OXFORD

INTERNATIONAL
AQA EXAMINATIONS

# INTERNATIONAL AS

# MATHEMATICS

# MA02

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

June 2022

Version 1.0 Final

226XMA02/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

**Copyright information**

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2022 Oxford International AQA Examinations and its licensors. All rights reserved.

**Key to mark scheme abbreviations**

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| √ **or ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **A2, 1** | 2 or 1 (or 0) accuracy marks |
| **–*x* EE** | Deduct *x* marks for each error |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **SCA** | Substantially correct approach |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **1(a)(i)** | $A = 4$ | **B1** | |
| | | **1** | |
| **1(a)(ii)** | $20 = 4 \times 3^{8k}$<br><br>$k = \frac{1}{8}\log_3(5)$ | **M1**<br><br>**A1** | **oe** Correct substitution of $(8,\ 20)$ and their $A$ from **part 1(a)(i)**.<br>**CSO**<br>Accept $k = 0.125\log_3(5)$ |
| | | **2** | |
| **1(b)** | $2x = \log_4(11)$<br>or<br>$2x = 1.72[971...]$<br>or<br>$2x\log_{[n]}(4) = \log_{[n]}(11)$<br><br>$x = 0.865$ | **M1**<br><br>**A1** | **oe PI** by $x = 0.86[4857...]$<br><br>**CAO** to 3 sf |
| | | **2** | |
| | **Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **2** | $\frac{1}{2}r^2\theta = 3$<br><br>$r\theta + 2r = 8$<br><br>$r^2 - 4r + 3 = 0$<br><br>or<br><br>$3\theta^2 - 20\theta + 12 = 0$<br><br>$r = 3$ or $\theta = \frac{2}{3}[= 0.66...]$<br><br>$r = 3$ and $\theta = \frac{2}{3}[= 0.66...]$ | **B1**<br><br>**B1**<br><br>**M1**<br><br>**A1**<br><br>**A1** | **oe PI** by later working<br><br>**oe PI** by later working<br><br>**oe**<br>Eliminates a variable to find a correct equation in either $r$ or $\theta$<br>**PI** by $r = 1$ and $r = 3$, or $\theta = \frac{2}{3}[= 0.66...]$ and $\theta = 6$<br>For $\theta = \frac{2}{3}$ allow $\theta = 0.67$<br>In both **A1** marks, allow $\theta = 0.67$<br>Condone sight of $r = 1$ and/or $\theta = 6$ in addition to correct value(s)<br><br>This pair of values and no others |
| | **Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **3(a)** | $[\log_a 2x =] \log_a 4^3 + \log_a 5$<br>or<br>$[\log_a 2x =] \log_a 64 + \log_a 5$ | **M1** | One log rule used |
| | $[\log_a 2x =] \log_a (4^3 \times 5)$<br>or<br>$[\log_a 2x =] \log_a (64 \times 5)$<br>or<br>$[\log_a 2x =] \log_a (320)$ | **M1** | Second log rule used |
| | $[\Rightarrow 2x = 320]$<br>$x = 160$ | **A1** | **CSO**<br>**AG** Be convinced |
| | | **3** | |
| **3(b)** | $\left[\begin{array}{l}\log_a y = 9 + \log_a 10 \\ \Rightarrow \log_a y - \log_a 10 = 9\end{array}\right]$ | | |
| | $\log_a \left(\frac{y}{10}\right) = 9$ | **M1** | Accept $\log_a a^9$ for $9$ |
| | $\frac{y}{10} = a^9$ | **M1** | Eliminates log provided an equation involving a single log seen |
| | $[y =] 10a^9$ | **A1** | **CSO** |
| | | **3** | |
| **3(b) ALT** | $\log_a y = \log_a a^9 + \log_a 10$ | **M1** | Substitutes $\log_a a^9$ for $9$ |
| | $[\log_a y =] \log_a (10a^9)$ | **M1** | Log rule used |
| | $[y =] 10a^9$ | **A1** | **CSO** |
| | | **3** | |
| | **Total** | **6** | |

