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MA03

(9660/MA03) Unit P2 Pure Mathematics

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$12\times(0.5)^3+b\times(0.5)^2+c\times0.5+6=10$ $12(-1.5)^3+b(-1.5)^2+c\times-1.5+6=-24$	M1 A1	**M1**: One correct substitution or use of long division **A1**: Both substitutions correct
	$b+2c=10$ $3b-2c=14$	m1	Attempt to solve **PI**
	$b=6$ $c=2$	A1	Both values correct

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	x | y 0.3 | $5^{0.7} = 3.085169....$ 0.9 | $5^{0.1} = 1.1746189....$ 1.5 | $5^{-0.5} = 0.447213595....$ 2.1 | $5^{-1.1} = 0.17026798....$ 2.7 | $5^{-1.7} = 0.06482626....$	**B1**	All 5 correct x values (and no extra used) **PI** by 5 correct y values
		M1	At least 4 correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula) **PI** by **AWRT** correct answer
	$\left[\int_0^3 5^{(1-x)}\,\mathrm{d}x \approx\right]$		
	$0.6\times\left(5^{0.7}+5^{0.1}+5^{-0.5}+5^{-1.1}+5^{-1.7}\right)$	**m1**	Correct sub into formula with $h = 0.6$ **oe** and at least 4 correct y values either listed, with + signs, or totalled **PI** by **AWRT** correct answer
	$= 2.965$	**A1**	**CAO**, must see this value exactly and no errors made
		4	

Q	Answer	Marks	Comments
2(b)(i)	$\mathrm{f}(x) = 5^{(1-x)} - 2x + 3$ $\mathrm{f}(1.6) = 0.18...$ $\mathrm{f}(1.7) = -0.075...$	**M1**	Or reverse Both values rounded or truncated to at least 1 sf
	Change of sign, $1.6 < \alpha < 1.7$	**A1**	Must have both statement and interval in words or symbols **or** comparing 2 sides: at 1.6, $5^{-0.6} > 2\times1.6 - 3$ at 1.7, $5^{-0.7} < 2\times1.7 - 3$ **(M1)** Conclusion as before **(A1)**
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$x_2 = 1.690$	**B1**	
	$x_3 = 1.665$	**B1**	
		2	

	Question 2 Total	**8**	

Q	Answer	Marks	Comments
3(a)	$\left[\frac{dy}{dx} =\right] -80x\left(3-4x^2\right)^9$	M1 A1	**M1**: $kx\left(3-4x^2\right)^9$ **A1**: **ACF** eg $10\times(-8x)(3-4x^2)^9$ is **M1 A1**
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{dy}{dx} = \ln(4x) + x\times\frac{4}{4x} + 2\times3\times\sec^2(3x)$	M1 M1 B1	Correct differentiation of $\ln 4x$ Correct use of product rule Correct differentiation of $\tan 3x$ (may be in terms of sin/cos)
	$\left[\frac{dy}{dx} =\right] 1+\ln(4x)+6\sec^2(3x)$	A1	All correct **ACF**
		4	

Q	Answer	Marks	Comments
3(c)	$\left[\frac{dy}{dx} =\right]$ $\frac{(1-4x)^3(3x-2)\times6-(3x-2)^2(1-4x)^2\times-12}{\left((1-4x)^3\right)^2}$	M1	$\frac{a(1-4x)^3(3x-2)-b(3x-2)^2(1-4x)^2}{(1-4x)^6}$ Maybe seen as 2 terms using product rule $a(3x-2)(1-4x)^{-3}-b(3x-2)^2(1-4x)^{-4}$
	$\left[=\frac{6\times(1-4x)(3x-2)+12\times(3x-2)^2}{(1-4x)^4}\right]$	A1	All correct
	$=\frac{6\times(3x-2)\left((1-4x)+2\times(3x-2)\right)}{(1-4x)^4}$	m1	$\frac{c(3x-2)\left((1-4x)+d(3x-2)\right)}{(1-4x)^4}$ **oe**
	$=\frac{6(3x-2)(2x-3)}{(1-4x)^4}$	A1	
		4	

	Question 3 Total	**10**	

Q	Answer	Marks	Comments
4(a)	$\overrightarrow{AB} = \begin{bmatrix} -4 \\ 2 \\ 5 \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
4(b)	$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{\left(2-\left(-2\right)\right)^2+\left(-3-\left(-1\right)\right)^2+\left(-1-4\right)^2}$	M1	oe
	$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{45}$	A1	oe
		2	

