OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Thursday 13 January 2022 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
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6	
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8	
9	
10	
11	
12	
13	
TOTAL	

Answer **all** questions in the spaces provided.

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1 (a) Use Simpson's rule with 5 ordinates (4 strips) to find an estimate for $\int_0^3 e^{-x^2}\,dx$

Give your answer to three decimal places.

[4 marks]

Answer _____

Do not write outside the box

1 (b) A curve is defined by the equation $y = e^{-x^2}$ for $x \geq 0$

The curve intersects the line $y = \frac{1}{2}(x+1)$ at a single point where $x = \alpha$

1 (b) (i) Show that α lies between 0.5 and 0.6

[2 marks]

1 (b) (ii) Show that the equation $e^{-x^2} = \frac{1}{2}(x+1)$ can be rearranged to $x = \sqrt{\ln\left(\frac{2}{(x+1)}\right)}$

[2 marks]

1 (b) (iii) Use the iterative formula

$$x_{n+1} = \sqrt{\ln\left(\frac{2}{(x_n+1)}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

10

Turn over ►

Do not write outside the box

2 (a) It is given that $y = (2x+1)^8 \cos 3x$

Find $\frac{dy}{dx}$

[2 marks]

$\frac{dy}{dx} =$

2 (b) It is given that $y = \frac{3x^3 - 1}{2x^3 + 5}$

Show that $\frac{dy}{dx} = \frac{px^2}{\left(2x^3 + 5\right)^2}$ where p is a constant.

[2 marks]

Do not write outside the box

2 (c) It is given that $2xy^2 - 1 = 3x^2y + y$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

[3 marks]

$\frac{\mathrm{d}y}{\mathrm{d}x} =$

7

Turn over ►

Do not write outside the box

3 The polynomial $f(x)$ is defined by $f(x) = 8x^3 + ax^2 + bx + 6$ where a and b are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 6

When $f(x)$ is divided by $(2x + 1)$ the remainder is 9

3 (a) Find the value of a and the value of b

[4 marks]

$a =$ _____ $b =$ _____

3 (b) Use the Factor Theorem to prove that $(2x + 3)$ is a factor of $f(x)$

[1 mark]

Do not write outside the box

3 (c) Simplify $\dfrac{f(x)}{4x^2+4x-3}$ giving your answer in the form

$$2x+p+\frac{q}{2(2x-1)}$$

where p and q are constants.

[4 marks]

Answer _____

9

Do not write outside the box

4 The curve C_1 satisfies the differential equation $y^2 \frac{dy}{dx} = 2x$

The curve C_2 satisfies the differential equation $2y \frac{dy}{dx} = x^2$

Both curves pass through the point (2, 3)

4 (a) Find an equation for C_1

[2 marks]

Answer _____

4 (b) Find an equation for C_2

[2 marks]

Do not write outside the box

Answer _____

4 (c) The acute angle between the tangents to C_1 and C_2 at the point $(2, 3)$ is θ

Find the exact value of $\tan\theta$

[3 marks]

Answer _____

7

Turn over ►

Do not write outside the box

5 (a) (i) Express $12\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give your value of α in radians to three significant figures.

[3 marks]

Answer _____

5 (a) (ii) Hence solve the equation

$$12\cos(x + 0.4) - 5\sin(x + 0.4) = 6.5 \quad \text{for} \quad -\pi < x < \pi$$

giving all values of x to two decimal places.

[3 marks]

Answer _____

Do not write outside the box

5 (b) Solve the equation

$$8\cot^2 y = 2\operatorname{cosec} y + 7 \quad \text{for} \quad -180° < y < 180°$$

giving all solutions to the nearest degree.

[5 marks]

Answer _____

11

Do not write outside the box

6 (a) Describe the **single** geometrical transformation that maps the graph of $y = \ln x$ onto the graph of $y = \ln(x+2)+1$

[2 marks]

6 (b) The function f is defined by

$$\mathrm{f}(x) = \ln(x+2)+1 \quad \text{for} \quad x > -2$$

6 (b) (i) Find an expression for $\mathrm{f}^{-1}(x)$

[3 marks]

Answer _____

6 (b) (ii) Describe the **single** geometrical transformation that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

[1 mark]

6 (b) (iii) State the range of $\mathrm{f}^{-1}(x)$

[1 mark]