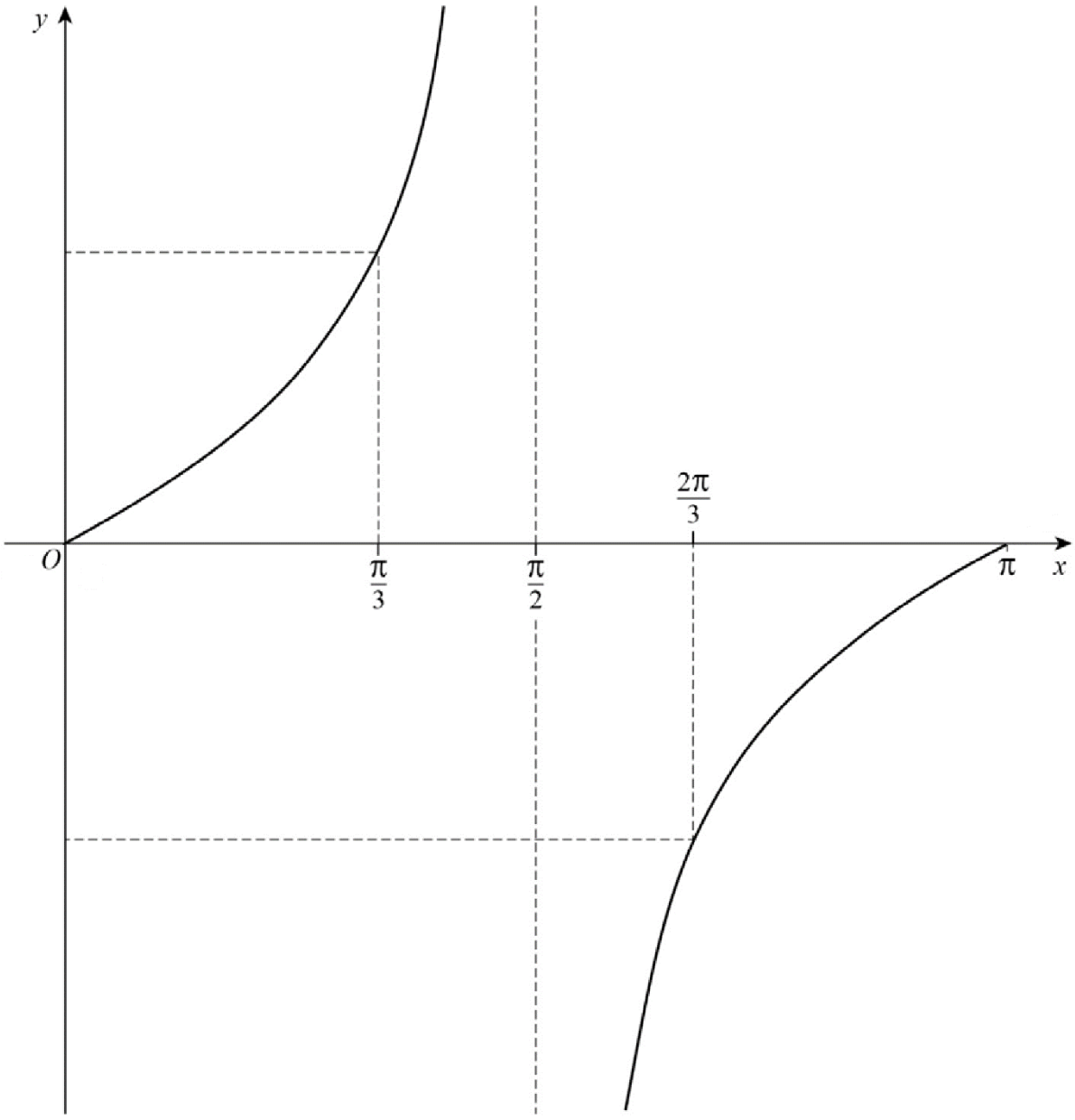
| Q | Answer | Marks | Comments |
|---|---|---|---|
| **4(a)** | $10^2 = 9^2 + 13^2 - 2\times 9\times 13\cos\theta$ | **M1** | **oe** cosine rule used with values correctly substituted<br>**PI** by next line |
| | $\cos\theta = \dfrac{9^2+13^2-10^2}{2\times 9\times 13} \quad \left[= \dfrac{25}{39} = 0.641...\right]$ | **M1** | **oe**<br>Correct rearrangement |
| | $\theta = \left[50.13165...^\circ =\right] \ 50.1^\circ$ | **A1** | **CSO**<br>**AWRT** $50.1^\circ$ |
| | | **3** | |

