OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL AS
MATHEMATICS

(9660/MA02) Unit PSM1 Pure Mathematics, Statistics and Mechanics

Wednesday 5 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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ANSWER IN THE SPACES PROVIDED

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Section A

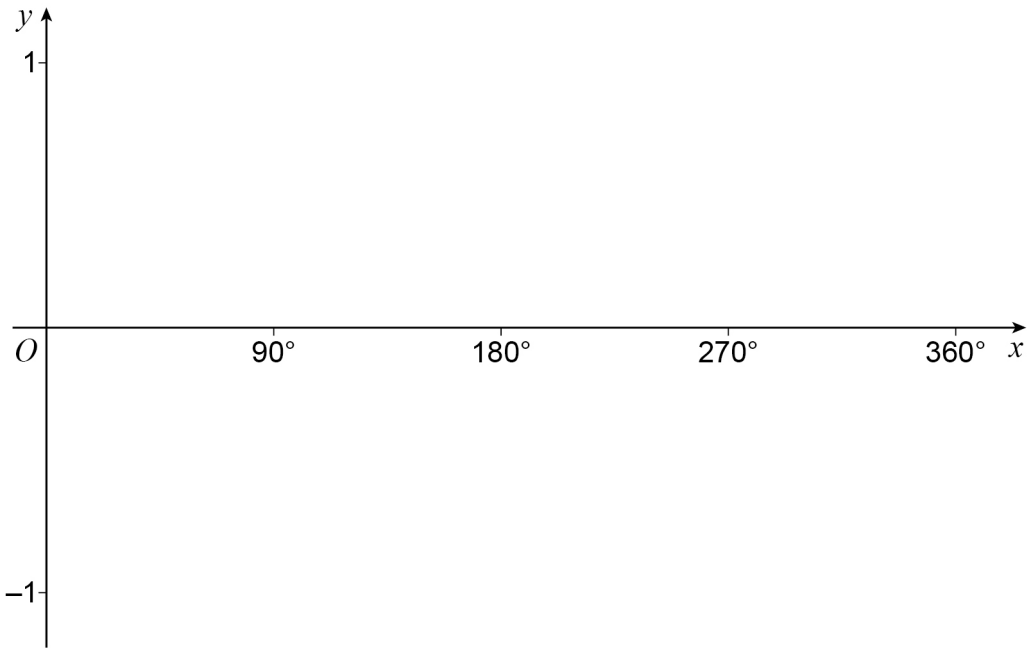
Pure Mathematics

Answer **all** questions in the spaces provided.

1 (a) On the axes, sketch the graph of $y = \cos\left(x + 60^\circ\right)$ for $0^\circ \leq x \leq 360^\circ$

Show on your graph the coordinates of any intercepts with the axes.

[3 marks]



1 (b) The period of $\mathrm{f}(x) = \cos\left(x + 60^\circ\right)$ is 4 times the period of $\mathrm{g}(x) = \tan(kx)$

where k is a constant.

Find the value of k

[2 marks]

$k =$ _____

5

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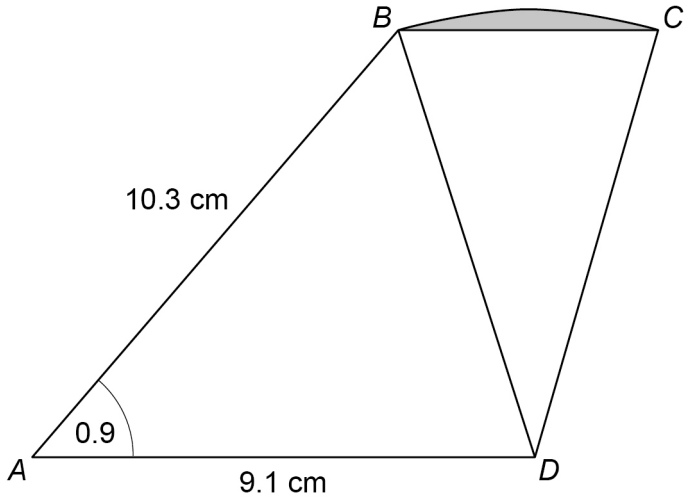
2 The diagram shows the wing of a model aircraft.

The wing consists of a triangle ABD and the sector BCD of a circle with centre D



The length $AB = 10.3$ cm

The length $AD = 9.1$ cm

The angle $BAD = 0.9$ radians

2 (a) Show that $BD = 8.5$ cm correct to one decimal place.

[2 marks]

2 (b) The arc $BC = 5.1$ cm

Show that angle $BDC = 0.6$ radians correct to one decimal place.

[1 mark]

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2 (c) Find the area of the shaded region bounded by the arc BC and the line segment BC

Give your answer to one decimal place.

