

**INTERNATIONAL A-LEVEL  
MATHEMATICS**

**MA03**

(9660/MA03) Unit P2 Pure Mathematics

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Mark scheme

January 2024

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Version: 1.0 Final



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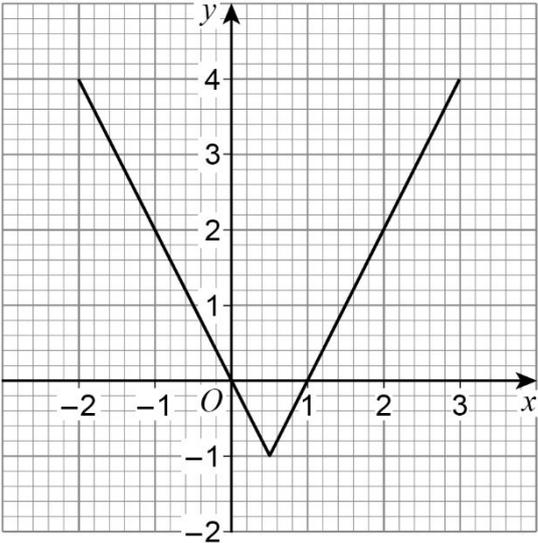
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**Key to mark scheme abbreviations**

<b>M</b>	Mark is for method
<b>m</b>	Mark is dependent on one or more M marks and is for method
<b>A</b>	Mark is dependent on M or m marks and is for accuracy
<b>B</b>	Mark is independent of M or m marks and is for method and accuracy
<b>E</b>	Mark is for explanation
<b>√ or ft</b>	Follow through from previous incorrect result
<b>CAO</b>	Correct answer only
<b>CSO</b>	Correct solution only
<b>AWFW</b>	Anything which falls within
<b>AWRT</b>	Anything which rounds to
<b>ACF</b>	Any correct form
<b>AG</b>	Answer given
<b>SC</b>	Special case
<b>oe</b>	Or equivalent
<b>A2, 1</b>	2 or 1 (or 0) accuracy marks
<b>-x EE</b>	Deduct x marks for each error
<b>NMS</b>	No method shown
<b>PI</b>	Possibly implied
<b>SCA</b>	Substantially correct approach
<b>sf</b>	Significant figure(s)
<b>dp</b>	Decimal place(s)
<b>ISW</b>	Ignore subsequent working



Q	Answer	Marks	Comments
2(a)		<p><b>B1</b></p> <p><b>B1</b></p>	<p>Correct for <math>-2 \leq x \leq 0.5</math></p> <p>Correct for <math>0.5 \leq x \leq 3</math></p>
		2	

Q	Answer	Marks	Comments
2(b)	$x \leq 0 \quad x \geq 2$	<p><b>M1</b></p> <p><b>A1</b></p>	<p>Either correct, condone strict inequalities</p> <p>Both correct and no extras</p>
		2	

	<b>Question 2 Total</b>	<b>4</b>	
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Q	Answer	Marks	Comments
3(a)(i)	$\left[ \frac{dy}{dx} = \right] ae^{-0.5x} \sin 3x + be^{-0.5x} \cos 3x$	M1	Use of product rule
	$\left[ \frac{dy}{dx} = \right] -0.5e^{-0.5x} \sin 3x + 3e^{-0.5x} \cos 3x$	A1	ISW
		2	

Q	Answer	Marks	Comments
3(a)(ii)	$\left[ \frac{dy}{dx} = \right] \frac{a(3 + \tan 5x) \times (1 - 2x)^2 - (1 - 2x)^3 b \sec^2 5x}{(3 + \tan 5x)^2}$	M1 A1	M1: Correct use of quotient/product rule A1: one numerator term and denom correct
	$\left[ \frac{dy}{dx} = \right] \frac{-6(3 + \tan 5x) \times (1 - 2x)^2 - (1 - 2x)^3 5 \sec^2 5x}{(3 + \tan 5x)^2}$	A1	ACF All correct (unsimplified) ISW
		3	