| | | | |
|---|---|---|---|
| **4(b)** | $\dfrac{\sin PRQ}{20}=\dfrac{\sin 35^\circ}{15}$ | **M1** | Correct equation with all known values substituted |
| | $\sin PRQ=\dfrac{20\sin 35^\circ}{15}\ [=0.76476...]$ | **M1** | Correct rearrangement<br>**PI** by correct value for $\sin PRQ$ |
| | $49.8[8640...]^\circ$<br>or<br>[Angle $PRQ$ =] $130.1[1359...]^\circ$ | **A1** | **PI** in later working.<br>**AWRT** $49.9^\circ$, Allow $49.8^\circ$<br>**PI** in later working.<br>**ft** $180$ minus their acute angle provided **M1** scored.<br>**AWRT** $130.1^\circ$ |
| | $\dfrac{1}{2}\times 20\times 15\times\sin\left(14.8[8640...]^\circ\right)$ | **m1** | **ft** $180^\circ-35^\circ-$ their obtuse angle or **ft** their acute angle $-35^\circ$<br>Dependent on at least one of the previous **M1** marks |
| | $38.5\ \left[\text{cm}^2\right]$ | **A1** | **CAO**<br>**AWRT** $38.5$ or $38.6$ |
| | | **5** | |

| | | | |
|---|---|---|---|
| **4(b) ALT** | $\left[\lvert PR\rvert=x\right]$<br>$15^2=x^2+20^2-2\times 20\times x\cos 35^\circ$<br>or<br>$225=x^2+400-\left(40\cos 35^\circ\right)x$ | **M1** | **oe**<br>Cosine rule used with values correctly substituted.<br>**PI** by next line. |
| | $x^2-\left(40\cos 35^\circ\right)x+175=0$<br>or<br>$x^2-\left(32.7[6608...]\right)x+175=0$ | **M1** | **oe**<br>Forms quadratic equation set equal to zero.<br>**PI** by correct value of $x$ |
| | $[x=]\ 6.7[1846...]$ | **A1** | **AWRT** $6.7$<br>**PI** in later working.<br>May also see $[x=]\ 26.0[4761...]$ |
| | $[\text{Area}=]\ \dfrac{1}{2}\times 20\times 6.7[1846...]\times\sin 35^\circ$ | **m1** | **ft** their $x$<br>Dependent on at least one of the previous **M1** marks |
| | $38.5\ \left[\text{cm}^2\right]$ | **A1** | **CAO**<br>**AWRT** $38.4$ or $38.5$ |
| | | **5** | |
| | **Total** | **8** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **5(a)** | $\cos^2 x - 2\cos x \tan x + \tan^2 x$<br>$+1+2\sin x+\sin^2 x\,[=5]$ | **M1** | Correct expansion of at least one bracket |
| | $\cos^2 x - 2\cos x \dfrac{\sin x}{\cos x} + \tan^2 x$<br>$+1+2\sin x+\sin^2 x = 5$<br>or<br>$\left[\left(\cos^2 x+\sin^2 x\right) - 2\cos x \tan x + \tan^2 x\right.$<br>$\left.+1+2\sin x = 5\right]$<br>$1-2\cos x \tan x + \tan^2 x$<br>$+1+2\sin x = 5$ | **M1** | Both brackets correctly expanded and then use of:<br>$\tan x = \dfrac{\sin x}{\cos x}$<br>or<br>$\cos^2 x+\sin^2 x = 1$<br>**oe**<br>Must be seen as a correct equation |
| | $1-2\sin x+\tan^2 x+1+2\sin x = 5$<br>$\Rightarrow \tan^2 x = 3$ | **A1** | **CSO**<br>**AG** Must see use of both identities<br>Be convinced |
| | | **3** | |
| **5(b)(i)** | $x = \dfrac{\pi}{3}$ | **B1** | One correct value<br>**oe** such as 2 sf decimals<br>Condone given in degrees |
| | $x = \dfrac{2\pi}{3}$ | **B1** | A second correct value and no others<br>**oe** such as 2 sf decimals<br>Condone given in degrees |
| | | **2** | |

