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MA02

Pure, Statistics and Mechanics Unit PSM1

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
$-x$ EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Mark	Comments
1(a)	$\dfrac{1}{2} \times 6 \times 6 \times \sin\theta = 14$	M1	
	$\sin\theta = \dfrac{7}{9}$ or $\theta = \sin^{-1}\left(\dfrac{7}{9}\right)$	M1	oe
	$\theta = 0.891$	A1	AG To score A1 must have sight of θ accurate to at least 4sf (0.8911…) in radians before final answer. $\sin\theta = 0.777$ ... and $\theta = 0.891$ only scores 0/3
1(b)	6×0.891	M1	
	(Length =) 5.35	A1	AWRT 5.35
	Total	**5**	

Q	Answer	Mark	Comments
2(a)	$m = \dfrac{1}{2}$	B1	Gradient of perpendicular to tangent. PI by later working.
	$y - 12 = \dfrac{1}{2}(x - 6)$ or $y = \dfrac{1}{2}x + 9$	M1	Correct equation of tangent OE
	$a = 2$	A1	AG. Substitutes $x = a$ and $y = 5a$ into equation of tangent and solves to find $a = 2$
2(a) ALT	$m = \dfrac{1}{2}$	(B1)	Gradient of perpendicular to tangent. PI by later working.
	$\dfrac{12 - 5a}{6 - a} = \dfrac{1}{2}$	(M1)	Correct expression for gradient of CP in terms of a equated to $\frac{1}{2}$.
	$a = 2$	(A1)	AG. Must come from completely correct working and needs intermediate line.
2(b)(i)	$\sqrt{(6-2)^2 + (12-10)^2}$	M1	oe.
	$r = \sqrt{20}$	A1	Accept $r = 2\sqrt{5}$
2(b)(ii)	$(x - 2)^2 + (y - 10)^2$	B1	
	... = 20	B1ft	ft their r^2 provided their r is a positive surd. Condone $\left(2\sqrt{5}\right)^2$ Ignore subsequent working after correct answer
2(c)	$Q(-2, 8)$	B2	B1 for answer given as coordinates with one coordinate correct.
	Total	9	

Q	Answer	Mark	Comments
3(a)	$\dfrac{CD}{\sin 25^\circ} = \dfrac{9}{\sin 40^\circ}$	M1	PI Use of Sine Rule.
	$CD = \sin 25^\circ \times \dfrac{9}{\sin 40^\circ}$ $CD = 5.92 \quad (= 5.9172\ldots)$	A1	AG AWRT 5.92 Correct intermediate working and/or more accurate answer must be seen for A1
3(b)	$(AC^2 =)$ $8^2 + 5.92^2 - 2 \times 8 \times 5.92 \times \cos 65^\circ$	M1	RHS of Cosine Rule used.
	$64 + 35.0143\ldots - 40.0121\ldots$	M1	Correct order of evaluation. Condone calculations in radians and award for AC^2 = AWRT 152 or AC = AWRT 12.3
	$(AC =) \sqrt{59.0022\ldots} = 7.6812\ldots = 7.68$	A1	AWRT 7.68
3(c)	$8 \sin x^\circ$ or $x \sin 65^\circ$	M1	Any x PI by later working
	$8 \sin 65^\circ$	A1	Accept $\dfrac{8 \sin 65^\circ}{\sin 90^\circ}$ If derived from an area formula, award for final answer AWRT 7.25 or 7.26
	$7.25 \quad (=7.2504\ldots)$	A1	AWRT 7.25
	Total	8	