Q	Answer	Marks	Comments
3(a)(iii)	$1 + \frac{1}{xy} \left( x \frac{dy}{dx} + y \right) = 3x^2 + 2y \frac{dy}{dx}$	M1 A1	M1: One correct use of implicit differentiation A1: All correct
	$\frac{dy}{dx} \left( \frac{1}{y} - 2y \right) = 3x^2 - 1 - \frac{1}{x}$	m1	Attempt to isolate $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{3x^2 - 1 - \frac{1}{x}}{\frac{1}{y} - 2y}$ or $\frac{dy}{dx} = \frac{y(3x^3 - x - 1)}{x(1 - 2y^2)}$	A1	ACF, ISW
		4	

Q	Answer	Marks	Comments
3(b)(i)	$\left[ \int \frac{x}{4x^2+5} dx = \right] a \ln(4x^2+5)$	M1	
	$\left[ \int \frac{x}{4x^2+5} dx = \right] \frac{1}{8} \ln(4x^2+5) \quad [+c]$	A1	ISW
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$u = x \quad dv = \cos x$ $du = 1 \quad v = \sin x$	M1	PI
	$\left[ \int x \cos x dx = \right] x \sin x - \int \sin x dx$ $= x \sin x + \cos x$	m1 A1	Correct use of integration by parts formula All correct
	$\left[ x \sin x + \cos x \right]_0^{\frac{\pi}{2}}$		
	$\left[ \int_0^{\frac{\pi}{2}} x \cos x dx \right] = \frac{\pi}{2} - 1$	A1	ACF, ISW
		4	

	<b>Question 3 Total</b>	<b>15</b>	
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Q	Answer	Marks	Comments												
4(a)	<table border="1" data-bbox="272 333 813 663"> <thead> <tr> <th><math>x</math></th> <th><math>y</math></th> </tr> </thead> <tbody> <tr> <td>0.06</td> <td><math>4^{-0.06} - 0.25 = 0.670187</math></td> </tr> <tr> <td>0.18</td> <td><math>4^{-0.18} - 0.25 = 0.529165</math></td> </tr> <tr> <td>0.30</td> <td><math>4^{-0.30} - 0.25 = 0.409754</math></td> </tr> <tr> <td>0.42</td> <td><math>4^{-0.42} - 0.25 = 0.308644</math></td> </tr> <tr> <td>0.54</td> <td><math>4^{-0.54} - 0.25 = 0.223029</math></td> </tr> </tbody> </table> <p data-bbox="237 768 826 808"><math>0.12(0.67... + 0.53... + 0.41... + 0.31... + 0.22...)</math></p> <p data-bbox="237 931 357 965"><math>= 0.2569</math></p>	$x$	$y$	0.06	$4^{-0.06} - 0.25 = 0.670187$	0.18	$4^{-0.18} - 0.25 = 0.529165$	0.30	$4^{-0.30} - 0.25 = 0.409754$	0.42	$4^{-0.42} - 0.25 = 0.308644$	0.54	$4^{-0.54} - 0.25 = 0.223029$	<p data-bbox="890 356 930 389"><b>B1</b></p> <p data-bbox="890 555 930 589"><b>M1</b></p> <p data-bbox="890 775 930 808"><b>m1</b></p> <p data-bbox="890 931 930 965"><b>A1</b></p>	<p data-bbox="994 322 1449 389">All 5 correct <math>x</math> values (and no extra used)</p> <p data-bbox="994 394 1302 427"><b>PI</b> by 5 correct <math>y</math> values</p> <p data-bbox="994 488 1505 622">At least 3 correct <math>y</math> values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula)</p> <p data-bbox="994 627 1361 660"><b>PI</b> by <b>AWRT</b> correct answer</p> <p data-bbox="994 721 1489 855">Correct sub into formula with <math>h = 0.12</math> <b>oe</b> and at least 3 correct <math>y</math> values either listed, with + signs, or totalled</p> <p data-bbox="994 860 1361 893"><b>PI</b> by <b>AWRT</b> correct answer</p> <p data-bbox="994 920 1489 987"><b>CAO</b> Must see this value exactly and no error seen</p>
$x$	$y$														
0.06	$4^{-0.06} - 0.25 = 0.670187$														
0.18	$4^{-0.18} - 0.25 = 0.529165$														
0.30	$4^{-0.30} - 0.25 = 0.409754$														
0.42	$4^{-0.42} - 0.25 = 0.308644$														
0.54	$4^{-0.54} - 0.25 = 0.223029$														
		<b>4</b>													

