Mark Scheme (Results)

January 2026

Pearson Edexcel International Advanced Level in
Pure Mathematics P3
WMA13/01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

   - bod – benefit of doubt
   - ft – follow through
     - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC – special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp – decimal places
   - sf – significant figures
   - ∗ – The answer is printed on the paper or ag- answer given

- ⎣ or d… – The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by ‘MR’ in the body of the script.

6. If a candidate makes more than one attempt at any question:
   a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$ leading to x = ...

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c,\quad q\neq 0$ leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n\to x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n\to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$y=\frac{5}{2}$ at point P	B1
	$y=\frac{3x-2}{(x-2)^2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)^2\times3-(3x-2)\times2(x-2)}{(x-2)^4}$	M1A1
	Sub $x=4$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(-\frac{7}{4}\right)$	M1
	$\frac{4}{7}=\frac{y-\frac{5}{2}}{x-4}\Rightarrow 8x-14y+3=0$	M1,A1
		(6 marks)

Notes

B1: Correct coordinate $y=\frac{5}{2}$ at P stated or used.

M1: Attempts to differentiate using the Quotient rule or Product rule to achieve an expression of the correct form.

Look for $\frac{(x-2)^2\times A-(3x-2)\times B(x-2)}{\left((x-2)^2\right)^2}$ or $A(x-2)^{-2}+(3x-2)\times-B(x-2)^{-3}$ where $A,B>0$

You may condone mising brackets for the M mark, but they must have been recovered (possibly implied by working) for the A mark.

Partial fractions may be used. Look for

$y=\frac{3x-2}{(x-2)^2}\Rightarrow y=\frac{P}{(x-2)}+\frac{Q}{(x-2)^2}(P=3,Q=4)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{R}{(x-2)^2}\pm\frac{S}{(x-2)^3}$ with appropriate signs for their P and Q.

A1: Any correct (unsimplified) form of the derivative. Accept from the product rule versions equivalent to $\frac{\mathrm{d}y}{\mathrm{d}x}=3(x-2)^{-2}-2(3x-2)(x-2)^{-3}$ or from partial fractions $\frac{\mathrm{d}y}{\mathrm{d}x}=-3(x-2)^{-2}-8(x-2)^{-3}$

FYI: Correct simplified expressions are $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3x^2+4x+4}{(x-2)^4}$ or $\frac{-3x-2}{(x-2)^3}$

M1: Substitutes $x=4$ into their derivative (any changed function) to find a numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$.

M1: Uses $x=4$ and their numerical value of y at $x=4$ with their numerical $-\frac{\mathrm{d}x}{\mathrm{d}y}$ to form an equation of a normal. If the form $y=mx+c$ is used then it must be a full method reaching a value for c.

A1: Accept $\pm A(8x-14y+3)=0$ where A is a non-zero integer coming from correct work. Terms may be in a different order but must be on one side and the "= 0" must be given. Watch for correct answers coming from incorrect versions of $\frac{\mathrm{d}y}{\mathrm{d}x}$ with eg. $(x-2)^2$ on the denominator, which will lose both A marks.

Question Number	Scheme	Marks
2. (a)	$0 < \mathrm{f}(x) < \frac{4}{5}$	M1A1
		(2)
(b)	$y = \frac{4}{3x+5} \quad \Rightarrow (3x+5)y = 4 \Rightarrow x = \mathrm{h}(y)$	M1
	$\Rightarrow x = \frac{4-5y}{3y}$ or $\Rightarrow y = \frac{4-5x}{3x}$ o.e.	A1
	$\mathrm{f}^{-1}(x) = \frac{4-5x}{3x} \qquad 0 < x < \frac{4}{5}$	A1ft
		(3)
(c)	$\mathrm{fg}(x) = \frac{4}{\frac{3}{x}+5}$	B1
		(1)
(d)	$\frac{3x+5}{4} = \frac{4}{\frac{3}{x}+5} \rightarrow ax^2 + bx + c = 0$	M1
	$15x^2 + 18x + 15 = 0$ or $5x^2 + 6x + 5 = 0$ oe	A1
	E.g. Uses $18^2 - 4 \times 15 \times 15$ or completing square, or applies formula.	dM1
	E.g. Discriminant < 0 hence there are no real solutions.	A1
		(4)
		(10 marks)

Notes

(a)

M1: One end correct, condoning $\leqslant$ instead of $<$. Accept alternative forms, e.g. interval notation. Condone use of *x* for this mark.

A1: Completely correct. Accept with f(*x*), f or *y* or Range, but A0 with *x* used. If interval notation is used, brackets must be correct for this mark, ie $\left(0, \frac{4}{5}\right)$.

(b)

M1: Sets *y* = f(*x*), multiplies both sides by denominator and reaches *x* as a function of *y*. Condone slips in expanding or in signs when rearranging after cross multiplying.
Alternatively swaps *x* any *y* and proceeds to *y* as a function of *x*.