| | | | |
|---|---|---|---|
| **5(b)(ii)** | Correct sketch of $y = \tan x$ for $0 \leq x \leq \pi$<br><br>$x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$ clearly indicated on horizontal axis or curve in approximately correct positions. | **B1**<br><br>**B1ft** | Must be of correct form<br>Condone asymptotes not drawn<br><br>**ft** their solution(s)<br>Condone given in degrees |



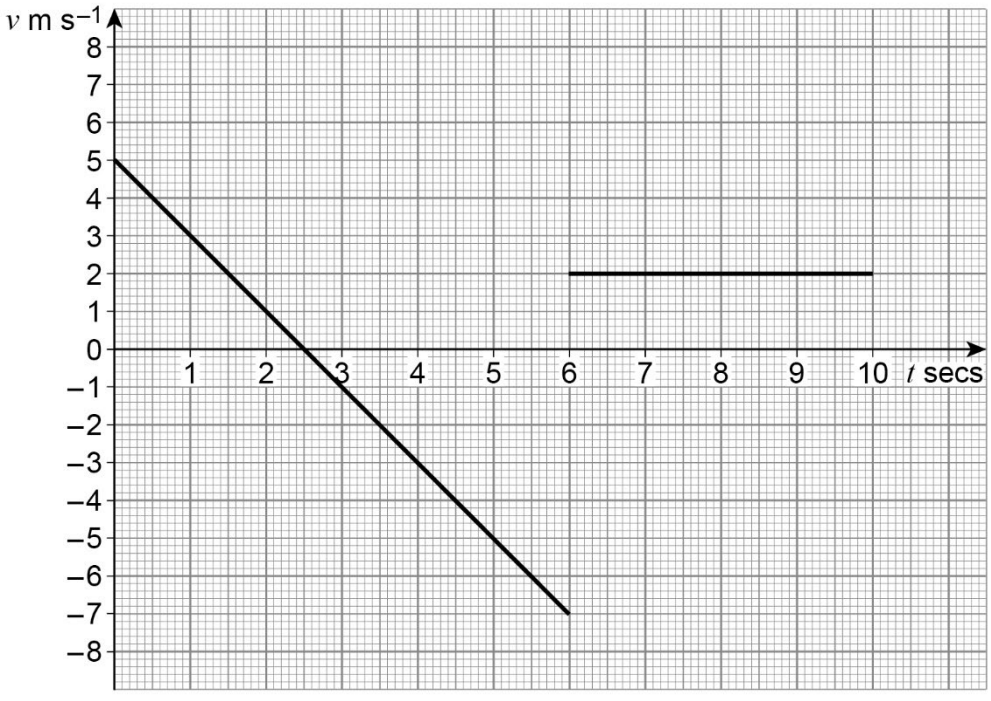

| | | | |
|---|---|---|---|
| | | **2** | |
| | **Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **6(a)** | $\left[\text{Radius} =\right]\ 2\sqrt{13}$ | **B1** | Accept $\sqrt{52}$<br>Mark at most accurate |
| | $\left[\text{Centre} =\right]\ (3, 1)$ | **B1** | |
| | | **2** | |
| **6(b)** | $m = \dfrac{7-1}{7-3}\ \left[= \dfrac{3}{2}\right]$ | **M1** | |
| | $(y-1) = \dfrac{3}{2}(x-3)$ | **A1** | **ACF** eg $2y-3x=-7$, $y=\dfrac{3}{2}x-\dfrac{7}{2}$, $(y-7)=\dfrac{3}{2}(x-7)$ |
| | | **2** | |
| **6(c)** | $\left[2\sqrt{13}+8\sqrt{13} =\right]\ 10\sqrt{13}$ | **B1** | **oe** Must be in surd form.<br>Distance between centres of circles.<br>**PI** |
| | $\sqrt{\left(10\sqrt{13}\right)^2-\left(8\sqrt{13}\right)^2}$<br>$\left[\begin{matrix}=\sqrt{1300-832}\\ =\sqrt{468}\end{matrix}\right]$ | **M1** | Method for finding distance from centre of $C_1$ to $Q$ |
| | $6\sqrt{13}$ | **A1** | **oe** Must be in surd form.<br>Distance from centre of $C_1$ to $Q$ |
| | $\left[PQ = 6\sqrt{13}-2\sqrt{13} =\right]\ 4\sqrt{13}$ | **B1** | At least one correct distance<br>**oe** Must be single term in surd form |
| | $\left[PQ = 6\sqrt{13}+2\sqrt{13} =\right]\ 8\sqrt{13}$ | **B1** | Both correct distances and no others<br>**oe** Must be single term in surd form |
| | | **5** | |
| | **Total** | **9** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **7(a)** | $P(A \cap B) = P(A) \times P(B) = 0.22 \times 0.13$<br><br>$= 0.0286$ | **M1**<br><br>**A1** | Uses independence of A and B<br><br>**oe** such as $\frac{143}{5000}$ |
| | | **2** | |
| **7(b)** | $P(A \cap C) = 0$ | **B1** | Uses fact A and C mutually exclusive |
| | | **1** | |
| **7(c)** | $P(B\|A) = P(B)$<br>or<br>$P(B\|A) = \left[\frac{P(A \cap B)}{P(A)} =\right] \frac{0.0286}{0.22}$<br><br>$= 0.13$ | **M1**<br><br>**A1** | Uses independence of A and B to find conditional probability<br>**ft** their answer to **(a)**<br>**PI** by correct final answer<br><br>**CAO** |
| | | **2** | |
| | **Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **8(a)** | Bernoulli $p = 0.5$ or Binomial $n = 1, p = 0.5$ | **B1**<br>**B1** | Accept $\mathrm{B}(n, p)$ for Binomial<br>Must state both parameter values for Binomial |
| | | **2** | |
| **8(b)(i)** | $\mathrm{E}(H) = 0.5$ | **B1** | **oe** |
| | | **1** | |
| **8(b)(ii)** | $\mathrm{Var}(H) = 0.25$ | **B1** | **oe** |
