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MA03

(9660/MA03) Unit P2 – Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments										
1(a)	<table border="1" data-bbox="277 383 815 689"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0.1</td> <td>$\sin(e^{0.1}) = 0.8935409$</td> </tr> <tr> <td>0.3</td> <td>$\sin(e^{0.3}) = 0.9756924$</td> </tr> <tr> <td>0.5</td> <td>$\sin(e^{0.5}) = 0.9969654$</td> </tr> <tr> <td>0.7</td> <td>$\sin(e^{0.7}) = 0.9034885$</td> </tr> </tbody> </table> <p data-bbox="277 763 770 808">$0.2 \times [0.89... + 0.97... + 0.99... + 0.90...]$</p> <p data-bbox="277 949 384 987">$= 0.754$</p>	x	y	0.1	$\sin(e^{0.1}) = 0.8935409$	0.3	$\sin(e^{0.3}) = 0.9756924$	0.5	$\sin(e^{0.5}) = 0.9969654$	0.7	$\sin(e^{0.7}) = 0.9034885$	<p data-bbox="927 376 967 405">B1</p> <p data-bbox="927 501 967 530">M1</p> <p data-bbox="927 757 967 786">m1</p> <p data-bbox="927 949 967 978">A1</p>	<p data-bbox="1034 353 1493 427">All 4 correct x values (and no extra used) PI by 4 correct y values</p> <p data-bbox="1034 517 1543 667">At least 3 correct y values in exact form or decimals, rounded or truncated to 2 dp or better (in table or formula) (PI by AWRT correct answer)</p> <p data-bbox="1034 763 1513 913">Correct sub into formula with $h = 0.2$ OE and at least 3 correct y values either listed, with + signs, or totalled. (PI by AWRT correct answer)</p> <p data-bbox="1034 949 1528 1010">CAO, must see this value exactly and no error seen</p>
x	y												
0.1	$\sin(e^{0.1}) = 0.8935409$												
0.3	$\sin(e^{0.3}) = 0.9756924$												
0.5	$\sin(e^{0.5}) = 0.9969654$												
0.7	$\sin(e^{0.7}) = 0.9034885$												
		4											
1(b)(i)	<p data-bbox="277 1099 576 1144">$f(x) = \sin(e^x) - 3x + 2$</p> <p data-bbox="277 1167 480 1211">$f(0.8) = 0.39...$</p> <p data-bbox="277 1227 512 1272">$f(0.9) = -0.069...$</p> <p data-bbox="277 1294 655 1339">Change of sign, $0.8 < \alpha < 0.9$</p>	<p data-bbox="927 1160 967 1189">M1</p> <p data-bbox="927 1294 967 1323">A1</p>	<p data-bbox="1034 1115 1174 1144">Or reverse</p> <p data-bbox="1034 1160 1543 1227">Both values rounded or truncated to at least 1sf</p> <p data-bbox="1034 1294 1543 1361">Must have both statement and interval in words or symbols</p> <p data-bbox="1034 1368 1326 1406">or comparing 2 sides:</p> <p data-bbox="1034 1413 1417 1458">at 0.8, $\sin(e^{0.8}) > 3 \times 0.8 - 2$;</p> <p data-bbox="1034 1480 1477 1525">at 0.9, $\sin(e^{0.9}) < 3 \times 0.9 - 2$ (M1)</p> <p data-bbox="1034 1547 1543 1585">Conclusion as before (A1)</p>										
		2											
1(b)(ii)	<p data-bbox="277 1671 416 1715">$x_2 = 0.931$</p> <p data-bbox="277 1720 416 1765">$x_3 = 0.856$</p>	<p data-bbox="927 1682 967 1711">B1</p> <p data-bbox="927 1727 967 1756">B1</p>											
		2											
	Total	8											

Q	Answer	Marks	Comments
2(a)	$\left[\frac{dy}{dx} = \frac{(2x+5) \times (-3) - (1-3x) \times 2}{(2x+5)^2} \right]$ $= \frac{-17}{(2x+5)^2}$	<p>M1</p> <p>A1</p>	<p>or use of product rule</p> <p>PI by correct answer</p>
		2	
2(b)	$\left[\frac{dy}{dx} = \frac{-17}{(2x+5)^2} \times \frac{(2x+5)}{(1-3x)} \right]$ $= \frac{-17}{(2x+5)(1-3x)}$	<p>M1</p> <p>A1</p>	<p>their (a) $\times \frac{2x+5}{1-3x}$</p> <p>oe such as $\frac{A}{1-3x} - \frac{B}{5+2x}$</p> <p>with $A < 0$ and $B > 0$</p> <p>ft their (a)</p> <p>ACF such as $\frac{-3}{1-3x} - \frac{2}{2x+5}$</p>
		2	
	Total	4	

