabelled. Apply isw (e.g. if exact answers are followed by decimals). Must follow M1 dM1.

Question Number	Scheme	Marks
6 (a)	$2\cos(\theta-60°)=3\sin\theta \Rightarrow 2\cos\theta\cos 60°+2\sin\theta\sin 60°=3\sin\theta$	M1, A1
	$\Rightarrow \cos\theta=(3-\sqrt{3})\sin\theta$ or $\Rightarrow 1+\sqrt{3}\tan\theta=3\tan\theta$	dM1
	$\Rightarrow \tan\theta=\dfrac{1}{3-\sqrt{3}}$	
	$\Rightarrow \tan\theta=\dfrac{1}{6}(3+\sqrt{3})$ *	A1*
		(4)
(b)	Deduces that $\theta=2x-10°$	B1
	$\tan(2x\pm\alpha)=\dfrac{1}{6}(3+\sqrt{3})\Rightarrow 2x\pm\alpha=\text{awrt}\,38$ or $218\Rightarrow x=...$	M1
	$x=\text{awrt}\,24.1°$, awrt $114.1°$.	A1
		(3) **(7 marks)**

(a)

M1 Attempts to use $\cos(\theta-60°)=\cos\theta\cos 60°\pm\sin\theta\sin 60°$ within the given equation
Condone the omission of a 2 on the second term and a slip on the 3 of $3\sin\theta$

A1 Correct equation in $\sin\theta$ and $\cos\theta$

dM1 Dependent on the first M1. Uses $\cos 60°=\dfrac{1}{2}$, $\sin 60°=\dfrac{\sqrt{3}}{2}$ AND attempts one of:

- Dividing all 3 terms by $\cos\theta$ or $\sin\theta$ to set up an equation in just $\tan\theta$; or
- Collecting terms in $\sin\theta$ to form an equation of the form $P\sin\theta\pm Q\cos\theta=0$ o.e.

A1* Correctly proceeds to given answer showing all necessary steps: **see the steps set out in the main scheme above**. It is not necessary to see the rationalising of the denominator. Withhold this mark for invisible brackets or for use of the wrong variable, e.g. x, or a missing variable, at any stage within the body of the solution.

(b) **Notes**

- **Correct answers from no working score B0 M0 A0**
- **This is a "hence" question, so starting from scratch also scores B0 M0 A0**

B1 Deduces that $\theta=2x-10°$

M1 Attempts correct method from $\tan(2x\pm\alpha)=\dfrac{1}{6}(3+\sqrt{3})\Rightarrow x=....$
Look for the correct order of operations, i.e. arctan, then add or subtract α, then divide by 2.
As a minimum look for $\tan(2x\pm\alpha)=\dfrac{1}{6}(3+\sqrt{3})\Rightarrow 2x\pm\alpha=\text{awrt}\,38$ or $218\Rightarrow x=...$
If $\tan(2x\pm\alpha)$ is expanded to $\dfrac{\tan 2x\pm\tan\alpha}{1\mp\tan 2x\tan\alpha}$ then $\tan 2x$ must be made the subject using the correct order of operations before arctan and division by 2.

Calculations must be in degrees.

A1 $x=\text{awrt}\,24.1°$, awrt $114.1°$ and no others in the range.

Question Number	Scheme	Marks
7 (a)	$\left(\frac{11}{3}, -4\right)$	B1 B1
		(2)
(b)	Attempts $3x-11-4=8$ and $-3x+11-4=8$	M1, dM1
	$x=-\frac{1}{3}, \frac{23}{3}$	A1
		(3)
(c)	$m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$	M1, A1, A1
		(3)
(d)	$a=-4,\ b=\frac{4}{3}$	B1ft, B1ft
		(2)
		(10 marks)

Note: Check working near to the question text or written on the diagram.