A1: Correct expression for *x* in terms of *y*, or if swapping first, *y* in terms of *x*. E.g. $y = \frac{4-5x}{3x}$,

$y = \frac{4}{3x} - \frac{5}{3}$ or $x = \frac{\frac{4}{y} - 5}{3}$

A1ft: A fully correct definition for f^{-1} with the correct rule and a correct domain or follow through domain for their (a). Must be in terms of *x* but accept with "*y* ="

E.g. $\mathrm{f}^{-1}(x) = \frac{1}{3}\left(\frac{4}{x} - 5\right) \quad 0 < x < \frac{4}{5}$ Accept equivalent forms for the rule as long as it is unambiguous e.g. $\frac{\frac{4}{x} - 5}{3}$ is acceptable. You may isw after an acceptable answer has been seen.

(c)

B1: $\text{fg}(x) = \dfrac{4}{\dfrac{3}{x}+5}$ - allow any correct form then isw.

(d)

M1: Sets their fg(x) from (c) equal to their attempt at 1/f(x) (condone a slip if intent is clear), and attempt at simplification to give a quadratic. Alternatively, sets $\text{g}^{-1}\text{fg}(x) = \text{f}(x)$ or $\text{gfg}^{-1}(x) = \text{f}(x)$ with $\text{g}^{-1}(x) = \dfrac{1}{x}$ and attempts the simplification to a quadratic, e.g. $\dfrac{3+5x}{4x} = \dfrac{4}{3x+5}$ or $\dfrac{3\left(\dfrac{1}{x}\right)+5}{4} = \dfrac{4}{3x+5}$ leading to a quadratic in x.

A1: Correct 3TQ formed (any multiple). The "=0" may be implied.

dM1: Uses discriminant or completes the square to e.g. $5(x+b)^2 - c + "15"$ or $\left(\sqrt{5}x+b\right)^2 - c + "15"$ or attempts formula on their quadratic. There must be some working or indication of what they are doing to show the result, not just a value or solutions stated.

For example, stating the value −576 (or −576 < 0) with no context is dM0 but writing "Discriminant = −576" is sufficient for dM1 as they have said what the −576 is. Similarly, if they state "Discriminant =…" for an incorrect quadratic without "b^2-4ac" the value would need to be correct for their quadratic to imply correct method but just a value with no context will be dM0 even if correct.

Simply stating the complex roots with no working is dM0, as is going direct to a factorisation $\left(x-\dfrac{-3+4\text{i}}{5}\right)\left(x-\dfrac{-3-4\text{i}}{5}\right)=0.$

A1: Completely correct work with reason and conclusion. Accept for the reason

- Correct discriminant < 0 (−576 < 0 or −64 < 0 for the quadratic shown in scheme) oe
- Correct completed square > 0 for all x
- Correct formula with correct complex (hence not real) roots stated with reference to "complex" or "not real" or contradiction due to square root of negative.

The conclusion may be minimal, e.g. // or QED.

Question Number	Scheme	Marks
3.(i)(a)	$\begin{array}{r} x-5 \\ x+2\overline{)x^2-3x+6} \\ \underline{x^2+2x} \\ -5x+6 \\ \underline{-5x-10} \\ 16 \end{array}$ $\frac{x^2-3x+6}{x+2}=Ax+B+\frac{C}{x+2}$ $\Rightarrow x^2-3x+6=(Ax+B)(x+2)+C$ $\Rightarrow 1=A,-3=2A+B,6=2B+C$ $\Rightarrow A=\ldots,\ B=\ldots,\ C=\ldots$	M1
	Quotient = x – 5 and Remainder = 16	A1
		(2)
(b)	$\int\frac{x^2-3x+6}{x+2}\,\mathrm{d}x=\int x-5+\frac{16}{x+2}\,\mathrm{d}x=\frac{1}{2}x^2-5x+16\ln(x+2)+c$	M1, B1ft, A1
		(3)
(ii)	$\int_0^{\frac{\pi}{6}}(\cos\theta-\sin\theta)^2\mathrm{d}\theta\ =\int_0^{\frac{\pi}{6}}(\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta)\mathrm{d}\theta$ **and** one of $\int\sin^2\theta+\cos^2\theta\ \mathrm{d}\theta=\int 1\ \mathrm{d}\theta\to\theta$ or $\int 2\sin\theta\cos\theta\ \mathrm{d}\theta=\int\sin2\theta\ \mathrm{d}\vartheta\to k\cos2\theta$ or other valid integration approaches for one of these	M1
	E.g. $\to\int 1-\sin2\theta\ \mathrm{d}\theta\to\theta+k\cos2\theta$ (oe)	dM1
	$\int_0^{\frac{\pi}{6}}(\cos\theta-\sin\theta)^2\mathrm{d}\theta\ =\left[\theta+\frac{1}{2}\cos2\theta\right]_0^{\frac{\pi}{6}}$ oe	A1
	$=\left(\frac{\pi}{6}+\frac{1}{2}\times\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)=\frac{\pi}{6}-\frac{1}{4}$	ddM1 A1
		(5)
		(10 marks)

Notes

(i)(a)

M1: Full method to find the quotient and remainder. May use long division to produce a linear quotient and a constant remainder or use a partial fraction method to set up correct form and find the constants.