Q	Answer	Marks	Comments
4(c)(i)	$\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \\ 5 \end{bmatrix} = 7$	M1	PI
	$\cos\theta = \dfrac{7}{\sqrt{14}\sqrt{45}}$	m1	Allow $\cos\theta = \dfrac{\pm 7}{\sqrt{14}\sqrt{45}}$
	$\theta = 73.8^\circ$	A1	AWRT 73.8°
		3	

Q	Answer	Marks	Comments
4(c)(ii)	$3+\lambda=2-4\mu$ $-15+3\lambda=-1+5\mu$	**M1**	Equating **i** components and equating **k** components
	$\lambda=3, \quad \mu=-1$	**A1**	Both correct
	$b-2\lambda=-3+2\mu$		
	$b=1$	**A1**	
		3	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5	$[\text{Vol} =]\ \pi\int_0^{\frac{\pi}{3}}(1+\sin x)^2\ \mathrm{d}x$	B1	Correct volume statement
	$\left[(1+\sin x)^2 = 1+2\sin x+\sin^2 x\right]$		
	$[\text{Vol} = \pi]\ \int(1+2\sin x+0.5-0.5\cos 2x)\ \mathrm{d}x$	M1	Correct use of double angle
	$=[\pi]\left(1.5x-2\cos x-0.25\sin 2x\right)$	A1	Correct integration
	$=[\pi]\left(\left(1.5\times\frac{\pi}{3}-2\times 0.5-0.25\times\frac{\sqrt{3}}{2}\right)-(0-2-0)\right)$	m1	Correct subst of limits into their expression (must be in the correct form, $ax+b\cos x+c\sin 2x$)
	$=\pi\left(\frac{\pi}{2}+1-\frac{\sqrt{3}}{8}\right)$	A1	oe

	Question 5 Total	**5**	

Q	Answer	Marks	Comments
6(a)	$x = 4\sin\left(\frac{y}{3}\right)$	**M1**	Interchange x and y
	$\frac{x}{4} = \sin\left(\frac{y}{3}\right)$	**M1**	Attempt to rearrange
	$\mathrm{f}^{-1}(x) = 3\sin^{-1}\left(\frac{x}{4}\right)$	**A1**	**ACF**
		3	

Q	Answer	Marks	Comments
6(b)(i)	$\left[\mathrm{gf}(x) =\right] \left\|4\sin\left(\frac{x}{3}\right)\right\|$ or $4\left\|\sin\left(\frac{x}{3}\right)\right\|$	**B1**	
		1	

Q	Answer	Marks	Comments
6(b)(ii)	$0 \leq \mathrm{gf}(x) \leq 4$	**M1 A1**	**M1**: Identifying 0 and 4 **A1**: Fully correct
		2	

Q	Answer	Marks	Comments
6(c)	Stretch + either I or II Parallel to y-axis I SF 4 II Stretch + either I or II Parallel to x-axis I SF 3 II	**M1** **A1** **M1** **A1**	Transformations in either order
		4	

	Question 6 Total	**10**	

Q	Answer	Marks	Comments
7(a)(i)	$[16\cos\theta - 30\sin\theta =]$ $R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $R = 34$ $\alpha = 1.08$ $34\cos(\theta + 1.08)$	 **B1** **B1** **B1**	 **AWRT** 1.08 **AWRT** 1.08
		3	

Q	Answer	Marks	Comments
7(a)(ii)	$[34\cos(y + 1.08 + 2) = 17]$ $y + 3.08 = \cos^{-1}(0.5)$ $y = -2.03, 2.16$	**M1** **A1 A1**	**PI** **AWRT** $-2.03, 2.16$
		3	

Q	Answer	Marks	Comments
7(b)	$2\tan^2 X = 4 + \sec X$ $2(\sec^2 X - 1) = 4 + \sec X$ $(2\sec X + 3)(\sec X - 2) = 0$ $[\sec X =] -1.5, 2$ $X = -60°,\ 60°, 132°, 228°$ $x = -45°, 75°, 147°, 243°$	 **M1** **A1** **A1** **B2,1**	X could be Y, $x - 15$ etc Correct use of trig identity (might use sin/cos) **oe** At least one correct **PI** by one correct final answer **B1** for three correct final answers
		5	