Answer _____

Do not write outside the box

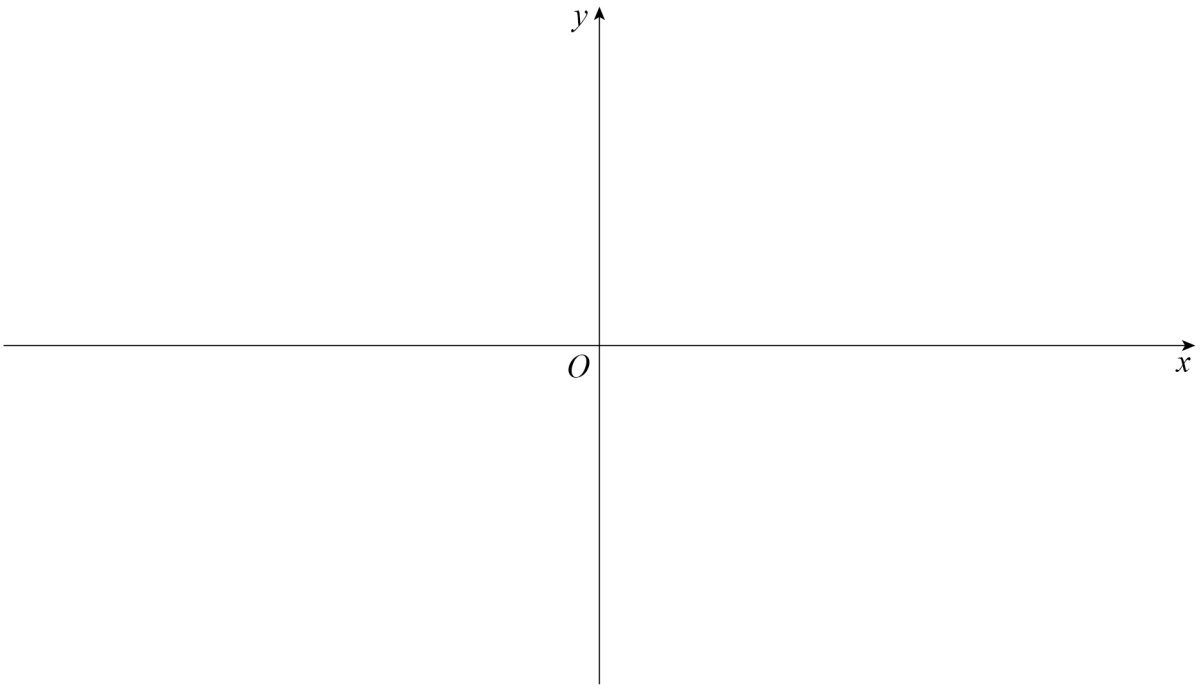
6 (c) A curve has equation

$$y = \ln(x+2)+1 \quad \text{for} \quad x > -2$$

6 (c) (i) Sketch the graph of the curve.

State, in an exact form, the coordinates of the points of intersection of the curve with the axes.

[3 marks]



6 (c) (ii) Find the equation of the tangent to the curve at the point where $x = -1$

[2 marks]

Answer _____

12

Do not write outside the box

7 (a) Use the substitution $u = e^{4x} + 1$ to find the exact value of $\int_0^{\ln 2} \frac{1}{e^{4x} + 1} \, dx$

Give your answer in the form $a \ln b$ where a and b are constants.

[8 marks]

Do not write outside the box

Answer _____

7 (b) Find $\int \frac{e^{4x}}{1+2e^{4x}}\,dx$

[2 marks]

Answer _____

10

Do not write outside the box

8 A curve is defined by the parametric equations

$$x = a\sec\theta \quad \text{and} \quad y = b\tan\theta \quad \text{for} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where a and b are non-zero constants.

8 (a) Find a Cartesian equation of the curve.

[2 marks]

Answer

8 (b) Find the equation of the normal to the curve at the point P where $\theta = \frac{\pi}{4}$

[5 marks]

Do not write outside the box

Answer

8 (c) The normal to the curve at P intersects the coordinate axes at the points A and B

Find, in terms of a and b, the exact value of the area of the triangle OAB where O is the origin.