A1: Correct quotient and remainder seen anywhere – even in the division. If using partial fractions with correct form shown allow for the correct constants found.

(b)

M1: $\int\frac{K}{x+2}\,\mathrm{d}x\to A\ln(x+2)$ (A constant) condoning a missing bracket.

B1ft: Correct integration of their linear quotient. Note $(x-5)\to\frac{1}{2}(x-5)^2$ is an equivalent correct expression.

A1: Fully correct answer including the constant. Note $\ln|x+2|$ is fine.

(ii)

Note: d θ need not be seen, and condone use of dx in the integrals.

M1: Attempts to multiply out (correct form for terms – at least three terms) and attempts a correct integration method on either the $\cos^2\theta+\sin^2\theta$ or the $\sin\theta\cos\theta$. Must use correct trig identities – but allow benefit of doubt throughout if e.g. $\cos^2\theta$ and $\sin^2\theta$ double angle identities are two correct identities but may have changed order.

There are a variety of methods that can be used:

$\cos^2\theta+\sin^2\theta=1\rightarrow\theta$ is the most common on these terms but also possible is

$\cos^2\theta+\sin^2\theta=\frac{1}{2}(1+\cos2\theta)+\frac{1}{2}(1-\cos2\theta)\rightarrow\frac{\theta}{2}+A\sin2\theta+\frac{\theta}{2}-A\sin2\theta$

For the other term $2\sin\theta\cos\theta=\sin2\theta\rightarrow K\cos2\theta$ is most common, but you may also see $2\sin\theta\cos\theta\rightarrow K\cos^2\theta$ or $K\sin^2\theta$ (or possibly attempts at integration by parts, but they would need to be complete attempts).

dM1: Multiplies out and applies correct integration technique to all terms of the integral.

A1: Fully correct integral, unsimplified. Possible forms are

$\theta+\frac{1}{2}\cos2\theta,\ \frac{\theta}{2}+\frac{1}{4}\sin2\theta+\cos^2\theta+\frac{\theta}{2}-\frac{1}{4}\sin2\theta,\ \theta-\sin^2\theta$ etc.

Condone a mix of x's and θ's as long as they are recovered by substitution of the limits, but A0 if they do not attempt the substitution.

ddM1: Applies the limits to their full attempt at the integral and evaluates trig expressions– depends on both previous Ms. Both limits must be applied, do not accept just "– 0" for the lower limit if it is not 0

A1: $\frac{\pi}{6}-\frac{1}{4}$ oe simplified expression, e.g $\frac{2\pi-3}{12}$

Question Number	Scheme		Marks
4	Differentiates wrt x $\frac{\mathrm{d}y}{\mathrm{d}x}=16\sec^2(2x)$	Gives $x=\frac{1}{2}\arctan\left(\frac{y}{8}\right)$	M1
	Inverts to get $\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{16\sec^2 2x}=\frac{1}{16(1+\tan^2 2x)}$ Or $\frac{\mathrm{d}y}{\mathrm{d}x}=16\left(1+\tan^2 2x\right)=16\times\left(1+\left(\frac{y}{8}\right)^2\right)$	$\frac{\mathrm{d}x}{\mathrm{d}y}=K\times\frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$	dM1
	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{A}{B+y^2}$	$\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{2}.\frac{1/8}{\left(1+\left(\frac{y}{8}\right)^2\right)}=\frac{A}{B+y^2}$	ddM1
	$=\frac{4}{64+y^2}$		A1
			(4)
			(4 marks)

Notes

Way 1:

M1: Achieves $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\lambda\sec^2(2x)$ or $1=\lambda\sec^2(2x)\frac{\mathrm{d}x}{\mathrm{d}y}$ (oe) In the second case the $\frac{\mathrm{d}x}{\mathrm{d}y}$ must be present, but for this mark allow if just "$=\lambda\sec^2 2x$" is given – but the $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be present for further marks to be scored.

If they change tan $2x$ to $\frac{\sin 2x}{\cos 2x}$ they can score this mark for $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{a\cos 2x\cos 2x\pm b\sin 2x\sin 2x}{(\cos 2x)^2}$

If other identities are seen progress is unlikely and must reach a derivative in terms of trig terms in $2x$. For instance, use of $y=\frac{16\tan x}{1-\tan^2 x}$ will not like make suitable porgress as they will be working in $\tan x$. Only if suitable identities to get in to $2x$ terms is made will they become elligible for this mark. Review any cases you feel worthy of merit and do not know how to score.

dM1: Either

- finds reciprocal of their function (or makes $\frac{\mathrm{d}x}{\mathrm{d}y}$ the subject) **and** uses $1+\tan^2 2x=\sec^2 2x$

  OR
- uses $1+\tan^2 2x=\sec^2 2x$ **and** replaces tan $2x$ by $y/8$ to get and expression in y only

ddM1: Full process (all three steps noted in the dM mark) to make $\frac{\mathrm{d}x}{\mathrm{d}y}$ the subject in terms of y and eliminate fractions to reach the correct form. Note, correct replacement of tan$2x$ by $y/8$ is required but there may be algebraic slips following that in simplifying to the correct form.