| | | **1** | |
| **8(c)(i)** | $\mathrm{Var}(K) = 2$ | **B1ft** | **ft** $8 \times$ their $\mathrm{Var}(H)$ |
| | | **1** | |
| **8(c)(ii)** | $K \sim \mathrm{B}(8,\ 0.5)$ | **B1** | May be seen in **(c)(i)**<br>**PI** in later working for example sight of $0.9648$ or $0.0313$ or $0.9961$ or $0.0039$ |
| | $\left[\mathrm{P}(K \geqslant 7) = 1 - \mathrm{P}(K \leqslant 6)\right]$<br>$= 1 - 0.9648$<br>or<br>$\left[\mathrm{P}(K = 7) + \mathrm{P}(K = 8)\right]$<br>$= 0.0313 + 0.0039$ or $= \frac{1}{32} + \frac{1}{256}$ | **M1** | Correct method to calculate probability |
| | $= 0.035$ | **A1** | **AWRT** |
| | | **3** | |
| | **Total** | **8** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 9(a) | $a + 0.2 + 0.34 + b = 1$ | M1 | Forms equation using sum of probabilities $= 1$<br>Implied by $a + b = 0.46$ |
| | $2a + 3 \times 0.2 + 7 \times 0.34 + 9b = 5.16$ | M1 | Forms equation using $\mathrm{E}(X) = 5.16$<br>Implied by $2a + 9b = 2.18$ |
| | $a = 0.28$ or $b = 0.18$ | A1 | At least one of $a$ or $b$ correct |
| | $a = 0.28$ and $b = 0.18$ | A1 | Both $a$ and $b$ correct |
| | | 4 | |
| 9(b) | $\mathrm{E}(X^2) = 2^2 \times a + 3^2 \times 0.2 + 7^2 \times 0.34 + 9^2 \times b$<br>$\left[= 34.16 \text{ or } \frac{854}{25}\right]$ | M1 | Applies $\mathrm{E}(X^2)$ formula with their $a$ and $b$<br>Implied by sight of $34.16$ |
| | $\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$<br>$= 34.16 - 5.16^2 \left[= 7.5344 \text{ or } \frac{4709}{625}\right]$ | M1 | Correctly uses $\mathrm{Var}(X)$ formula with values substituted |
| | $= 2.745$ | A1 | **AWRT** |
| | | 3 | |
| | **Total** | 7 | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **10(a)** | $\left[s = 2 + 5t - t^2 \Rightarrow\right]$<br>$\left[v = \frac{\mathrm{d}s}{\mathrm{d}t} =\right] 5 - 2t$ | **M1** | Correct expression in terms of $t$ for velocity in the first $6$ seconds.<br>**PI** by (part of the) correct line on diagram for first phase of motion. |
| | $t = 0 \Rightarrow v = 5$ and $t = 6 \Rightarrow v = -7$ | **A1** | **PI** by fully correct line on diagram for first phase of motion. |
| | Fully correct line segment from $(0, 5)$ to $(6, -7)$ | **B1** | Ignore velocities not given on diagram if straight-line segment correct. |
| | $\left[\frac{4-(-4)}{10-6} =\right] 2$ | **B1** | Correct gradient for line segment between $6$ and $10$ seconds on displacement-time graph.<br>**PI** by correct line segment on velocity-time graph. |
| | Correct line segment from $(6, 2)$ to $(10, 2)$ | **B1ft** | Ignore velocity not given on diagram if straight-line segment correct.<br>**ft** their gradient for line segment between $6$ and $10$ seconds on displacement-time graph provided a correct method for the calculation of the gradient is seen. |
| |  | | |
| | | **5** | |