	Question 7 Total	**11**	

Q	Answer	Marks	Comments
8	$\int \ln(2x+1)\mathrm{d}x$		
	$u=\ln(2x+1),\ \dfrac{\mathrm{d}v}{\mathrm{d}x}=1$	**M1**	Attempt at integration by parts
	$\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{2}{2x+1},\ v=x$	**A1**	All 4 terms correct
	$\int \ln(2x+1)\mathrm{d}x = x\ln(2x+1)-\int x\dfrac{2}{2x+1}\,\mathrm{d}x$	**m1**	Correct substitution into integration by parts formula
	$= x\ln(2x+1)-\int 1-\dfrac{1}{2x+1}\,\mathrm{d}x$	**B1**	Dividing **PI**
	$= x\ln(2x+1)-x+\dfrac{1}{2}\ln(2x+1)$	**A1**	All correct
	$\left[\int_0^1 \ln(2x+1)\,\mathrm{d}x=\right]\left(\ln 3-1+\dfrac{1}{2}\ln 3\right)$ $-\left(0-0+\dfrac{1}{2}\ln 1\right)$	**m1**	Substitution limits into their expression (must be in correct form eg $(ax+b)\ln(2x+1)+cx$) or could have changed limits
	$=\dfrac{3}{2}\ln(3)-1$	**A1**	**ACF**

	Question 8 Total	7	

Q	Answer	Marks	Comments
9	$\frac{\tan\beta+\tan\alpha}{\tan\alpha\tan\beta}=-2 \quad \tan\alpha\tan\beta=-1,$		
	$\tan\beta=\frac{-1}{\tan\alpha}$ or $\tan\beta=2-\tan\alpha$	**B1**	Isolating $\tan\beta$ or $\tan\alpha$
	$\tan\alpha-\frac{1}{\tan\alpha}=2$	**M1**	Equation in $\tan\beta$ or $\tan\alpha$
	$\tan^2\alpha-2\tan\alpha-1=0$	**m1**	Quadratic equation in $\tan\alpha$ or $\tan\beta$
	$\tan\alpha=\frac{2\pm\sqrt{8}}{2}=1\pm\sqrt{2}$	**A1**	**oe** final answer

	Question 9 Total	**4**	

Q	Answer	Marks	Comments
10(a)	$\int\left(\frac{1}{4}\mathrm{e}^{2x}-3\mathrm{e}^{-2x}\right)\mathrm{d}x=\frac{1}{8}\mathrm{e}^{2x}+\frac{3}{2}\mathrm{e}^{-2x}\ [+c]$	**M1 A1**	**M1**: $a\mathrm{e}^{2x}+b\mathrm{e}^{-2x}$ **A1**: Fully correct
		2	

Q	Answer	Marks	Comments
10(b)(i)	$\frac{1}{4}\mathrm{e}^{2x}-\frac{9}{4}=3\mathrm{e}^{-2x}-2$	**B1**	
	$\left[\mathrm{e}^{2x}-9=12\mathrm{e}^{-2x}-8\right]$		
	$\mathrm{e}^{2x}-1-12\mathrm{e}^{-2x}=0$	**M1**	Set up quadratic equation, must =0
	$\left(\mathrm{e}^{2x}-4\right)\left(\mathrm{e}^{2x}+3\right)=0$		**oe**
	$\mathrm{e}^{2x}\neq -3$	**E1**	Rejection of $\mathrm{e}^{2x}=-3$
	$\mathrm{e}^{2x}=4,\quad 2x=\ln 4$ or $\mathrm{e}^{x}=2\ \Rightarrow\ x=\ln 2$	**A1**	**AG** Must be convincingly shown
		4	

Q	Answer	Marks	Comments
10(b)(ii)	$\text{Area} = \left\lvert \int_{0.5\ln 1.5}^{\ln 2} \left(\frac{1}{4}\left(e^{2x} - 9\right) - \left(3e^{-2x} - 2\right) \right) dx \right\rvert$	**B1**	Complete correct statement **PI**
	$= \left\lvert \left[\frac{1}{8}e^{2x} - \frac{9}{4}x + \frac{3}{2}e^{-2x} + 2x \right]_{0.5\ln 1.5}^{\ln 2} \right\rvert$	**M1** **A1**	Attempt at integration All 4 terms correct
	$= \left\lvert \left(\frac{1}{8}e^{2\ln 2} - \frac{1}{4}\ln 2 + \frac{3}{2}e^{-2\ln 2} \right) - \left(\frac{1}{8}e^{\ln 1.5} - \frac{1}{8}\ln 1.5 + \frac{3}{2}e^{-\ln 1.5} \right) \right\rvert$	**m1**	Correct substitution of correct limits into their expressions of the correct form
	$= \frac{5}{16} - \frac{1}{8}\ln\left(\frac{3}{8}\right)$	**A1**	**oe** eg $\frac{5}{16} + \frac{1}{8}\ln\left(\frac{8}{3}\right)$
		5	