(a)

B1 For one correct coordinate

B1 For $\left(\frac{11}{3}, -4\right)$. Allow missing brackets, or $x=\frac{11}{3}, y=-4$. Accept equivalent fractions for $\frac{11}{3}$

(b)

M1 For a full attempt at either $3x-11-4=8$ or $-3x+11-4=8$. Must lead to a value for x.

dM1 Dependent on the first M1. For a full attempt at both $3x-11-4=8$ and $-3x+11-4=8$ leading to values for x.

A1 $x=-\frac{1}{3}, \frac{23}{3}$ only. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.

Alt

M1 Reaches $|3x-11|=12$, squares both sides and solves a quadratic equation. Must lead to a value for x.

dM1 Reaches two values for x following a valid method for solving their quadratic equation.

A1 $x=-\frac{1}{3}, \frac{23}{3}$. If one of these has clearly been selected as a final answer and the other rejected, withhold this mark.

(c)

M1 Finds any of the three critical values for m. This need not be presented as an inequality.

A1 Two of $m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$. Do not give this mark if there are contradictions, e.g. $m=-\frac{12}{11}, m>-\frac{12}{11}$ unless the correct answer is clearly indicated. Accept set notation.

A1 All three of $m \geq 3, \quad m=-\frac{12}{11}, \quad m<-3$ with no contradictions (see above). Accept set notation.

(d) **Note: If *a*=… and *b*=… are not stated, they could be embedded in $y=a\mathrm{f}(x-b)$. The stated values take precedence.**

B1ft Either $a = -4$ or $b = \frac{4}{3}$ but follow through on their vertex, so allow $a = \frac{16}{"-4"}$ or $b = 5 - "\frac{11}{3}"$

B1ft Both $a = -4$ and $b = \frac{4}{3}$ but follow through on their vertex, so allow $a = \frac{16}{"-4"}$ and $b = 5 - "\frac{11}{3}"$

Question Number	Scheme	Marks
8	$x = e^{2\tan y} \Rightarrow \frac{dx}{dy} = e^{2\tan y} \times 2\sec^2 y$	B1
	$\Rightarrow \frac{dx}{dy} = 2x\left(1+\tan^2 y\right) = 2x\left(1+\left(\frac{\ln x}{2}\right)^2\right)$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{4}{2x\left(4+\left(\ln x\right)^2\right)}$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{2}{x\left(4+\left(\ln x\right)^2\right)}$	A1
		(4) **(4 marks)**

Note: Condone incorrect notations for $\left(\ln x\right)^2$ such as $\ln x^2$ and $\ln^2 x$ in working, but not in the final answer.

B1 Correct expression for $\frac{dx}{dy}$ as in scheme or in an equivalent form

The following two M marks require their $\frac{dx}{dy}$ to be in the form $k e^{2\tan y} \sec^2 y$.

M1 Either:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; OR
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\frac{dx}{dy}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; AND
- Uses $\sec^2 y = 1+\tan^2 y$ to get $\frac{dy}{dx}$ in terms of x.

A1 $\frac{dy}{dx} = \frac{2}{x\left(4+\left(\ln x\right)^2\right)}$. Condone missing $\frac{dy}{dx}$ on LHS as long as it appears somewhere in working and it is clear that it refers to their final answer.

Alt for the two M marks using cos

The following two M marks require their $\frac{dx}{dy}$ to be in the form $\frac{k e^{2\tan y}}{\cos^2 y}$.

M1 Either:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; OR
- Forms an expression for $\cos^2 y$ using a valid method and gets $\frac{dx}{dy}$ in terms of x.

M1 Both:

- Attempts the reciprocal of their $\frac{dx}{dy}$ to form an expression for $\frac{dy}{dx}$; AND
- Forms an expression for $\cos^2 y$ using a valid method and gets $\frac{dy}{dx}$ in terms of x.

Alt using arctan. Note that B1 M1 M0 A0 is not possible via this route

B1 $x = e^{2\tan y} \Rightarrow y = \arctan\left(\frac{1}{2}\ln x\right) \Rightarrow \frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{2}\ln x\right)^2} \times ...$ where … could be 1.