Q	Answer	Mark	Comments
4(a)	$10(1 - \sin^2 x) = 7\sin x - 2$ or $10 - 10\sin^2 x = 7\sin x - 2$	M1	Clearly uses $\sin^2 x + \cos^2 x = 1$ to obtain a quadratic equation in $\sin x$ Condone notation slips
	$10\sin^2 x + 7\sin x - 12 = 0$	A1	CSO. AG. Be convinced. Notation must be correct
4(b)	$(2\sin x \pm 3)(5\sin x \pm 4)$ or $\dfrac{-7 \pm \sqrt{7^2 - 4 \times 10 \times -12}}{2 \times 10}$	M1	Method to find roots PI
	$\dfrac{4}{5}$ or $-\dfrac{3}{2}$	A1	Finds at least one root
	$(2\sin x + 3) = 0 \implies$ $\sin x = -\dfrac{3}{2}$. Has no solutions.	E1	'Solution' $\sin x = -\frac{3}{2}$ considered and rejected. If reason given, it must be valid
	$(5\sin x - 4) = 0 \implies$ $\sin x = \frac{4}{5}$ is the only possible value. Megan is correct	E1	Conclusion stated. Dependent on previous E mark being awarded
4(c)	$\theta = 13°$ or $\theta = 87°$	B1	AWRT 13° or AWRT 87°.
	$\theta = 13°$ and $\theta = 87°$	B1	AWRT 13°. AWRT 87°. Ignore extra values given only if outside the range $0° \le \theta \le 360°$. SC1 for AWRT $x = 53°$ and $x = 127°$ only,
	Total	**8**	

Q	Answer	Mark	Comments
5(a)	$\log_5\left(\frac{5^{3a}}{5^b}\right)$ or $\log_5(5^{3a-b})$	M1	Attempt to write numerator as a power of 5. PI by $3a - b$.
	$3a - b = 7$	A1	
	$a + b = 3$	B1	
	$a = \frac{5}{2}$ and $b = \frac{1}{2}$	B1	oe. Both values given.
5(b)	$2 = \log_3(9)$	B1	Seen or used at any stage.
	$\log_3(x + 7)^2$	M1	A law of logs used.
	$\log_3\left(\frac{(x + 7)^2}{5x - 1}\right) = 2$	M1	A second law of logs used leading to an equation including a single log term in x. Accept … $= \log_3(9)$
	$x^2 - 31x + 58 = 0$	A1	Correct 3TQ
	$(x - 2)(x - 29) = 0$	M1	Or correct use of completing the square or the quadratic formula to find at least one root
	$x = 2$ and $x = 29$	A1	CSO
	Total	**10**	

Q	Answer	Mark	Comments
6(a)	<table><tr><td>12</td><td>18</td><td>3</td><td>33</td></tr><tr><td>13</td><td>**10**</td><td>7</td><td>**30**</td></tr><tr><td>5</td><td>2</td><td>0</td><td>7</td></tr><tr><td>**30**</td><td>30</td><td>10</td><td>**70**</td></tr></table>	B1	All values in bold
6(b)(i)	$\dfrac{k}{70}$	M1	ft their 70, 0 < k < their 70
	$\dfrac{7}{70}$ oe or 0.1	A1ft	ft their 70
6(b)(ii)	18 + 10 + 2 + 5	M1	Adds correct values from table ft their 10
	$\dfrac{35}{70}$ oe or 0.5	A1ft	ft their 70
6(b)(iii)	$\dfrac{k}{30}$	M1	ft their 30, 0 < k < their 30
	$\dfrac{12}{30}$ oe or 0.4	A1ft	ft their 30
	Total	**7**	

Q	Answer	Mark	Comments
7(a)	$(0^2 + 4)a + (1^2 + 4)b + (2^2 + 4)0.125 + (3^2 + 4)0.125 = 6.225$ or $b + 2^2 \times 0.125 + 3^2 \times 0.125 + 4 = 6.225$	M1	Forms correct equation using $E(X^2 + 4) = 6.225$
	$a + b + 0.125 + 0.125 = 1$	M1	Forms correct equation using sum of probabilities = 1
	$a = 0.15$ or $b = 0.6$	A1	One of a or b Accept $a = 3/20$ oe or $b = 3/5$ oe
	$a = 0.15$ and $b = 0.6$	A1	Both a and b Accept $a = 3/20$ oe and $b = 3/5$ oe
7(b)	$E(6Y - 9) = 6E(Y) - 9$	M1	Applies expectation formula Implied by $6 \times 6.225 - 9$ or correct final answer
	$= 28.35$	A1	Accept 567/20 oe
	Total	**6**	