A1: cao and depends on all previous marks.

Way 2:

M1: Expresses x as $x=\lambda\arctan\left(\frac{y}{8}\right)$

dM1: Differentiates to achieve a form $K\times\frac{1}{\left(1+\left(\frac{y}{8}\right)^2\right)}$

ddM1: Differentiates their x **correctly** *using the chain rule* **and** eliminates fractions correctly to achieve the correct form.

A1: cao and depends on all previous marks.

Question Number	Scheme	Marks
5 (a)	Way 1: $\dfrac{\cot^2x}{1+\cot^2 x}\equiv\dfrac{\cos^2x/\sin^2 x}{\operatorname{cosec}^2x}\equiv\dfrac{\cos^2x/\sin^2 x}{1/\sin^2 x}\equiv\cos^2 x$	M1, M1, A1
	Way 2: $\dfrac{\cot^2x}{1+\cot^2 x}\equiv\dfrac{\cos^2x}{\sin^2x+\cos^2x}\equiv\dfrac{\cos^2x}{1}\equiv\cos^2 x$	M1, M1 A1
	Way 3: Consider $\cos^2x(1+\cot^2 x)=\cos^2x\operatorname{cos ec}^2x=\dfrac{\cos^2 x}{\sin^2 x}=\cot^2x$ Hence $\dfrac{\cot^2x}{1+\cot^2x}\equiv\cos^2x$	M1 M1 A1
	Way 4: Consider $\cos^2x(1+\cot^2 x)=\cos^2x(1+\dfrac{\cos^2 x}{\sin^2 x})=\dfrac{\cos^2 x(\sin^2 x+\cos^2 x)}{\sin^2 x}=\dfrac{\cos^2 x}{\sin^2 x}=\cot^2x$ Hence $\dfrac{\cot^2x}{1+\cot^2x}\equiv\cos^2x$	M1 M1 A1
		(3)
(b)	$\cos^2x=8(2\cos^2 x-1)+2\cos x$	M1
	$15\cos^2 x+2\cos x-8=0$	A1
	So cos$x$ = 2/3 or $-4/5$ leading to at least one value for x	dM1
	$\Rightarrow x=48.2°$ or $143.1°$ or $216.9°$ or $311.8°$	A1, A1
		(5)
		(8 marks)

Notes

(a)

Way 1 and Way 3:

M1: See one of $1+\cot^2 x=\operatorname{cosec}^2x$, $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$ used.

M1: See two of $1+\cot^2 x=\operatorname{cosec}^2x$, $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$, $\operatorname{cosec}^2x={}^{1}/_{\sin^2 x}$ used.

A1: See all three of $1+\cot^2 x=\operatorname{cosec}^2x$, $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$, $\operatorname{cosec}^2x={}^{1}/_{\sin^2 x}$ used and **conclusion reached** – which may be reaching $\cos^2x$ if working from LHS to RHS.

Way 2:

M1: See one of $\sin^2 x(1+\cot^2 x)=\sin^2x+\cos^2x$, $\cot^2x\sin^2 x=\cos^2x$ used **or implied**

M1: See two of $\sin^2 x(1+\cot^2 x)=\sin^2x+\cos^2x$, $\cot^2x\sin^2 x=\cos^2x$, $\sin^2x+\cos^2x=1$ used **or implied**

A1: See all three of $\sin^2 x(1+\cot^2 x)=\sin^2x+\cos^2x$, $\cot^2x\sin^2 x=\cos^2x$, $\sin^2x+\cos^2x=1$used **or implied** and **conclusion reached**. In this method multiplication of numerator and denominator may be implied and $\sin^2 x(1+\cot^2 x)=\sin^2x+\cos^2x$ **may not be explicit**.

Way 4:

M1: See $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$ used

M1: See both $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$ and $(1+\dfrac{\cos^2 x}{\sin^2 x})=\dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ used

A1: See all three of $\cot^2x={}^{\cos^2x}/_{\sin^2 x}$ $(1+\dfrac{\cos^2 x}{\sin^2 x})=\dfrac{(\sin^2 x+\cos^2 x)}{\sin^2 x}$ and $\sin^2 x+\cos^2 x=1$used and **conclusion reached**.

Note: for conclusions, if working from one side direct to the other, accept successfully reaching the other side as the conclusion. If multiplying or not starting directly from one side some concluding statement will be required (e.g. "hence proved").

There will be some combinations of these methods or variations on these. **A complete method with no errors scores** M1M1A1

In general score M1 for one valid and appropriate identity used (possibly implied), M1 for two valid and appropriate (different) identities applied within a single proof and A1 for a complete process using all necessary identities and if not proving from one side to the other directly a conclusion will also be needed.