Q	Answer	Marks	Comments
3(a)	$[16 \sin \theta + 30 \cos \theta =]$ $R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ $\alpha = 1.08$ $R = 34$ $[34 \sin(\theta + 1.08)]$	M1 A1 B1	Implied by $16 = R \cos \alpha$ and $30 = R \sin \alpha$
		3	
3(b)(i)	[Min value =] -34	B1ft	
		1	
3(b)(ii)	[$\theta =$] 3.63	B1	oe such as -2.65 Accept 3.6 Accept $1.5\pi - 1.08$ $[\pm 2n\pi]$
		1	
	Total	5	

Q	Answer	Marks	Comments
4(a)(i)	$18(-0.5)^3 + b(-0.5)^2 + c(-0.5) - 4 = 0$ $18\left(\frac{1}{3}\right)^3 + b\left(\frac{1}{3}\right)^2 + c\left(\frac{1}{3}\right) - 4 = -5$ $b - 2c = 25$ $b + 3c = -15$ $b = 9, \quad c = -8$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>At least one correct substitution or M1 use of long division $9x^2 + \left(\frac{b-9}{2}\right)x$</p> <p>Both substitutions correct or A1 for $9x^2 + \left(\frac{b-9}{2}\right)x + \frac{1}{2}\left(c - \frac{b-9}{2}\right)$</p> <p>Attempt to solve their simultaneous equations</p> <p>Both values correct</p>
		4	
4(a)(ii)	$f(x) = (2x+1)(9x^2 - 4)$ $= (2x+1)(3x+2)(3x-2)$	<p>M1</p> <p>A1</p>	<p>PI</p> <p>Condone $p = 3$ and $q = 2$</p>
		2	
4(b)	$\frac{f(x)}{(3x+2)(x^2-2)} = \frac{(2x+1)(3x-2)}{(x^2-2)}$ $= \frac{6x^2 - x - 2}{x^2 - 2}$ $= \frac{6x^2 - 12 - x + 10}{x^2 - 2}$ $= 6 + \frac{10 - x}{x^2 - 2}$	<p>M1</p> <p>A1</p>	<p>Substitutes their $f(x)$ and correctly cancels the factor of $3x + 2$ in numerator and denominator</p> <p>Be convinced</p>
		2	
	Total	8	

Q	Answer	Marks	Comments
5(a)	$3 \sec^2 Y = 2 - 4 \tan Y$ $3(1 + \tan^2 Y) = 2 - 4 \tan Y$ $3 \tan^2 Y + 4 \tan Y + 1 = 0$ $\tan Y = -1, -\frac{1}{3}$ $Y = -0.785 \left(\text{or } -\frac{\pi}{4} \right), -0.322$ $x = 0.11, 1.68; 0.34, 1.91$	<p>M1</p> <p>m1</p> <p>A1</p> <p>B2,1</p>	<p>Correct use of trig identity</p> <p>Attempt to solve their quadratic</p> <p>At least one correct Y value PI by a correct value of x Condone Y value(s) given in degrees, e.g. $-45^\circ, -18.4\dots^\circ$</p> <p>2 dp or better B1: at least 3 correct values for x B2: all 4 correct values for x and no others</p> <p>Allow use of $X, 2x - 1$ etc in place of Y throughout</p>
		5	
5(b)	$\frac{\sin 4x(1 - \cos 2x)}{\cos 2x(1 - \cos 4x)} = \frac{2 \sin 2x(1 - \cos 2x)}{1 - \cos 4x}$ $\frac{2 \sin 2x(1 - \cos 2x)}{1 - \cos 4x}$ $= \frac{2 \sin 2x(1 - 1 + 2 \sin^2 x)}{1 - 1 + 2 \sin^2 2x}$ $\frac{2 \sin 2x(1 - 1 + 2 \sin^2 x)}{1 - 1 + 2 \sin^2 2x} = \frac{2 \sin^2 x}{\sin 2x}$ $\frac{2 \sin^2 x}{\sin 2x} = \frac{\sin x}{\cos x} = \tan x$	<p>M1</p> <p>m1</p> <p>m1</p> <p>A1</p>	<p>Use of $\sin 4x = 2 \sin 2x \cos 2x$</p> <p>Use of $\cos 4x$ trig identity</p> <p>Cancelling of $\sin 2x$ oe</p> <p>AG Be convinced</p>
		4	
	Total	9	