Q	Answer	Marks	Comments
4(b)(i)	$x = 4^{-y} - 0.25$	M1	Interchange $x$ and $y$
	$-y \ln 4 = \ln(x + 0.25)$	M1	Attempt to isolate PI
	$f^{-1}(x) = \frac{\ln(x+0.25)}{-\ln 4}$ or $\frac{\ln(x+0.25)}{\ln 0.25}$	A1	ACF eg $-\log_4(x+0.25)$
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$-0.25 < x \leq 0.75$ or $x \in (-0.25, 0.75]$	B2	If B2 not awarded, award B1 for at least one of the two limits correct or $-0.25 \leq x < 0.75$
		2	

Q	Answer	Marks	Comments
4(c)	Reflection [in the line] $y = x$	B1	
		1	

	<b>Question 4 Total</b>	<b>10</b>	
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Q	Answer	Marks	Comments
5(a)(i)	$[10\sin\theta - 24\cos\theta =]$ $R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$  $R = 26$  $\alpha = 1.18$  $26\sin(\theta - 1.18)$	<b>M1</b>  <b>B1</b>  <b>A1</b>	PI
		<b>3</b>	

Q	Answer	Marks	Comments
5(a)(ii)	[Min value =] -26	B1ft	
		<b>1</b>	

Q	Answer	Marks	Comments
5(a)(iii)	12.17	B1	Condone 12.18
		<b>1</b>	

Q	Answer	Marks	Comments
5(a)(iv)	$[26\sin((x - 1.18) - 0.6) = 6.5]$ $\sin(x - 1.78) = 0.25$ $[x - 1.78 =] 0.253$  $x = -1.62, 2.03$	<b>M1</b>  <b>A1, A1</b>	Attempt to solve $\sin((x - \text{their } 1.18) - 0.6) = \frac{6.5}{\text{their } 26}$  Condone -1.61
		<b>3</b>	

Q	Answer	Marks	Comments
5(b)	Let $Y = 2y - 10^\circ$ $16 \tan^2 Y - 14 = 4 \sec Y$ $16(\sec^2 Y - 1) = 4 \sec Y + 14$ $8 \sec^2 Y - 2 \sec Y - 15 = 0$ $\sec Y = -1.25, 1.5$ $Y = 143.1^\circ$ and $48.2^\circ$ $y = -67^\circ, -19^\circ, 29^\circ, 77^\circ$	  <b>M1</b>   <b>m1</b>   <b>A1</b>  <b>B2,1</b>	<b>oe</b>  Correct use of trig identity   Attempt to solve <i>their</i> quadratic   <b>PI</b>  <b>AWRT</b> the correct values Award <b>B1</b> for 3 correct answers
		<b>5</b>	
	<b>Question 5 Total</b>	<b>13</b>	