E,g. $\frac{\cot^2 x}{1+\cot^2 x} = \frac{\cos^2 x / \sin^2 x}{\operatorname{cosec}^2 x} = \frac{\cos^2 x}{\sin^2 x \operatorname{cosec}^2 x} = \cos^2 x$ scores M1 for use of $\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$, M1 for use of $1+\cot^2 \mathrm{x} = \operatorname{cosec}^2 x$ (or vice versa) and A for the complete proof with all appropriate identities used.

Note: Condone poor notation for the M marks, but for the final A the notation should be correct throughout the proof, allowing a single notational slip if recovered.

(b)

M1: Uses part (a) to replace left hand side and uses double angle formula for cos $2x$ on right hand side to get an equation in cos$x$ only.

Condone $\cos 2x = \pm 2\cos^2 x \pm 1$ for the double angle identity. If $\cos 2x = \cos^2 x - \sin^2 x$ is used this mark is not scored until the $\sin^2 x$ is replaced by $1 - \cos^2 x$. Condone a slip or an omission on either of the coefficients 8 and 2.

A1: Correct three term quadratic reached. Terms should all be gathered on one side unless an appropriate completing the square method is shown to solve to imply it. The = 0 may be implied.

dM1: Solves their 3TQ in cos$x$ by any method – factorising, formula or completion of square or just writing down answers (must be correct for their quadratic if no method shown – you may need to check - an invalid solutions for cos$x$ may be omitted). You must see a value for cos$x$ (or e.g. value for a if as substitution a = cos$x$ has been used) appearing before sight of a value for x. No value for cos x seen will be dM0A0A0 (even if correct answer appear).

A1: Two correct answers in the range. Must be in degrees.

A1: All four answers in the range, $x = awrt\, 48.2°$ or $143.1°$ or $216.9°$ or $311.8°$

Any extra solutions in the range withhold the last A mark.

Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

Question Number	Scheme	Marks
6 (a)	$1000 < V \leqslant 23000$	B1,B1
		(2)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 18000 \times -0.2\mathrm{e}^{-0.2t} + 4000 \times -0.1\mathrm{e}^{-0.1t}$	M1
	$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right\|_{t=10} = 18000 \times -0.2\mathrm{e}^{-2} + 4000 \times -0.1\mathrm{e}^{-1} = (-)634$ (awrt)	dM1A1
		(3)
(c)	$(15000 = 18000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000 \Rightarrow 0 = 9\mathrm{e}^{-0.2t} + 2\mathrm{e}^{-0.1t} - 7)$	
	$0 = (9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$	M1A1
	$9\mathrm{e}^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1
		(4)
		(9 marks)

Notes

(a)

B1: Achieves the value of one end, either 23000 or 1000 associated to the correct end. Condone for this mark the incorrect omission or inclusion of the boundary. Eg $V < 23000$ but do not condone $V \geq 23000$ or just 23000 with no inequality (or similar with the 1000).

B1: Completely correct solution. Accept $1000 < V \leq 23000$, $1000 < Range \leq 23000$, (1000, 23000]. The inequalities must be correct, $1000 < V < 23000$ is B0.

(b)

M1: Differentiates to form $\frac{\mathrm{d}V}{\mathrm{d}t} = A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t} + (0)$ where $A \neq 18000, B \neq 4000$

dM1: Substitutes $t = 10$ into their $\frac{\mathrm{d}V}{\mathrm{d}t}$. It is dependent upon having scored the previous M1.

A1: Correct solution only. Accept awrt ± 634 following a correct $\frac{\mathrm{d}V}{\mathrm{d}t} = -3600\mathrm{e}^{-0.2t} - 400\mathrm{e}^{-0.1t}$. Do not isw if they change to 6.34 as their final answer.

Watch for students who substitute $t = 10$ into their V first and then differentiate. This is M0dM0A0.

Watch for students who achieve +634 following $\frac{\mathrm{d}V}{\mathrm{d}t} = 3600\mathrm{e}^{-0.2t} + 400\mathrm{e}^{-0.1t}$. This is M1dM1A0

(c)

M1: Sets up a 3TQ in $\mathrm{e}^{\pm 0.1t}$ from the information **AND** makes a valid attempt to factorise or solve the quadratic. May be implied by reaching $\mathrm{e}^{\pm 0.1t} = \frac{7}{9}$ or $\frac{9}{7}$ or root of their quadratic either way up. May use a subsitution and solve a quadratic in their variable.

A1: Correct factors $(9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$ or $(7\mathrm{e}^{0.1t} - 9)(\mathrm{e}^{0.1t} + 1)$ or correct solutions associated to the correct exponential terms, though they may omit the negative solution.

dM1: Dependent upon the previous M1. Proceeds via correct ln work to find the exact value for t E.g. This is scored for setting the $a\mathrm{e}^{\pm 0.1t} - b = 0$ and proceeding 'correctly' to $t = \ldots$ or if quadratic solved by other means it is for proceeding from $\mathrm{e}^{\pm 0.1t} = K$ correctly to achieve a value for t.