Q	Answer	Marks	Comments
6(a)	$= 1 + \left(-\frac{1}{3}\right) \times (-x) + \frac{\left(-\frac{1}{3}\right) \times \left(-\frac{4}{3}\right) \times (-x)^2}{2}$ $+ \frac{\left(-\frac{1}{3}\right) \times \left(-\frac{4}{3}\right) \times \left(-\frac{7}{3}\right) \times (-x)^3}{6}$ $= 1 + \frac{1}{3}x + \frac{2}{9}x^2 + \frac{14}{81}x^3$	<p>M1 A1</p> <p>A1</p>	<p>M1: At least 3 terms correct (unsimplified)</p> <p>A1: All terms correct (unsimplified)</p>
		3	
6(b)(i)	$\sqrt[3]{\frac{1}{1-2x}} = (1-2x)^{-\frac{1}{3}}$ $= 1 + \frac{1}{3} \times 2x + \frac{2}{9} \times (2x)^2 + \frac{14}{81} \times (2x)^3$ $= 1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3$	<p>M1</p> <p>A1</p>	<p>Substitutes $2x$ in to their (a)</p>
		2	
6(b)(ii)	$-0.5 < x < 0.5$	B2	<p>oe such as $x < 0.5$</p> <p>B1 for $-0.5 \leq x \leq 0.5$</p>
		2	
6(c)	<p>$[x = 0.1] \quad 1 + \frac{2}{3} \times 0.1 + \frac{8}{9} \times 0.1^2 + \frac{112}{81} \times 0.1^3$</p> <p>$[= 1.0769\dots]$</p> <p>$[x = 0.1] \quad \frac{1}{\sqrt[3]{1-0.2}} = \frac{1}{\sqrt[3]{0.8}} = \sqrt[3]{\frac{10}{8}}$</p> <p>$\sqrt[3]{10} \quad [= 2 \times 1.0769\dots] = 2.154$</p>	<p>B1ft</p> <p>M1</p> <p>A1</p>	<p>Substitutes $x = 0.1$ into their (b)(i)</p> <p>oe</p> <p>AWRT 2.154 from correct use of binomial expansion Value calculated using this binomial expansion is 2.153876543</p>
		3	
	Total	10	

Q	Answer	Marks	Comments
7(a)	Translation $\begin{bmatrix} 0 \\ k \end{bmatrix}$ or Stretch + either I or II $k = 1$ [Followed by] Stretch + either I or II I: Parallel to y-axis II: SF $\frac{1}{3}$	M1 A1 M1 A1	
		4	
7(b)	$x = \frac{1 + \cos y}{3}$ $3x - 1 = \cos y$ $[f^{-1}(x) =] \cos^{-1}(3x - 1)$	M1 A1	Interchanging x and y
		2	
	Total	6	

Q	Answer	Marks	Comments
8(a)(i)	$6 = a(x^2 + 1) + x(bx)$ $a = 6, b = -6$ $\frac{6}{x^3 + x} = \frac{6}{x} - \frac{6x}{x^2 + 1}$	M1 A1	
		2	
8(a)(ii)	$\int \frac{6}{x^3 + x} [dx] = \int \frac{6}{x} - \frac{6x}{x^2 + 1} [dx]$ $= 6 \ln x - 3 \ln(x^2 + 1)$ $\left[\int_1^2 \frac{6}{x^3 + x} dx \right]$ $= (6 \ln 2 - 3 \ln 5) - (0 - 3 \ln 2)$ $= \ln \left(\frac{512}{125} \right)$	M1 A1 M1 A1	ft their a and b from (a)(i) Substitutes $x = 1$ and $x = 2$ into their integration, provided it is in the form $= a \ln x + b \ln(x^2 + 1)$ ACF such as $\ln \left(\frac{2^9}{5^3} \right)$
		4	