Q	Answer	Marks	Comments
6(a)	$f(x) = -x^2 + \ln(12 + 24x)$ $\frac{dy}{dx} = -2x + \frac{24}{12 + 24x}$	<b>M1</b>	
	$\left[ \frac{dy}{dx} = 0 \right]$ $2x = \frac{24}{12 + 24x}$ $48x^2 + 24x - 24 = 0$	<b>m1</b>	Attempt to solve $\frac{dy}{dx} = 0$
	$x = 0.5$ $[-1 \therefore \text{reject}]$	<b>A1</b>	
	$\left[ x = 0.5, y = -\frac{1}{4} + \ln 24 \right]$		
	$x = 1.5, y = -\frac{9}{4} + \ln 48$	<b>B1</b>	<b>PI</b> by 1.62 AWRT
	$-\frac{9}{4} + \ln 48 \leq f(x) \leq -\frac{1}{4} + \ln 24$	<b>A1</b>	Must be in an exact form
		<b>5</b>	

Q	Answer	Marks	Comments
6(b)(i)	$g(x) = -x^2 + \ln(12 + 24x) - 2x$ $g(1.1) = 0.24$ $g(1.2) = -0.13$	<b>M1</b>	or reverse Both values rounded or truncated to at least 1sf
	Change of sign, $1.1 < \alpha < 1.2$	<b>A1</b>	Must have both statement and interval in words or symbols <b>or</b> comparing 2 sides: $f(1.1) = 2.44 > 2.2$ $f(1.2) = 2.27 < 2.4$ ( <b>M1</b> ) Conclusion as before ( <b>A1</b> )
		<b>2</b>	

Q	Answer	Marks	Comments
6(b)(ii)	$\ln(12+24x) = x^2 + 2x - 1 + \ln(12+24x) = x^2 + 2x + 1$ $(x+1)^2 = 1 + \ln(12+24x)$ $x = -1 + \sqrt{1 + \ln(12+24x)}$	<b>B1</b>	<b>AG</b> No errors seen including correct use of brackets Must be convincingly shown
		<b>1</b>	

Q	Answer	Marks	Comments
6(b)(iii)	$x_2 = 1.156$ $x_3 = 1.164$	<b>B1, B1</b>	If 0 scored, <b>SC1</b> for <b>AWRT</b> 1.156 and 1,164
		<b>2</b>	

Q	Answer	Marks	Comments
6(c)(i)	Translation $\begin{bmatrix} 0 \\ -\ln 12 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ \ln \frac{1}{12} \end{bmatrix}$	<b>B1</b>  <b>B1</b>	Allow $\begin{bmatrix} 0 \\ -2.48 \end{bmatrix}$ <b>AWRT</b> -2.48
		<b>2</b>	

Q	Answer	Marks	Comments
6(c)(ii)	$A - 1.5 \ln 12$	<b>B1ft</b>	<b>oe</b>
		<b>1</b>	

	<b>Question 6 Total</b>	<b>13</b>	
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Q	Answer	Marks	Comments
7(a)	$2x - 2y \frac{dy}{dx} = 6 \frac{dy}{dx} - 2$	M1	LHS or RHS correct
	$\frac{dy}{dx} = \frac{2x+2}{2y+6}$	A1	
	OR		
	$y = -3 \pm ((x+1)^2 - 12)^{0.5}$ $\frac{dy}{dx} = k((x+1)^2 - 12)^{-0.5} (x+1)$	(M1)	
	$\frac{dy}{dx} = \frac{x+1}{((x+1)^2 - 12)^{0.5}}$	(A1)	
At (3, -1) $\frac{dy}{dx} = 2$	m1	Attempt to find gradient at (3, -1) PI by further work	
$y+1 = 2(x-3)$ or $y = 2x-7$	A1	ACF	
		4	

Q	Answer	Marks	Comments
7(b)(i)	$x = \frac{1 + \sqrt{17} \cos \theta}{2}, \quad \cos \theta = \frac{2x-1}{\sqrt{17}}$	M1	Rearranges for $k \cos \theta$ and $k \sin \theta$
	$y = -1 + \sqrt{17} \sin \theta, \quad \sin \theta = \frac{y+1}{\sqrt{17}}$		
	$\cos^2 \theta + \sin^2 \theta = 1$ $(2x-1)^2 + (y+1)^2 = 17$	A1	ACF, eg $\left(\frac{2x-1}{\sqrt{17}}\right)^2 + \left(\frac{y+1}{\sqrt{17}}\right)^2 = 1$ $4x^2 - 4x + y^2 + 2y = 15$
		2	