A1: $t = 10\ln\left(\frac{9}{7}\right)$. Accept other simplest form alternatives such as $-10\ln\left(\frac{7}{9}\right)$ or $\ln\left(\frac{9}{7}\right)^{10}$ and withhold if any extra solutions are given. ISW a correct answer as long as no extra solutions are given.

NB Answers only will score no marks in part (c).

Question Number	Scheme	Marks
7.	$\sqrt{5}\cos\theta - 2\sin\theta \equiv R\cos(\theta + \alpha)$	
(a)	$R = 3$	B1
	$\tan\alpha = \dfrac{2}{\sqrt{5}}, \Rightarrow \ \alpha =$ awrt 0.7297 (4 sf)	M1, A1
		(3)
(b)	$3\cos(\theta + 0.7297) = 0.5 \Rightarrow \cos(\theta + 0.7297) = \dfrac{0.5}{3}$	M1
	$\Rightarrow \theta_1 = 0.674$ (3 sf)	A1
	$\theta_2 + "0.7297" = "-1.4033" \Rightarrow \theta_2 = ...$	dM1
	$\Rightarrow \theta_2 = -2.13$ (3 sf)	A1
		(4)
(c)	E.g. $\begin{matrix} 3A + B = 33 \\ -3A + B = -15 \end{matrix}$ or $\begin{matrix} 3A + B = -15 \\ -3A + B = 33 \end{matrix}$ or $B = \dfrac{33-15}{2} = ...$ $\Rightarrow A = ...$ or $B = ...$	M1
	$A = 8, B = 9$ and $A = -8, B = 9$	A1A1A1
		(4)
		(11 marks)

Notes

(a)

B1: For $R = 3$ stated or seen in their expression. $\sqrt{9}$ is B0. ± 3 is B0.

M1: $\tan\alpha = \pm\dfrac{2}{\sqrt{5}}$, $\tan\alpha = \pm\dfrac{\sqrt{5}}{2} \Rightarrow \alpha = ...$

Also allow $\cos\alpha = \pm\dfrac{\sqrt{5}}{3}$ or $\pm\dfrac{2}{3}$, $\sin\alpha = \pm\dfrac{2}{3}$ or $\pm\dfrac{\sqrt{5}}{3} \Rightarrow \alpha = ...$, where “3” is their R

Note the M may be scored if they find α in degrees (41.8°).

A1: $\alpha =$ awrt 0.7297 (4 sf). No need to state the full expression for this mark.

(b)

M1: Uses part (a) and proceeds to $\cos(\theta + "0.7297") = \dfrac{0.5}{"3"}$. Must see some working, but may be implied by $\theta \pm "0.7297" = 1.4033$ or $\theta \pm "0.7297" = \cos^{-1}\left(\dfrac{0.5}{\text{their } 3}\right)$

A1: Achieves a correct value for θ of awrt 0.674

dM1: Correct method for a second angle, usually given for $\theta_2 +$ their $0.7297 = -$ their $1.4033 \Rightarrow \theta_2 = ...$ May be implied but the previous M must have been gained. If working in degrees as long as they are consistently in degrees.

A1: $\Rightarrow \theta_2 =$ awrt -2.13 (3 sf) but withhold if there are extra solutions in the range and first A scored.

For candidates who work consistently in degrees in (a) and (b) allow awrt 38.6° for the first A in (b) or awrt – 122° for the second A mark if the first A was lost due to error (i.e. max one A mark).

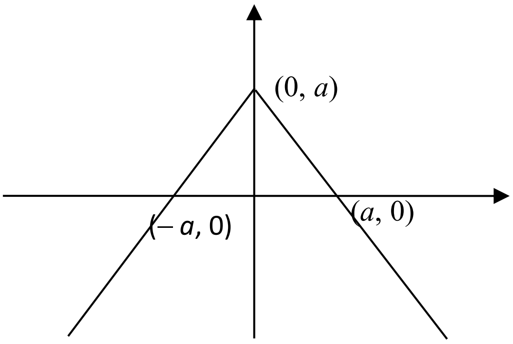
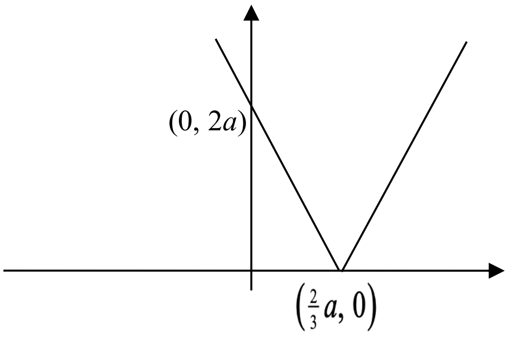
(c)

M1: Correct method to find at least one value. E.g. writes down at least one pair of simultaneous equations (or in-equations) of form $-15 = kA + B$, $33 = -kA + B$ or uses B is mean value of end points other suitable working or equation and finds at least one of $A = ...$ or $B =$ Note correct value for A or B implies M1A1 (with no incorrect working for it)

A1: At least one of $A = 8$, $B = 9$ or $A = -8$

A1: At least two of $A = 8$, $B = 9$ or $A = -8$

A1: For all three of $A = 8$, $B = 9$ and $A = -8$ and no other values.