Q	Answer	Mark	Comments
8(a)	$\binom{10}{4} p^4 (1-p)^{10-4}$	M1	Expression for P(X = 4) with $0 < p < 1$
	= 0.054	A1	AWRT
8(b)	P(X ≥ 1) > 0.95 or P(X ≥ 1) = 0.95	M1	Correct inequality or equation Condone ≥
	P(X = 0) ≤ 0.05 or P(X = 0) = 0.05	M1	Correct inequality or equation Condone <
	$\left(\dfrac{5}{6}\right)^n \leq 0.05$ or $\left(\dfrac{5}{6}\right)^n = 0.05$	A1	Replace P(X = 0) with correct probability Condone < oe
	$n \geq \log_{\frac{5}{6}} 0.05$ or $n = \log_{\frac{5}{6}} 0.05$ or $n \geq \dfrac{\log 0.05}{\log \frac{5}{6}}$ or $n = \dfrac{\log 0.05}{\log \frac{5}{6}}$	M1	Uses logarithms to solve for n oe Implied by sight of 16.43
	n = 17	A1	Do not award without more accurate answer seen first
8(b) ALT	Trial or error – last two marks $\left(\dfrac{5}{6}\right)^{16} = 0.0541$ and $\left(\dfrac{5}{6}\right)^{17} = 0.0451$	(M1)	Both n = 16 and n = 17 into their expression
	n = 17	(A1)	
	Total	**7**	

Q	Answer	Mark	Comments
9	$\frac{1}{2} \times 18 \times 12 = 108$	M1	Calculates area of first triangle PI by later working.
	$\frac{1}{2} \times 12 \times 3 = 18$	M1	Calculates area of second triangle PI by later working. Condone $\frac{1}{2} \times 12 \times -3$
	Total distance travelled = 126	A1	PI by later working
	$\frac{126}{30}$	M1	Their total distance divided by 30
	Average speed = 4.2	A1	CSO
	Total	**5**	

Q	Answer	Mark	Comments
10(a)	$6 \times \pm 12 + 4.5 \times \pm 8 = 6 \times \pm 3 \pm 4.5v$	M1	Four-term equation for COM ignoring signs but with correct values substituted.
	$6 \times 12 + 4.5 \times (-8) = 6 \times 3 + 4.5v$	A1	Correct equation simplified or unsimplified.
	$v = 4$	A1	CSO
10(b)	(A) $\ I = 6 \times 12 - 6 \times 3$ or (B) $\ I = 4.5 \times 4 - 4.5 \times (-8)$	M1	Allow subtraction either way round If *B* used ft v from part (a)
	$I = 54$	A1	CSO. Must be positive value.
	Total	**5**	

Q	Answer	Mark	Comments
11(a)	$T - mg = 1.4m$	M1	Correct three term equation of motion ignoring signs with g or 9.8.
		A1	Correct three term equation of motion with g or 9.8.
	$bmg - T = 1.4bm$	M1	Correct three term equation of motion ignoring signs with g or 9.8.
		A1	Correct three term equation of motion with g or 9.8.
	$bg - g = 1.4b + 1.4$	M1ft	oe. Attempt to eliminate T and combine their equations of motion to form an equation in b with m eliminated. g or 9.8
	$b = \dfrac{4}{3}$	A1	oe CSO Condone 1.33… or better.
11(b)	$v^2 = 2 \times 1.4 \times 2.5$	M1	Correct use of $v^2 = u^2 + 2as$ or complete method to find speed of A when B hits the surface
	$v = \sqrt{7}$ or $v = 2.65$	A1	CSO AWRT 2.65
	$0 = \sqrt{7} - gt$	m1	Correct use of $v = u + at$. g or 9.8 ft their $\sqrt{7}$. Dependent on first M mark
	$t = 0.27$	A1	CSO AWRT Do not ignore subsequent working
	Total	**10**	