Q	Answer	Marks	Comments
7(b)(ii)	$4(2x-1) + 2(y+1) \frac{dy}{dx} = 0$	<b>M1</b>	Attempt at implicit differentiation
	$\frac{dy}{dx} = \frac{4-8x}{2y+2}$	<b>A1</b>	All correct
	OR		
	$\frac{dy}{d\theta} = \sqrt{17}\cos\theta, \quad \frac{dx}{d\theta} = \frac{-\sqrt{17}\sin\theta}{2}$	<b>(M1)</b>	Attempt at parametric differentiation
	$\frac{dy}{dx} = -2 \cot\theta$	<b>(A1)</b>	All correct
	$\theta = \cos^{-1}\left(\frac{1}{\sqrt{17}}\right), \quad x=1, \quad y=3 \quad \frac{dy}{dx} = -0.5$	<b>m1</b>	
	$y-3 = 2(x-1) \quad \text{or} \quad y = 2x+1$	<b>A1</b>	<b>ACF</b>
		<b>4</b>	

Q	Answer	Marks	Comments
7(c)	49:1	<b>B1ft</b>	ft their answers for part (a) and part (b)(ii) only if both equations have a gradient of 2
		<b>1</b>	

	<b>Question 7 Total</b>	<b>11</b>	
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Q	Answer	Marks	Comments
8	$y = \frac{1}{10-2x} \quad 10-2x = \frac{1}{y}$ $x = 5 - \frac{1}{2y}$ $\left[ x^2 = 25 - \frac{5}{y} + \frac{1}{4y^2} \right]$ $[\text{Volume} =] \pi \int_{0.1}^1 \left( 25 - \frac{5}{y} + \frac{1}{4y^2} \right) dy$ $\int_{0.1}^1 \left( 25 - \frac{5}{y} + \frac{1}{4y^2} \right) dy = 25y - 5\ln y - \frac{1}{4y}$ $\left[ 25y - 5\ln y - \frac{1}{4y} \right]_{0.1}^1 = (25 - 0.25)$ $\quad \quad \quad - (2.5 - 5\ln 0.1 - 2.5)$ $\quad \quad \quad = 24.75 + 5\ln 0.1$ $[\text{Volume} =] (24.75 - 5\ln 10)\pi$	<p><b>M1</b> <b>A1</b></p> <p><b>B1ft</b></p> <p><b>M1</b> <b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Isolating 'x' Correct</p> <p><b>PI</b> by later work</p> <p>At least 2 terms integrated correctly All terms integrated correctly</p> <p>Correctly substituting limits into their integration must be in form <math>py - q\ln y - \frac{r}{y}</math></p> <p><b>ACF</b></p>

	<b>Question 8 Total</b>	<b>7</b>	
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Q	Answer	Marks	Comments
9(a)	$\frac{dy}{dx} = k(3x + 4y)$	M1	$\frac{dy}{dx} = (3x + 4y)$ or $\frac{dy}{dx} \propto (3x + 4y)$
		A1	All correct
		2	

Q	Answer	Marks	Comments
9(b)(i)	$\frac{dy}{dx} = 4xe^{2y} - e^{2y} = e^{2y}(4x - 1)$ $\int e^{-2y} dy = \int 4x - 1 dx$  $-\frac{1}{2}e^{-2y} = 2x^2 - x + c$  (1,0) $-\frac{1}{2} = 1 + c$ $c = -1.5$  $\left[ -\frac{1}{2}e^{-2y} = 2x^2 - x - 1.5 \right]$  $\left[ e^{-2y} = -4x^2 + 2x + 3 \right]$  $y = -\frac{1}{2}\ln(3 + 2x - 4x^2)$	M1	Separate variables
		m1	Attempt to integrate eg $ae^{\pm 2y} = 2x^2 + kx$ oe
		m1	Attempt to find $c$ from equation as above
		A1	
		M1 A1	Attempt to isolate $y$ ie $y = a\ln(f(x))$ Must have scored first M1 ACF
		6	