Q	Answer	Marks	Comments
9(a)	$\frac{1}{(30-x)(10-x)} = \frac{A}{30-x} + \frac{B}{10-x}$ $1 = A(10-x) + B(30-x)$ $A = -\frac{1}{20}, B = \frac{1}{20}$ $\frac{1}{(30-x)(10-x)} = -\frac{1}{20(30-x)} + \frac{1}{20(10-x)}$	<p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p>	
		2	

<p>9(b)</p> $\frac{dx}{dt} = k(30-x)(10-x)$ $\int \frac{1}{(30-x)(10-x)} dx = \int \frac{-1}{20(30-x)} + \frac{1}{20(10-x)} dx$ $= \frac{1}{20} \ln(30-x) - \frac{1}{20} \ln(10-x)$ $\frac{1}{20} \ln\left(\frac{30-x}{10-x}\right) = kt + c$ $[t=0, x=0 \Rightarrow] \quad c = \frac{1}{20} \ln 3$ $\frac{1}{20} \ln\left(\frac{30-x}{10-x}\right) = kt + \frac{1}{20} \ln 3$ $[t=2, x=6 \Rightarrow] \quad \frac{1}{20} \ln\left(\frac{24}{4}\right) = 2k + \frac{1}{20} \ln 3$ $k = \frac{1}{40} \ln 2$ $\frac{1}{20} \ln\left(\frac{30-x}{10-x}\right) = \left(\frac{1}{40} \ln 2\right)t + \frac{1}{20} \ln 3$ $\ln\left(\frac{30-x}{10-x}\right) = \frac{t}{2} \ln 2 + \ln 3$ $\left(\frac{30-x}{10-x}\right) = 3 \times 2^{0.5t}$ $x = \frac{30 \times (2^{0.5t} - 1)}{3 \times 2^{0.5t} - 1}$		<p>B1</p> <p>M1</p> <p>m1</p> <p>A1ft</p> <p>m1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>Uses their partial fractions to separate variables</p> <p>Attempt to integrate</p> <p>ft their A and B from (a)</p> <p>Attempt to find c</p> <p>Attempt to find k</p> <p>Both c and k correct</p> <p>Attempt to solve</p> <p>ACF</p>
		<p>9</p>	
	<p>Total</p>	<p>11</p>	

Q	Answer	Marks	Comments
10(a)	$\frac{dx}{dt} = -3\cos^2 t \sin t$	M1	At least one derivative correct
	$\frac{dy}{dt} = -2\cos t \sin^2 t + (2 + \cos^2 t)\cos t$ $\left[= -2\cos t(1 - \cos^2 t) + 2\cos t + \cos^3 t = 3\cos^3 t \right]$	A1	Both derivatives correct
	$\frac{dy}{dx} = \frac{-3\cos^3 t}{3\cos^2 t \sin t}$	M1	Correct use of trig identity
	$\frac{dy}{dx} = -\frac{\cos t}{\sin t} = -\cot t$	A1	AG Be convinced
		4	
10(b)	gradient at $t = p$ is $-\frac{1}{-\cot p} \left[= \frac{\sin p}{\cos p} = \tan p \right]$	M1	Must be in terms of p
	$y - (2 + \cos^2 p)\sin p = \frac{\sin p}{\cos p}(x - \cos^3 p)$	A1	ACF $y = (\tan p)x + 2\sin p$
		2	
10(c)	$[x = 0 \Rightarrow] y - (2 + \cos^2 p)\sin p = \frac{\sin p}{\cos p}(-\cos^3 p)$	M1	Maybe seen in (b)
	$y = 2\sin p + \sin p \cos^2 p - \sin p \cos^2 p = 2\sin p$	A1	
	$[y = 0 \Rightarrow] -(2 + \cos^2 p)\sin p = \frac{\sin p}{\cos p}(x - \cos^3 p)$	M1	Maybe seen in (b)
	$x = -2\cos p - \cos^3 p + \cos^3 p = -2\cos p$	A1	
	$[AB =] \sqrt{4\sin^2 p + 4\cos^2 p}$ $[AB =] 2$	M1 A1	CAO
		6	
	Total	12	