Question Number	Scheme		Marks
8(a) **(i)**		Correct ∧ shape, any position	B1
		Correct diagram with intercepts correctly placed	B1
(ii)		Correct ∨ shape, any position	B1
		Correct diagram with intercepts correctly placed	B1
			(4)
(b)	$a-x=3x-2a \Rightarrow x=\ldots$ **or** $a-x=-3x+2a \Rightarrow x=\ldots$		M1
	$x=\frac{3}{4}a$ **or** $x=\frac{1}{2}a$		A1
	$a-x=3x-2a \Rightarrow x=\ldots$ **and** $a-x=-3x+2a \Rightarrow x=\ldots$		dM1
	$x=\frac{3}{4}a$ **and** $x=\frac{1}{2}a$		A1
			(4)
			(8 marks)

Notes

(a)(i)

B1: For a correct ∧ shape in any position. Condone if not quite symmetric.

B1: Fully correct sketch, and must continue below the *x* axis, with correct intercepts in terms of *a* either marked on the graph or correctly stated. No need for the 0 coordinate if labelled on the graph. Condone coordinates the wrong way round (eg (*a*,0) instead of (0, *a*)) only if they are in the correct positions on the graph. Condone if not quite symmetric.

(ii)

B1: For a correct v shape in any position, condone if it does not cross the *y* axis. Condone if not quite symmetric. Condone a Y shape as benefit of doubt as an attempt to form the graph.

B1: Fully correct sketch, must continue into second quadrant with same conditions for intercepts as above. Condone if not quite symmetric.

If multiple graphs are drawn on the same axes in (a), if they are labelled score for the graph labelled with correct function/part number. If there is no labelling you may apply bod for the first B only.

(b)

M1: Attempts to solve either correct equation. Must come from correct modulus work (bod if no incorrect work is seen). Watch out for $a-|x|=|3x-2a| \Rightarrow a-|x|=3|x|-2|a| \Rightarrow 3a=4x \Rightarrow x=\frac{3}{4}a$ which obtains a correct answer from incorrect work.

A1: One correct value from a correct equation.

dM1: Attempt to solve both correct equations, which may be amidst other work – again both must come from correct work.

A1: Both values correct and no other values. Any other values must have been rejected.

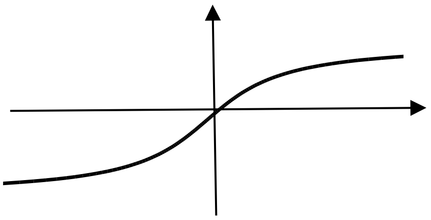
NB: Attempts via squaring may be seen. This will work since both solutions are positive (can be deduced from the sketch) and so $x > 0$ gives $|x| = x$. Score as follows

M1: Attempts to square producing a correct equation.
$x>0 \Rightarrow |x|=x \Rightarrow (a-|x|)^2=(3x-2a)^2 \Rightarrow a^2-2ax+x^2=9x^2-12ax+4a^2$

A1: Solves to produce at least one correct solution $\Rightarrow 8x^2-10ax+3a^2 \Rightarrow x=\frac{3}{4}a$ or $x=\frac{1}{2}a$

dM1: Attempts to solve their correct equation (look for factorisation or formula attempts, though may contain errors simplifying an initially correct equation).

A1: $x=\frac{3}{4}a$ **and** $x=\frac{1}{2}a$ and no other x values. Ignore any y values found for these coordindates.

Question Number	Scheme	Marks
9 (a)	 Curve thro (0,0) in quad 1 and 3 and is increasing	M1
	Gradient $\to 0$ as $x \to \pm\infty$	A1
		(2)
(b)	$3\arctan(x+1)-\pi=0 \Rightarrow \arctan(x+1)=\frac{\pi}{3} \Rightarrow x+1=\tan\left(\frac{\pi}{3}\right)$	M1
	$\Rightarrow (x+1)=\tan\left(\frac{\pi}{3}\right) \Rightarrow x=\tan\left(\frac{\pi}{3}\right)-1=\sqrt{3}-1$	A1
		(2)
(c)	$f(x)=\pm\left(\arctan x-4+\frac{1}{2}x\right) \Rightarrow f(5)=\mp 0.126..,\ f(6)=\pm 0.405...$	M1
	Since f(x) is continuous and changes sign over the interval (5,6), so there is a root in this interval [so $5<\alpha<6$]	A1
		(2)
(d)	$x_1=8-2\arctan 5$	M1
	$x_1=$ cao 5.2532, $\alpha=$ awrt 5.2358	A1, A1
		(3)
		(9 marks)

Notes

(a)

M1: Any mostly increasing curve (allowing tolerance of slight curvature at the ends) through (0,0) and that lies entirely in quadrants 1 and 3 with no clear vertical asymptote(s).