Q	Answer	Marks	Comments
9(b)(ii)	$y = 0, \quad 1 = 3 + 2x - 4x^2$ $\left[ 2x^2 - x - 1 = 0 \right]$ $\left[ (2x + 1)(x - 1) = 0 \right]$  $x = -0.5$	M1	<i>their</i> $(3 + 2x - 4x^2) = 1$
		A1	
		2	

	<b>Question 9 Total</b>	<b>10</b>	
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Q	Answer	Marks	Comments
<b>10(a)</b>	$\frac{x^2}{(3-x)(3+2x)(3-2x)} = \frac{A}{3-x} + \frac{B}{3+2x} + \frac{C}{3-2x}$ $x^2 = A(3+2x)(3-2x) + B(3-x)(3-2x) + C(3-x)(3+2x)$ $x = 3: 9 = A(9)(-3) \quad A = -\frac{1}{3}$ $x = 1.5: 2.25 = C(1.5)(6) \quad C = \frac{1}{4}$ $A = -\frac{1}{3} \quad B = \frac{1}{12} \quad C = \frac{1}{4}$	<b>B1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>	Correctly combining RHS <b>PI</b>  Attempt at finding one constant, could equate coefficients  Two constants correct  All correct
		<b>4</b>	

Q	Answer	Marks	Comments
<b>10(b)(i)</b>	$(3-x)^{-1} = 3^{-1} \left(1 - \frac{x}{3}\right)^{-1}$ $\frac{1}{3} \left( 1 + (-1) \times \left(-\frac{x}{3}\right) + \frac{(-1) \times (-2) \times \left(-\frac{x}{3}\right)^2}{2} \right)$ $= \frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2$	<b>M1</b>  <b>A1</b>	At least 2 terms correct (unsimplified)  All correct, simplified
		<b>2</b>	

Q	Answer	Marks	Comments
<b>10(b)(ii)</b>	$ x  < 3$ or $-3 < x < 3$	<b>B1</b>	
		<b>1</b>	

Q	Answer	Marks	Comments
<b>10(c)</b>	$(3-2x)^{-1} = \frac{1}{3} \left( 1 + \frac{2}{3}x + \frac{4}{9}x^2 \right)$ $(3+2x)^{-1} = \frac{1}{3} \left( 1 - \frac{2}{3}x + \frac{4}{9}x^2 \right)$ <p>[f(x)=]</p> $-\frac{1}{3} \left( \frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2 \right) + \frac{1}{12} \times \frac{1}{3} \left( 1 - \frac{2}{3}x + \frac{4}{9}x^2 \right)$ $+ \frac{1}{4} \times \frac{1}{3} \left( 1 + \frac{2}{3}x + \frac{4}{9}x^2 \right)$ $= \frac{1}{27}x^2$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b> <b>A1ft</b></p> <p><b>A1</b></p>	<p>At least one expansion correct (unsimplified)</p> <p>Both expansions correct (unsimplified)</p> <p>Attempt at finding f(x) All correct, <b>ft</b> their A, B and C and their binomial expansions</p> <p>Must have scored first 4 marks</p>
		<b>5</b>	