	Question 10 Total	**11**	

Q	Answer	Marks	Comments
11(a)	$8e^{4x} + 12y\frac{dy}{dx} = e^x \times 3y^2\frac{dy}{dx} + e^x y^3$ $\frac{dy}{dx} = \frac{e^x y^3 - 8e^{4x}}{12y - 3e^x y^2}$	**M1 A1** **A1**	**M1**: Attempt at implicit differentiation **A1**: Fully correct **ACF**
		3	

Q	Answer	Marks	Comments
11(b)	$\left[\frac{dy}{dx} = 0\right]$ $e^a b^3 = 8e^{4a}$	**M1**	Equating numerator to 0 and rearrange
	$b^3 = 8e^{3a} \Rightarrow b = 2e^a$	**A1**	**AG** Must be convincingly shown
		2	

Q	Answer	Marks	Comments
11(c)	$2e^{4a} + 6\left(2e^a\right)^2 = e^a\left(2e^a\right)^3$ or $\frac{1}{8}b^4 + 6b^2 = \frac{1}{2}b.b^3$	**M1**	Equation in one variable
	$2e^{4a} + 24e^{2a} = 8e^{4a}$	**A1**	$\frac{1}{8}b^4 + 6b^2 = \frac{1}{2}b^4$
	$4e^{2a} = e^{4a}$		
	$e^{2a} = 4 \qquad 2a = \ln 4$		
	$a = \ln 2$ $b = 4$	**A1** **A1**	**oe** **CAO**
		4	

	Question 11 Total	**9**	

Q	Answer	Marks	Comments
12(a)	$u^2 = x-2 \quad 2u\dfrac{\mathrm{d}u}{\mathrm{d}x} = 1$	**B1**	$u = \sqrt{x-2} \quad \mathrm{d}u = \dfrac{1}{2u}\mathrm{d}x$
	$\int x\sqrt{(x-2)}\ \mathrm{d}x = \int\left(u^2+2\right)\times u \times 2u\,\mathrm{d}u$	**M1**	All in terms of u
	$= \int\left(2u^4 + 4u^2\right)\mathrm{d}u$	**A1**	
	$= \dfrac{2}{5}u^5 + \dfrac{4}{3}u^3$	**m1**	Attempt to integrate $pu^4 + qu^2$
	$= \dfrac{2}{5}(x-2)^{\frac{5}{2}} + \dfrac{4}{3}(x-2)^{\frac{3}{2}} \quad [+c]$	**A1**	**ACF** eg $\dfrac{2}{15}(x-2)^{\frac{3}{2}}(3x+4)$
		5	

Q	Answer	Marks	Comments
12(b)	$\int \frac{1}{\sqrt{2y}}\,\mathrm{d}y = \int x\sqrt{(x-2)}\,\mathrm{d}x$	**M1**	Separate variables
		m1	Attempt to integrate both sides
	$\frac{1}{\sqrt{2}}2y^{0.5} = \frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{\frac{3}{2}} + c$	**A1**	All correct **oe** $\frac{1}{\sqrt{2}}2y^{0.5} = \frac{2}{3}x(x-2)^{\frac{3}{2}} - \frac{4}{15}(x-2)^{\frac{5}{2}} + c$
	At (3, 2): $2 = \frac{2}{5} + \frac{4}{3} + c$	**m1**	Attempt to find c (must have scored **M1m1**)
	$c = \frac{4}{15}$	**A1**	
	$\frac{1}{\sqrt{2}}2y^{0.5} = \frac{2}{5}(x-2)^{\frac{5}{2}} + \frac{4}{3}(x-2)^{\frac{3}{2}} + \frac{4}{15}$		
	$\sqrt{2}y^{0.5} = \frac{1}{15}\left(6(x-2)^{\frac{5}{2}} + 20(x-2)^{\frac{3}{2}} + 4\right)$	**m1**	Attempt to isolate y: $y = \frac{(\textit{their}\,\mathrm{RHS})^2}{2}$ (must have scored **M1m1m1**) oe
	$y = \frac{2}{225}\left(3(x-2)^{\frac{5}{2}} + 10(x-2)^{\frac{3}{2}} + 2\right)^2$	**A1**	**ACF** eg $\frac{2}{225}\left((x-2)^{\frac{3}{2}}(3x+4)+2\right)^2$ $\frac{2}{225}\left(5x(x-2)^{\frac{3}{2}} - 2(x-2)^{\frac{5}{2}} + 2\right)^2$
		7	