M2 $\frac{dy}{dx} = \frac{1}{1+\left(\frac{1}{2}\ln x\right)^2} \times \frac{1}{2x}$

A1 $\frac{dy}{dx} = \frac{2}{x\left(4+(\ln x)^2\right)}$

Alt using implicit differentiation

B1 $x = e^{2\tan y} \Rightarrow \ln x = 2\tan y \Rightarrow \frac{1}{x}\frac{dx}{dy} = 2\sec^2 y$ OR $x = e^{2\tan y} \Rightarrow \ln x = 2\tan y \Rightarrow \frac{1}{x} = 2\sec^2 y \frac{dy}{dx}$

Other variations are possible. Then as main method

Question Number	Scheme	Marks
	$\text{h}(x)=8+3\sin x(\cos x-2\sin x)$	
9 (a)	$\text{h}(x)=8+3\sin x\cos x-6\sin^2 x$	
	$\text{h}(x)=8+\frac{3}{2}\sin 2x+3\cos 2x-3$ o.e.	M1, dM1
	$R\cos\alpha=\frac{3}{2},\ R\sin\alpha=3$	A1
	Let $\frac{3}{2}\sin 2x+3\cos 2x=R\sin(2x+\alpha)$	
	$R^2=\left(\text{"}\frac{3}{2}\text{"}\right)^2+\text{"}3\text{"}^2\Rightarrow R=...$	ddM1
	$\tan\alpha=\dfrac{\text{"}3\text{"}}{\left(\text{"}\frac{3}{2}\text{"}\right)}\Rightarrow\alpha=...$	ddM1
	$(\text{h}(x)=)5+\frac{3}{2}\sqrt{5}\sin(2x+1.107)$	A1
		(6)
(b)	$5-\frac{3}{2}\sqrt{5}\leqslant\text{h}(x)\leqslant 5+\frac{3}{2}\sqrt{5}$	B1ft
		(1)
(c)	$2x+\text{"}1.107\text{"}=\frac{7\pi}{2}\Rightarrow x=...$	M1
	$x=$ awrt 4.94	A1
		(2) **(9 marks)**

(a)

M1 Multiplies out bracket and attempts to use **either** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

dM1 Dependent on the first M1. Multiplies out bracket and attempts to use **both** of

- $\sin 2x=2\sin x\cos x$
- $\cos 2x=1-2\sin^2 x$ o.e.

Expands $R\sin(2x+\alpha)$ and compares coefficients to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

Alt for the first two marks using the given form of the answer, expanding and equating coefficients.

M1 Expands $R\sin(2x+\alpha)$ to $R\sin 2x\cos\alpha+R\cos 2x\sin\alpha$ and uses **either** $\sin 2x=2\sin x\cos x$ or $\cos 2x=1-2\sin^2 x$

dM1 Dependent on the first M1. Expands using **both** $\sin 2x=2\sin x\cos x$ and $\cos 2x=1-2\sin^2 x$ to reach $2R\sin x\cos x\cos\alpha+R\sin\alpha-2R\sin^2 x\sin\alpha$

And compares coefficients of $\sin x\cos x$ and $\sin^2 x$ to reach two equations in R and α. In some cases, the two equations may not be explicitly stated, but implied by further work.

A1 $R\cos\alpha=\frac{3}{2}$ and $R\sin\alpha=3$ or equivalent equations. Note that a value of R may be embedded if it has already been calculated by inspection of coefficients or construction of a triangle. In some cases, the two equations may not be explicitly stated, but implied by further work.

The following two M marks are both dependent on the first two M marks, i.e. we require a correct method leading to $R\cos\alpha="\frac{3}{2}"$ and $R\sin\alpha="3"$.

ddM1 $R^2=\left("\frac{3}{2}"\right)^2+"3"^2\Rightarrow R=...$ May be implied by their R value, even if R is inexact.

ddM1 $\tan\alpha=\frac{"3"}{\left("\frac{3}{2}"\right)}\Rightarrow\alpha=...$ but condone $\tan\alpha=\frac{\left("\frac{3}{2}"\right)}{"3"}\Rightarrow\alpha=...$ Alternatively α can be found using $\sin\alpha=\frac{"3"}{R}$ or $\cos\alpha=\frac{"3/2"}{R}$ with their R.