| | | | |
|---|---|---|---|
| **10(b)** | $\frac{1}{2}\times 2.5\times 5 = 6.25$ [metres]<br><br>$\frac{1}{2}\times 3.5\times 7 = 12.25$ [metres]<br><br>$4\times 2 = 8$ [metres]<br><br>[Total distance = ] $26.5$ [metres] | **M1**<br><br>**A1** | Correct values or expressions for at least two relevant areas.<br>**ft** their graph.<br><br>**CAO** |
| | | **2** | |
| | **Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **11(a)** | $19 = u \times 2 + \frac{1}{2} \times a \times 2^2$ | **M1** | **oe** Use of $s = ut + \frac{1}{2}at^2$ with correct values substituted.<br>Simplified or unsimplified |
| | $19 = 2u + 2a$ | **A1** | Correct equation in $u$ and $a$ simplified.<br>**PI** in later working. |
| | $112 = u \times 8 + \frac{1}{2} \times a \times 8^2$ | **M1** | **oe** Use of $s = ut + \frac{1}{2}at^2$ with correct values substituted.<br>Simplified or unsimplified |
| | $14 = u + 4a$ | **A1** | **oe**<br>Correct equation in $u$ and $a$ simplified.<br>**PI** in later working. |
| | $a = 1.5$ and $u = 8$ | **B1** | **CAO** |
| | | **5** | |
| **11(a) ALT** | $\left[v_{BC} = u + 2a \Rightarrow\right]$<br>$19 = \frac{1}{2}\left(u + \left(u + 2a\right)\right) \times 2$<br>or<br>$19 = u \times 2 + \frac{1}{2} \times a \times 2^2$ | **M1** | **oe**<br>Correct use of $v = u + at$ and $s = \frac{1}{2}(u+v)t$ with correct values and their $v_{BC}$ substituted.<br>Or use of $s = ut + \frac{1}{2}at^2$ with correct values substituted.<br>Simplified or unsimplified |
| | $19 = 2u + 2a$ | **A1** | **oe**<br>Correct equation in $u$ and $a$ simplified.<br>**PI** in later working. |
| | $\left[v_{CD} = \left(u + 2a\right) + a \times 6 = u + 8a \Rightarrow\right]$<br>$93 = \frac{1}{2}\left(\left(u + 2a\right) + \left(u + 8a\right)\right) \times 6$<br>or<br>$93 = \left(u + 2a\right) \times 6 + \frac{1}{2} \times a \times 6^2$ | **M1** | **oe**<br>Correct use of $v = u + at$ and $s = \frac{1}{2}(u+v)t$ with correct values and their $v_{CD}$ and $v_{BC}$ substituted.<br>Or use of $s = ut + \frac{1}{2}at^2$ with correct values and $v_{BC}$ substituted.<br>Simplified or unsimplified |
| | $31 = 2u + 10a$ | **A1** | **oe**<br>Correct equation in $u$ and $a$ simplified.<br>**PI** in later working. |
| | $a = 1.5$ and $u = 8$ | **B1** | **CAO** |
| | | **5** | |