A1: Fully correct sketch with the correct curvature. Asymptotes need not be drawn but the curve must be flattening out at either end. Do not allow for curves that clearly turn back on themselves.

(b)

M1: Proceeds to $\arctan(x+1)=\frac{\pi}{3}$ codoning sign/coefficient slip rearranging and attempts to take tan of both sides. Accept seeing $\pm\sqrt{3}$ or $\pm\frac{1}{\sqrt{3}}$ (awrt 1.73 or 0.577) as evidence if method not shown.

A1: $\sqrt{3}-1$ only. A0 if other solutions are given.

(c)

M1: Substitutes $x=5$ and $x=6$ into $\pm\left(\arctan x-4+\frac{1}{2}x\right)$ and achieves one value correct to at least 1 sf rounded or truncated (evidence of clearly incorrect values that happen to round to 1s.f. is M0). A tighter range may be used, but must include the root.

A1: For achieving

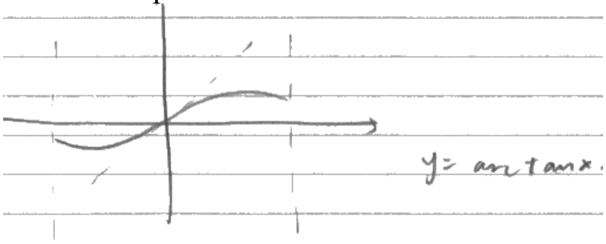
- both values correct (to at least one s.f. as per the M)
- a correct reason e.g. change of sign (accept <0, >0 shown) **and** reference to being a continuous function (condone poor spelling)
- a minimal conclusion, e.g. root, or so $5<\alpha<6$

(d)

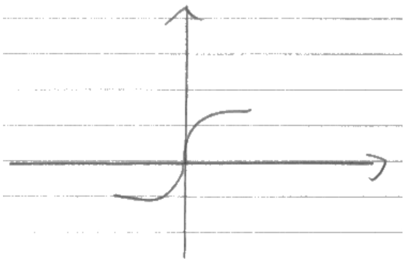
M1: Applies the iteration formula once to achieve a correct expression for x_1 or awrt 5.3 or awrt 5.2 before finding the root.

A1: 5.2532 cao A1: awrt 5.2358 provided the M is scored.

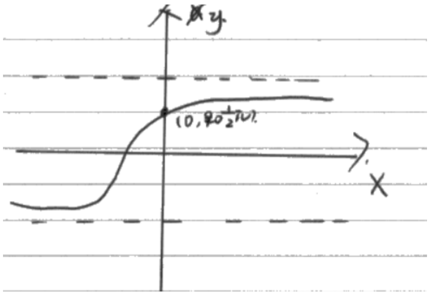
Some examples of sketches and how to score.



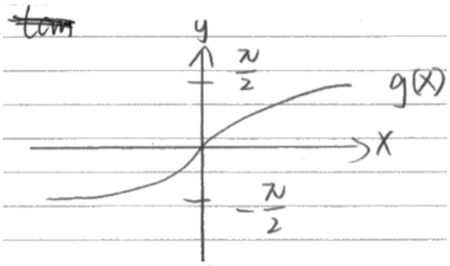
Scores M1A0 for the attempt, allowing tolerance at the ends for the curvature for the M mark, but it is clearly turning back on itself (at both ends) so A0.



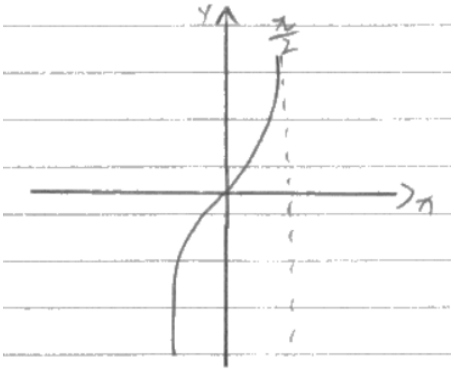
M1 A1 Mostly increasing curve in quadrants 1 and 3 only with no vertical asymptotes and this is a borderline case on the A mark with benefit of doubt applied to the “levelling off” on the left hand end. There is a slight curving back on itself but could be consider as pen slip.



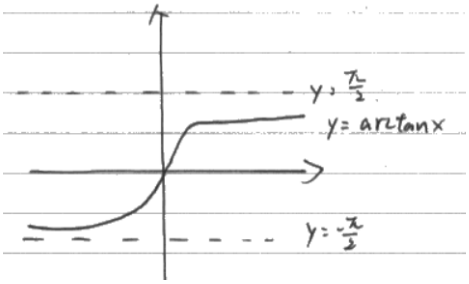
M0A0 Does not pass through (0,0)



M1A1 Suitable sketch showing all the necessary features with ends levelling sufficiently as *x* tends to infinity.



M0A0 There is a clear vertical asymptote



M1A0 Mostly increasing curve in quadrants 1 and 3 only with no vertical asymptotes, tolerable curvature for the M1, but a clear sudden change in gradient is no suitable for the A1.