A1 $(\mathrm{h}(x)=)5+\frac{3}{2}\sqrt{5}\sin(2x+1.107)$. A full expression must be given for h(x), not just values of P, R and α. This expression must be seen in part (a).

(b) **Note: this mark can be scored following incorrect working in part (a).**

B1ft $5-\frac{3}{2}\sqrt{5}\leqslant \mathrm{h}(x)\leqslant 5+\frac{3}{2}\sqrt{5}$ o.e. allowing follow through on their P and R

$\left[5-\frac{3\sqrt{5}}{2},5+\frac{3\sqrt{5}}{2}\right]$ and $\left\{\mathrm{h}(x):5-\frac{3\sqrt{5}}{2}\leqslant \mathrm{h}(x)\leqslant 5+\frac{3\sqrt{5}}{2}\right\}$ are suitable alternatives.

Condone y or h in place of h(x) but not f(x) or x.

(c) **Note: full marks can be scored in part (c) following incorrect working in part (a).**

M1 Attempts to solve $2x+"1.107"=\frac{7\pi}{2}$ using the correct order of operations, leading to a value for x.

Alternatively solves $\sin(2x+"1.107")=-1$ using the correct order of operations, and selects the second positive value of x.

Or differentiates to reach $2\cos(2x+"1.107")=0$ and solves to find the fourth positive solution.

A1 $x=$ awrt 4.94

Question Number	Scheme	Marks
10 (a)	$\left(\dfrac{dN_A}{dt}\right) = 900 \times 0.2e^{0.2\times5} = \text{awrt } 489 \text{ or } 180e$	M1, A1
		(2)
(b)	States or implies that $P = 2\,000$	B1
	$11570 = 8000 + 2000e^{k\times4} \Rightarrow 2000e^{4k} = 3570$	M1
	$\Rightarrow k = \text{awrt } 0.145$	dM1, A1
		(4)
(c)	$8000 + 900e^{0.2T} = 8000 + "2000"e^{0.145T}$	
	$e^{(0.2-"0.145")T} = \dfrac{"2000"}{900}$ or $\ln\left(900e^{0.2T}\right) = \ln\left("2000"e^{0.145T}\right)$	M1
	$\ln 900 + 0.2T = \ln"2000" + 0.145T$	
	$T = \dfrac{1}{(0.2-"0.145")}\ln\left(\dfrac{"2000"}{900}\right)$	dM1
	$T = 14.5$	A1
		(3)
		(9 marks)

(a) **Note: if a correct answer is seen without any working, please send to review.**

M1 $\dfrac{dN_A}{dt} = Ce^{0.2\times5} = \ldots$ where $C \neq 900$. Withhold this mark if they substitute $t = 5$ before differentiation.

A1 awrt 489 or exact 180e. isw

(b)

B1 States or implies that $P = 2\,000$

M1 Proceeds to $Ae^{4k} = B$ where $A \times B > 0$

dM1 Dependent on previous M1. Correct attempt to solve using ln, following the correct order of operations. Proceeding from $Ae^{4k} = B$ to a value for k with no ln work is dM0 A0.

A1 $k = \text{awrt } 0.145$. Exact answer only is A0.

(c) **Condone use of *t* in place of *T* throughout this part.**

M1 Cancels 8000 on each side, then **either**:

- uses correct index work to achieve an equation in $e^{(0.2-"0.145")T}$ that could be rearranged to the form $e^{(0.2-"0.145")T} = K$ where $K > 0$; **or**
- takes ln of both sides and uses the log addition law on both sides, giving an equation involving two terms in *T* and at least one constant term.

dM1 Dependent on previous M1. Correct ln work leading to a positive value or positive exact expression for *T*. Proceeding from $e^{(0.2-"0.145")T} = K$ to a value for *T* with no ln work is dM0 A0.

A1 *T* = awrt 14.5. Exact answer only is A0.