	<b>Question 10 Total</b>	<b>12</b>	
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Q	Answer	Marks	Comments
11	$2u \frac{du}{dx} = -2x \quad \text{or} \quad \frac{du}{dx} = \frac{-x}{(9-x^2)^{0.5}}$ $\text{or} \quad \frac{dx}{du} = -\frac{u}{(9-u^2)^{0.5}}$ $\left[ \int \frac{x^3}{(9-x^2)^{0.5}} dx = \int \frac{x^2 \times x dx}{(9-x^2)^{0.5}} \right]$ $= \int \frac{(9-u^2)2u du}{u \cdot -2}$ $= \int u^2 - 9 du$ $\left[ \int \frac{x^3}{(9-x^2)^{0.5}} dx = \right] \frac{u^3}{3} - 9u$ $\int_0^1 \dots dx = \int_3^{\sqrt{8}} \dots du$ $= \left( \frac{8\sqrt{8}}{3} - 9\sqrt{8} \right) - (9 - 27)$ $= 18 - \frac{38}{3}\sqrt{2}$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>All in terms of <math>u</math>, condone omission of <math>du</math></p> <p>Must see <math>du</math> here, or earlier</p> <p>Correct integral</p> <p>Change of limits, maybe seen earlier (may change back to <math>x</math> and not change limits)</p> <p>Correctly substituting correct limits for <math>u</math> into their integral of the form <math>au^3 + bu</math></p>

	<b>Question 11 Total</b>	<b>7</b>	
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Q	Answer	Marks	Comments
12(a)	$\overrightarrow{AB} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -2 \\ 6 \\ 15 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix}$ $4 - 6\lambda = -3p - 2$ $2 + 4\lambda = 2p + 6$ $3 + 12\lambda = 6p + 15$ $3p - 6\lambda = -6, \quad p - 2\lambda = -2$ $2p - 4\lambda = -4, \quad p - 2\lambda = -2$ $6p - 12\lambda = -12, \quad p - 2\lambda = -2$ <p>Equations are consistent, hence <math>P</math> lies on <math>AB</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>m1</b></p> <p><b>A1</b></p>	<p>Correct</p> <p>Equate 3 pairs of equations or solve a pair of equations <b>oe</b></p> <p><b>CSO</b> Should be convincingly shown</p>
		<b>4</b>	

Q	Answer	Marks	Comments
12(b)(i)2	$\overrightarrow{CP} = \begin{bmatrix} -3p-2 \\ 2p+6 \\ 6p+15 \end{bmatrix} - \begin{bmatrix} -1 \\ 10 \\ 6 \end{bmatrix}$ $\overrightarrow{CP} = \begin{bmatrix} -3p-1 \\ 2p-4 \\ 6p+9 \end{bmatrix}$ $\begin{bmatrix} -3p-1 \\ 2p-4 \\ 6p+9 \end{bmatrix} \cdot \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} = 0$ $18p + 6 + 8p - 16 + 72p + 108 = 0$ $98p = -98$ $p = -1$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>m1</b></p> <p><b>A1</b></p>	<p><b>A1:</b> Correct direction vector <b>oe</b></p> <p>Correct scalar product</p>
		<b>4</b>	

Q	Answer	Marks	Comments
12(b)(ii)	$AB = \sqrt{(4 - -2)^2 + (2 - 6)^2 + (3 - 15)^2}$	B1	Ft their $p$
	$AB = 14$		
	$CP = \sqrt{(-1 - 1)^2 + (10 - 4)^2 + (6 - 9)^2}$	M1	
	$CP = 7$	A1	
	Area $ABC = 49$	A1	
		4	

Q	Answer	Marks	Comments
12(c)	$AC^2 = (4 - -1)^2 + (2 - 10)^2 + (3 - 6)^2 = 98$	M1	$\text{or } \cos BAC = \frac{\begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 8 \\ 3 \end{bmatrix}}{14 \times 7\sqrt{2}} = \frac{1}{\sqrt{2}}$
	$\sin BAC = \frac{7}{\sqrt{98}}$		
	Angle $BAC = 45^\circ$	A1	oe
		2	

	<b>Question 12 Total</b>	<b>14</b>	
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