	Question 12 Total	**12**	

Q	Answer	Marks	Comments
13(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{(t-1)2-2t}{(t-1)^2}$	**M1**	Either correct
	$\frac{\mathrm{d}y}{\mathrm{d}t} = 1+\frac{1}{t^2}$	**A1**	Both correct **ACF**
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(1+\frac{1}{t^2}\right)(t-1)^2}{-2}$	**m1**	Correct **ft**
	When $t=2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{5}{8}$	**A1**	
	$x=4,\ y=\frac{3}{2}$	**B1**	
	$y-\frac{3}{2} = -\frac{5}{8}(x-4)$	**M1**	Correct equation
	$8y-12 = -5x+20$		
	$5x+8y=32$	**A1**	Allow integer multiples
		7	

Q	Answer	Marks	Comments
13(b)	$x=\frac{2t}{t-1}$		
	$(t-1)x=2t$		
	$t=\frac{x}{x-2}$	**M1**	Attempt to isolate t
	$y=t-\frac{1}{t},\quad y=\frac{x}{x-2}-\frac{x-2}{x}$	**M1**	Attempt to eliminate t
	$y=\left[\frac{x^2-x^2+4x-4}{x(x-2)} = \frac{4x-4}{x(x-2)} =\right]\frac{4(x-1)}{x(x-2)}$	**A1**	
		3	

	Question 13 Total	**10**	

Q	Answer	Marks	Comments
14(a)	$4x+4=P(b-ax)+Q(a-bx)$	B1	Correctly eliminating fractions
	$x=\frac{b}{a},\ \frac{4b}{a}+4=Q\left(a-\frac{b^2}{a}\right)$	M1	Attempt at finding P or Q
	$Q\left(a^2-b^2\right)=4b+4a$		
	$Q=\frac{4b+4a}{\left(a^2-b^2\right)}\qquad\left[=\frac{4}{a-b}\right]$	A1	
	$x=\frac{a}{b},\ \frac{4a}{b}+4=P\left(b-\frac{a^2}{b}\right)$		
	$P\left(b^2-a^2\right)=4b+4a$		
	$P=\frac{4b+4a}{\left(b^2-a^2\right)}\qquad\left[=\frac{4}{b-a}\right]$	A1	Both correct
		4	

Q	Answer	Marks	Comments
14(b)	$(a-bx)^{-1}=\frac{1}{a}\left(1-\frac{b}{a}x\right)^{-1}$	M1	
	$=\frac{1}{a}\left(1+\frac{b}{a}x+\frac{b^2}{a^2}x^2\right)$	A1	oe
		2	

Q	Answer	Marks	Comments
14(c)(i)	$(b-ax)^{-1} = \frac{1}{b}\left(1+\frac{a}{b}x+\frac{a^2}{b^2}x^2\right)$ $\frac{4}{b-a}\left(\frac{1}{a}\left(1+\frac{b}{a}x+\frac{b^2}{a^2}x^2\right)-\frac{1}{b}\left(1+\frac{a}{b}x+\frac{a^2}{b^2}x^2\right)\right)$ $=\frac{4}{b-a}\left(\left(\frac{1}{a}-\frac{1}{b}\right)+\left(\frac{b}{a^2}-\frac{a}{b^2}\right)x+\left(\frac{b^2}{a^3}-\frac{a^2}{b^3}\right)x^2\right)$	**M1** **A1**	Correct expansion **oe** **ACF**
		2	

Q	Answer	Marks	Comments
14(c)(ii)	$\frac{4}{a}\left(\frac{1}{a}+\frac{2}{a}x+\frac{4}{a}x^2-\frac{1}{2a}-\frac{1}{4a}x-\frac{1}{8a}x^2\right)$ $=\frac{4}{a^2}\left(\frac{1}{2}+\frac{7}{4}x+\frac{31}{8}x^2\right)$ $=\frac{1}{a^2}\left(2+7x+\frac{31}{2}x^2\right)$	**M1** **A1**	
		2	

	Question 14 Total	**10**	