Q	Answer	Marks	Comments
11(a)	$A(-2.5, 0), B(0, 5)$	B1	
		1	
11(b)(i)	$\left[\frac{dy}{dx} = 2e^{-x} - (5+2x)e^{-x} \right]$ $\left[\frac{dy}{dx} = -3e^{-x} - 2xe^{-x} \right]$	M1 A1	M1: $ae^{-x} + bxe^{-x}$ A1: $-3e^{-x} - 2xe^{-x}$ ACF
		2	
11(b)(ii)	$-3e^{-x} - 2xe^{-x} = 0$ $-e^{-x}(3+2x) = 0$ $\left[e^{-x} \neq 0 \Rightarrow \right] 3+2x = 0$ $(-1.5, 2e^{1.5})$	M1 A1	ft their [simplified] derivative oe
		2	
11(b)(iii)	$\left[\frac{d^2y}{dx^2} = 3e^{-x} + 2xe^{-x} - 2e^{-x} \right]$ $x = -1.5, \frac{d^2y}{dx^2} = -2e^{1.5} [= -8.96...] < 0$ <p>Hence [local] maximum</p>	B1 E1	or considers first derivative either side of $x = -1.5$, e.g. $x < -1.5 \Rightarrow \frac{dy}{dx} > 0$ $x > -1.5 \Rightarrow \frac{dy}{dx} < 0$ Must have been awarded B1
		2	

11(c)	$\int (5+2x)e^{-x} dx = -(5+2x)e^{-x} + \int 2e^{-x} dx$ $= -5e^{-x} - 2xe^{-x} - 2e^{-x}$ $= -7e^{-x} - 2xe^{-x}$ $\int_{-2.5}^0 (5+2x)e^{-x} dx = [-7e^{-x} - 2xe^{-x}]_{-2.5}^0$ $= (-7) - (-7e^{2.5} + 5e^{2.5})$ $= -7 + 2e^{2.5}$ <p>[Area of Triangle =] $0.5 \times 5 \times 2.5 \left[= \frac{25}{4} \right]$</p> <p>[Area =] $-7 + 2e^{2.5} - 0.5 \times 5 \times 2.5$</p> <p>[Area =] $-13.25 + 2e^{2.5}$</p>	M1 m1 A1 M1 B1 A1	oe Use of integration by parts Condone omission of dx throughout Complete use of integration by parts Their integral of the form $ae^{-x} + bxe^{-x}$ evaluated between their -2.5 and 0 May be seen at any point in their solution oe such as $2e^{2.5} - \frac{53}{4}$
		6	
	Total	13	

Q	Answer	Marks	Comments
12	$y = x \ln(x + y)$ $\frac{dy}{dx} = \ln(x + y) + \frac{x}{x + y} \left(1 + \frac{dy}{dx} \right)$ $(x + y) \frac{dy}{dx} = (x + y) \ln(x + y) + x \left(1 + \frac{dy}{dx} \right)$ $x \frac{dy}{dx} + y \frac{dy}{dx} = (x + y) \frac{y}{x} + x + x \frac{dy}{dx}$ $y \frac{dy}{dx} = (x + y) \frac{y}{x} + x$ $\frac{dy}{dx} = (x + y) \frac{1}{x} + \frac{x}{y}$ $\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{x}{y}$	<p>M1 A1</p> <p>m1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>M1: Attempt at implicit differentiation A1: All correct</p> <p>Eliminates the fraction</p> <p>Expands & eliminates logarithm or correctly isolates $\frac{dy}{dx}$ term</p> <p>Expands & eliminates logarithm and correctly isolates $\frac{dy}{dx}$ term</p> <p>AG Be convinced</p>
	Total	6	

