OXFORD
INTERNATIONAL
AQA EXAMINATIONS

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I declare this is my own work.

INTERNATIONAL A-LEVEL
MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Thursday 14 January 2021 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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12	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x - 5 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{25}{x+4} \qquad \text{for all real values of } x, \quad x \neq -4$$

The composite function fg is denoted by h

1 (a) Find $\mathrm{h}(x)$ giving your answer as a single fraction.

[2 marks]

Answer ______________________

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1 (b) The inverse of h is h^{-1}

1 (b) (i) Find $\mathrm{h}^{-1}(x)$

[3 marks]

Answer ____________________

1 (b) (ii) Find the range of h^{-1}

[1 mark]

Answer ____________________

6

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2 The line l_1 has equation $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ -4 \\ c \end{bmatrix}$

2 (a) In the case where l_1 and l_2 intersect, find

2 (a) (i) the value of c

[3 marks]

Answer ________________

2 (a) (ii) the coordinates of the point of intersection.

[1 mark]

Answer ________________

2 (b) In the case where l_1 and l_2 are perpendicular, find the value of c

[3 marks]

Answer ________________

7

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3 It is given that $y = 3\sin\theta - 3\cos\theta$

3 (a) Express y in the form $R\sin(\theta - \alpha)$ where R is a surd and $0^\circ < \alpha < 90^\circ$

[2 marks]

Answer

3 (b) Hence find

3 (b) (i) the greatest value of y^2

[1 mark]

Answer

3 (b) (ii) the least value of y^2

[1 mark]

Answer

3 (b) (iii) the values of θ in the interval $-90^\circ < \theta < 90^\circ$ for which $y = -\dfrac{3\sqrt{6}}{2}$

[3 marks]

Answer

7

4 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 1 + 2\cos x$

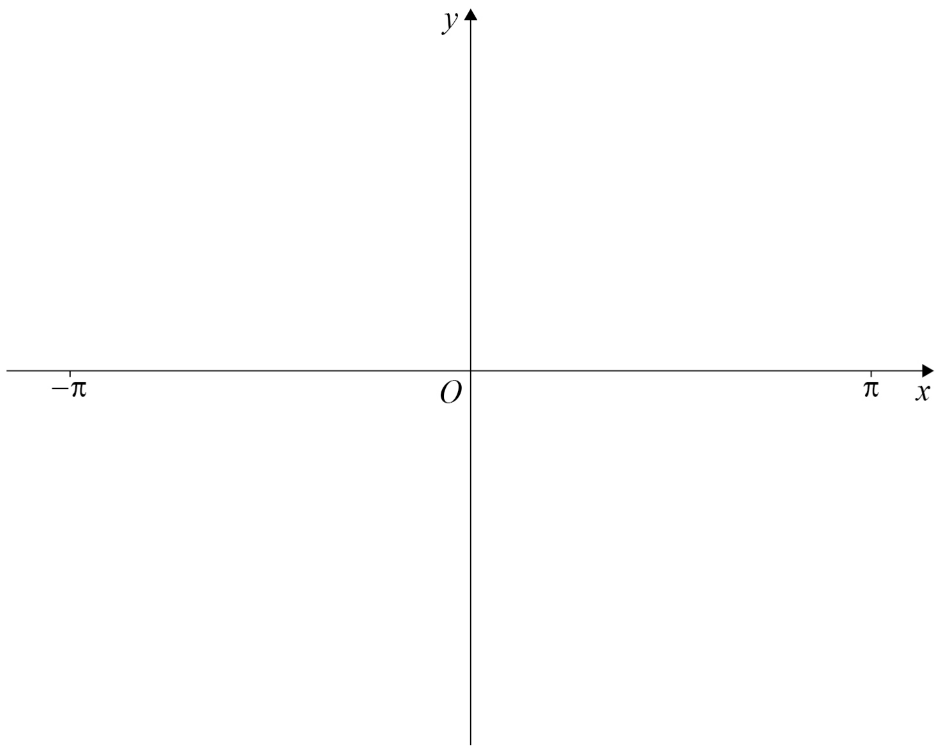
[4 marks]

4 (b) Sketch the graph of the curve with equation

$$y = 1 + 2\cos x \quad \text{for} \quad -\pi < x < \pi$$

indicating the value of y where the curve crosses the y-axis.

[2 marks]



4 (c) The region bounded by the curve $y = 1 + 2\cos x$ and the x-axis from $-\frac{2}{3}\pi$ to $\frac{2}{3}\pi$ is rotated through 2π radians about the x-axis to form a solid.

Find the exact value of the volume of the solid generated, giving your answer in the form $\pi\left(k\pi + p\sqrt{q}\right)$ where k, p and q are constants.

$\left[\text{you are given } \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\right]$

[6 marks]

Answer ______________________

12

Turn over ►

Do not write outside the box

5 (a) Find the binomial expansion of $\left(1+x^2\right)^{\frac{1}{2}}$ up to and including the term in x^4

[2 marks]

Answer ____________________

5 (b) By integrating each term in your answer to **part (a)**, find an approximate value of

$$\int_0^{0.5} \sqrt{\left(1+x^2\right)}\ \mathrm{d}x$$

giving your answer to five decimal places.