| | | | |
|---|---|---|---|
| **11(b)** | $\left[v = u + at \Rightarrow v = 8 + 1.5 \times 8 \Rightarrow\right]\ \ v = 20$<br><br>$\left[mv = 1400 \times 20 \Rightarrow\right]$ Momentum $= 28000$ | **B1ft**<br><br>**B1ft** | **ft** their $u$ and $a$ from **(a)**<br><br>**ft** their velocity at *D* |
| | | **2** | |
| **11(b) ALT** | $\left[v = u + at \Rightarrow v = 8 + 1.5 \times 8 \Rightarrow\right] v = 20$<br>or<br>$\left[v = u + at \Rightarrow v = 11 + 1.5 \times 6 \Rightarrow\right] v = 20$<br><br>$\left[mv = 1400 \times 20 \Rightarrow\right]$<br>Momentum $= 28000\ \left[\text{kg m s}^{-1}\right]$ | **B1ft**<br><br>**B1ft** | **ft** their $v_{CD}$, $u$ and $a$ from **(a)**<br>or<br>**ft** their $v_{BC}$ for $11$ and their $u$ and $a$ from **(a)**<br><br>**ft** their velocity at *D* |
| | | **2** | |
| | **Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **12(a)** | $\left[R_{AC} =\right]\ 8.5g\ \ [= 83.3\ \text{N}]$ | **B1** | Correct normal reaction between *A* and *C*.<br>**PI** by correct $F_{max(AC)}$ |
| | $\left[F_{max(AC)} =\right]\ 1.7g\ \ [= 16.66\ \text{N}]$ | **B1** | Correct maximum possible frictional force between *A* and *C* |
| | $\left[F_{max(AB)} =\right]\ 2.1g\ \ [= 20.58\ \text{N}]$ | **B1** | Correct maximum possible frictional force between *A* and *B* |
| | $1.7g < 2.1g$ or $16.66 < 20.58$<br>so box *A* will slide. | **E1** | Comparison of their maximum frictional forces and correct concluding statement. |
| | | **4** | |
| **12(b)** | $19 - 1.7g = 8.5a$<br><br>$a = 0.275$ | **M1**<br><br>**A1ft** | **oe**<br>Correct equation of motion, (using their maximum frictional force between *A* and *C*).<br>**AWRT** 0.275, allow $\frac{117}{425}$<br>**ft** their maximum frictional force between *A* and *C* provided it is less than $19$ newtons. |
| | | **2** | |
| | **Total** | **6** | |