<p>12 ALT 1</p>	$\left[\frac{y}{x} = \ln(x+y) \Rightarrow \right] e^{\frac{y}{x}} = x+y$ $\frac{d}{dx} \left(e^{\frac{y}{x}} \right) = \frac{d}{dx} (x+y)$ $e^{\frac{y}{x}} \left(x \frac{dy}{dx} - y \right) = 1 + \frac{dy}{dx}$ $(x+y) \left(x \frac{dy}{dx} - y \right) = x^2 \left(1 + \frac{dy}{dx} \right)$ $x^2 \frac{dy}{dx} - xy + yx \frac{dy}{dx} - y^2 = x^2 + x^2 \frac{dy}{dx}$ $yx \frac{dy}{dx} = y^2 + x^2 + xy$ $\frac{dy}{dx} = \frac{y^2 + x^2 + xy}{yx}$ $\frac{dy}{dx} = \frac{y}{x} + \frac{x}{y} + 1$	<p>M1</p> <p>m1 A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>PI</p> <p>m1: Attempt at implicit differentiation A1: All correct</p> <p>Eliminates exponential term or correctly isolates $\frac{dy}{dx}$ term</p> <p>Eliminates exponential term and correctly isolates $\frac{dy}{dx}$ term</p> <p>AG Be convinced</p>
Total		6	
<p>12 ALT 2</p>	$\frac{y}{x} = \ln(x+y)$ $\frac{d}{dx} \left(\frac{y}{x} \right) = \frac{d}{dx} (\ln(x+y))$ $\frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x+y} \left(1 + \frac{dy}{dx} \right)$ $(x+y) \left(x \frac{dy}{dx} - y \right) = x^2 \left(1 + \frac{dy}{dx} \right)$ $\frac{dy}{dx} (x(x+y) - x^2) = xy + y^2 + x^2$ $\frac{dy}{dx} = \frac{xy + y^2 + x^2}{xy}$ $\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{x}{y}$	<p>M1</p> <p>m1 A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>PI</p> <p>m1: Attempt at implicit differentiation A1: All correct</p> <p>Eliminates fractions or correctly isolates $\frac{dy}{dx}$ term</p> <p>Eliminates fractions and correctly isolates $\frac{dy}{dx}$ term</p> <p>AG Be convinced</p>
Total		6	

Q	Answer	Marks	Comments
13(a)	$[AB =]\sqrt{(16-2)^2 + (-1-(-3))^2 + (-1-7)^2}$ $[AB =]\sqrt{14^2 + 2^2 + (-8)^2}$ $AB = \sqrt{264}$	M1 A1	oe oe such as $2\sqrt{66}$ Condone 16.2[48...]
		2	
13(b)(i)	$2+14\lambda = 9+5\mu$ $-3+2\lambda = -2-4\mu$ $\lambda = 0.5, \quad \mu = 0$ $7-8\lambda = q+5\mu$ $q = 3$	M1 A1 A1	May use B instead Equating x and y
		3	
13(b)(ii)	$\begin{bmatrix} 14 \\ 2 \\ -8 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -4 \\ 5 \end{bmatrix} = 22$ $\cos \theta = \frac{\pm 22}{\sqrt{66}\sqrt{264}} \left[= \pm \frac{1}{6} \right]$ $[\theta =] 80.4^{[^\circ]}$	M1 A1 M1 A1	M1: Use of scalar product with direction vectors of l and AB A1: Correctly finds 22 ft their 22 from the scalar product between the two correct vectors
		4	

<p>13(c)</p>	$C(9+5c, -2-4c, 3+5c) \text{ or } \overrightarrow{OC} = \begin{bmatrix} 9+5c \\ -2-4c \\ 3+5c \end{bmatrix}$ $\overrightarrow{CD} = \begin{bmatrix} 10+5c \\ -4-4c \\ 5c \end{bmatrix}$ $\begin{bmatrix} 10+5c \\ -4-4c \\ 5c \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -4 \\ 5 \end{bmatrix} = 0$ $50 + 25c + 16 + 16c + 25c = 0$ $C(4, 2, -2) \text{ or } \overrightarrow{OC} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$ $BC^2 = (4-16)^2 + (2-(-1))^2 + (-2-(-1))^2 = \sqrt{154}$ $AC^2 = (4-2)^2 + (2-(-3))^2 + (-2-7)^2 = \sqrt{110}$ $AB^2 = AC^2 + BC^2 \text{ [so right-angled triangle]}$	<p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>oe</p> <p>$66c = -66, \quad c = -1$</p> <p>or finding $\overrightarrow{BC} \cdot \overrightarrow{AC} = -24 + 15 + 9$</p> <p>Note $\overrightarrow{AC} = \begin{bmatrix} 2 \\ 5 \\ -9 \end{bmatrix}$ and $\overrightarrow{BC} = \begin{bmatrix} -12 \\ 3 \\ -1 \end{bmatrix}$</p> <p>$\overrightarrow{BC} \cdot \overrightarrow{AC} = 0$ [so right-angled triangle]</p>
		<p>6</p>	
	<p>Total</p>	<p>15</p>	