[3 marks]

Answer

10

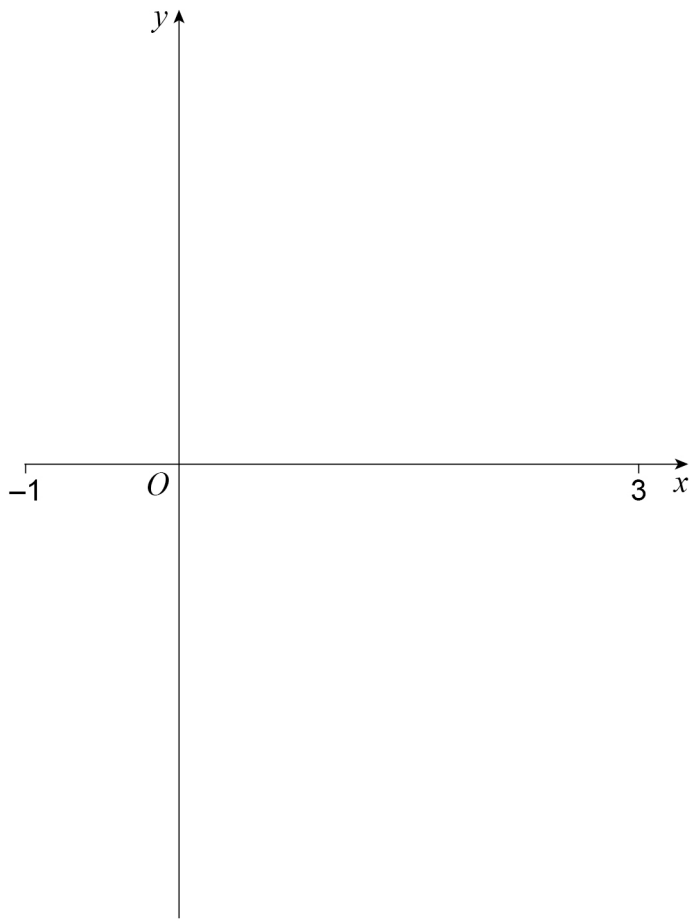
Do not write outside the box

9 The function f is defined by

$$\mathrm{f}(x) = \left|4 - x^2\right| - 3 \quad \text{for} \quad -1 \leq x \leq 3$$

9 (a) (i) Sketch the graph of $y = \mathrm{f}(x)$

[3 marks]



9 (a) (ii) Write down the range of f

[1 mark]

Answer _____

Do not write outside the box

9 (a) (iii) Solve $f(x) = -2$

[2 marks]

Answer _____

9 (b) The function g is defined by

$$g(x) = \frac{1}{x-1} \quad \text{for} \quad x \neq 1$$

Solve $fg(x) = -2$ giving your answers in an exact form.

[3 marks]

Answer _____

9

Turn over ►

Do not write outside the box

10 (a) By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$ show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

[3 marks]

10 (b) Use the result from **part (a)** and integration by parts to find

$$\int x\cos^3 2x \, \mathrm{d}x$$

[6 marks]

Do not write outside the box

Answer _____

9

Do not write outside the box

11 (a) Express $\dfrac{12}{(2-x)(1-2x)^2}$ in the form $\dfrac{A}{2-x}+\dfrac{B}{1-2x}+\dfrac{C}{(1-2x)^2}$

[4 marks]

Answer

11 (b) Find the binomial expansion of $(2-x)^{-1}$ up to and including the term in x^2

[1 mark]

Answer

Do not write outside the box

11 (c) Using your answers to **parts (a)** and **(b)** show that

$$\frac{12}{(2-x)(1-2x)^2} = D + Ex + Fx^2$$

for small values of x stating the values of the constants $D,\ E$ and F

[5 marks]

Answer _____

10

Do not write outside the box

12 The point A has coordinates $(-2, -2, 3)$

The line l has equation $\mathbf{r} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$

Find the shortest distance from A to the line l

[6 marks]

Do not write outside the box

Answer _____

6

Turn over for the next question

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13 Bacteria is grown in a laboratory.

The mass of bacteria M, in milligrams, after t days, satisfies the equation

$$M = \frac{A}{1 + 2e^{kt}}$$

where A and k are non-zero constants.

When $t = 0$, $M = 10$

When $t = 1$, $M = 15$

13 (a) Find the value of A and the exact value of k

[3 marks]

$A =$ _____ $k =$ _____

13 (b) Find the mass of bacteria after 5 days.

Give your answer to the nearest milligram.

[2 marks]

Answer _____

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13 (c) Find the value of t when the mass of bacteria is 18 milligrams.

Give your answer in an exact form.

[2 marks]

Answer _____

13 (d) Find the rate of change of the mass of bacteria when $t = 4$

Give your answer in the form $\frac{a}{b}\ln c$ where a, b and c are integers.

[3 marks]

Answer _____

10

END OF QUESTIONS

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