[4 marks]

Answer ____________________

Do not write outside the box

5 (c) Use Simpson's rule with **four** strips to find an estimate for

$$\int_0^{0.5} \sqrt{\left(1+x^2\right)}\,\mathrm{d}x$$

giving your answer to five decimal places.

[4 marks]

Answer ____________________

10

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6 It is given that $\alpha + \beta = 45^\circ$ where α and β are both positive.

6 (a) Find $\tan\beta$ in terms of $\tan\alpha$

[2 marks]

Answer ______________________________

6 (b) Show that $(1+\tan\alpha)(1+\tan\beta) = 2$

[2 marks]

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6 (c) Find the exact value of $\tan 22.5^{\circ}$

[3 marks]

Answer ______________________

7

Turn over for the next question

Do not write outside the box

7 (a) A curve has equation $y = \sin(\ln(2x))$, $0 < x < 2\pi$

The curve intersects the line $y = 3 - 4x$ at a single point where $x = \alpha$

7 (a) (i) Show that α lies between 0.6 and 0.7

[2 marks]

7 (a) (ii) The equation $\sin(\ln(2x)) = 3 - 4x$ can be rearranged to generate the iterative formula

$$x_{n+1} = \frac{3 - \sin(\ln(2x_n))}{4}$$

Use $x_1 = 0.6$ to find the values of x_2 and x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ ______ $x_3 =$ ______

7 (b) A curve has equation $y = \cos(\ln(3x))$, $0 < x < 2\pi$

Find the coordinates of a stationary point of the curve.

[3 marks]

Answer ______

7 (c) It is given that $y = A\sin(\ln(2x)) + B\cos(\ln(3x))$, $0 < x < 2\pi$ where A and B are constants.

Show that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

[4 marks]

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11

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8 (a) Given that $\cot x = \dfrac{\cos x}{\sin x}$, use the quotient rule to show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$$

[2 marks]

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8 (b) The curve C has the equation $x = \frac{3}{4}\cot\left(2y - \frac{\pi}{2}\right)$

8 (b) (i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ giving your answer in terms of y

[2 marks]

Answer ____________________

8 (b) (ii) Find the gradient of the normal to C at the point $\left(\frac{3}{4}, \frac{3\pi}{8}\right)$

[3 marks]

Answer ____________________

7

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9 (a) Given that $x^3 + y^3 = 3xy$ show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$

[2 marks]

9 (b) A curve has the equation $x^3 + y^3 = 3xy$

9 (b) (i) Find the coordinates of the stationary point of the curve in the interval $0 < x < 2^{\frac{2}{3}}$

Give your answer in exact form.

[3 marks]

Answer ______________________

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9 (b) (ii) Determine the nature of this stationary point.

[4 marks]

Answer ______________________

9

Turn over ►

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10 The difference between the temperature of an object and the temperature of the surrounding air is x °C at t minutes.

The rate at which this difference in temperature decreases is proportional to x

The surrounding air temperature is a constant 20 °C

When $t = 0$ the temperature of the object is 90 °C

When $t = 5$ the temperature of the object is 70 °C

10 (a) Explain briefly why this information can be represented by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -kx \qquad k > 0$$

[1 mark]

10 (b) Find the temperature of the object when $t = 15$ giving your answer to one decimal place

[6 marks]

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Answer ____________________

10 (c) Find the value of t when the temperature of the object is $40\,^{\circ}\mathrm{C}$ giving your answer to one decimal place.

[2 marks]

Answer ____________________

9

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11 (a) Find the exact value of $\int_0^6 x\mathrm{e}^{-0.5x}\,\mathrm{d}x$

[5 marks]

Answer

11 (b) Use the substitution $u^2 = x + 1$ to find

$$\int_8^{15} \frac{\sqrt{x+1}}{x-3}\,\mathrm{d}x$$

giving your answer in the form $a \ln b$ where a and b are constants.

[9 marks]

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Answer ______________________

14

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12 (a) Express $\dfrac{4x^2+5}{(1-x)(2-x)(5-2x)}$ in the form $\dfrac{A}{1-x}+\dfrac{B}{2-x}+\dfrac{C}{5-2x}$

[4 marks]

Answer ______________________

12 (b) Find the binomial expansion of $(2-x)^{-1}$ up to and including the term in x^2

[2 marks]

Answer ______________________

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12 (c) Use your answers to **parts (a)** and **(b)** to show that

$$\frac{4x^2+5}{(1-x)(2-x)(5-2x)} = D + Ex + Fx^2$$

for small values of x, stating the rational values of D, E and F

[4 marks]

10

Turn over ►

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13 A curve is defined by the parametric equations

$$x = ct, \quad y = \frac{c}{t} \qquad \text{where } t > 0 \text{ and } c \text{ is a constant.}$$

The tangent at the point $P\left(cp, \dfrac{c}{p}\right)$ on the curve meets the x-axis at A and the y-axis at B

The normal at the point P meets the line $y = x$ at C and the line $y = -x$ at D

13 (a) Find a Cartesian equation of the curve.

[1 mark]

Answer ______________________________

13 (b) Show that P is the mid-point of AB and the mid-point of CD

[7 marks]

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13 (c) Prove that A, B, C and D are the vertices of a square.

[3 marks]

11

END OF QUESTIONS

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