[5 marks]

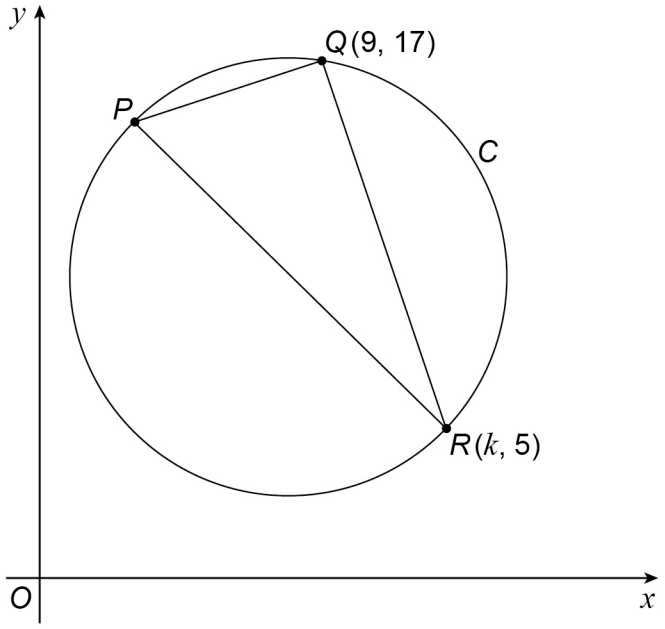
Answer _____

8

Turn over for the next question

3 The diagram shows the circle C and the triangle PQR

The points P, $Q(9, 17)$ and $R(k, 5)$, where k is a constant, lie on C



3 (a) PR is a diameter of C and the gradient of PQ is $\frac{1}{3}$

Show that $k = 13$

[3 marks]

3 (b) The centre of C has coordinates $(8, 10)$

3 (b) (i) Find the length of the radius of C giving your answer in exact form.

[2 marks]

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Answer

3 (b) (ii) Hence state the equation of C giving your answer in the form

$$(x-a)^2+(y-b)^2=c$$

where a, b and c are integers

[1 mark]

Answer

3 (c) The circle D has equation

$$x^2-8x+y^2+6y-d=0$$

where d is a constant.

The radius of D is equal in length to the radius of C

3 (c) (i) Find the coordinates of the centre of D

[2 marks]

Answer

3 (c) (ii) Describe the transformation which maps D onto C

[2 marks]

10

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4 (a) Solve the equation

$$\sin 2\alpha = 0.7$$

in the interval $0^\circ < \alpha < 90^\circ$, giving your answers to the nearest degree.

[2 marks]

Answer _____

4 (b) (i) Given that

$$\frac{10}{\sin x} - 3\tan x = \frac{11\cos x}{\sin x}$$

where $0^\circ < x < 90^\circ$, show that

$$8\cos^2 x - 10\cos x + 3 = 0$$

[3 marks]

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4 (b) (ii) Hence solve the equation

$$\frac{10}{\sin\left(\theta - 15^\circ\right)} - 3\tan\left(\theta - 15^\circ\right) = \frac{11\cos\left(\theta - 15^\circ\right)}{\sin\left(\theta - 15^\circ\right)}$$

in the interval $15^\circ < \theta < 90^\circ$

Give any non-exact answer to the nearest 0.1°

[4 marks]

Answer _____

9

Turn over ►

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5 (a) The point with coordinates $(p, 9)$ lies on the curve with equation $y = 7^{x-5}$

Find the value of p

Give your answer in the form $a + b\log_7 c$ where a, b and c are prime numbers.

[3 marks]

Answer _____

5 (b) It is given that

$$8 + \log_n k + 4\log_n (2y) = 6\log_n \left(n^2 y\right)$$

where n, k and y are positive constants.

Express y in terms of k and n in a form **not** involving logarithms.

Simplify your answer.

[5 marks]

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Answer _____

8

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The discrete random variable X has probability distribution function

$$\mathrm{P}(X = x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

6 (a) Show that $k = \frac{1}{30}$

[2 marks]

6 (b) Find $\mathrm{P}(X \geq 3)$

[1 mark]

Answer _____

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6 (c) Find the exact value of $\mathrm{E}\left(\frac{1}{X^3}\right)$

[2 marks]

Answer _____

6 (d) The mean of X is $\frac{10}{3}$

The random variable Y is independent of X and has mean 10

Find $\mathrm{E}(X-Y)$

[2 marks]

Answer _____

7

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7 Art and Business are optional subjects that students can study in a particular school.

The headteacher of the school selects a student at random.

The event that the student studies Art is represented by A

The event that the student studies Business is represented by B

It is given that $P(A) = 0.2$, $P(B) = 0.35$ and $P(A \cup B) = 0.48$

7 (a) Find $P(A \cap B)$

[2 marks]

Answer _____

7 (b) State with a reason whether A and B are mutually exclusive.

[1 mark]

Do not write outside the box

7 (c) Find the probability that the student studies Business, given that the student studies Art.

[2 marks]

Answer _____

7 (d) State with a reason whether A and B are independent.

[1 mark]

6

Turn over for the next question

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8 The discrete random variable X has a Bernoulli distribution with parameter p

8 (a) (i) State $\mathrm{E}(X)$ in terms of p

[1 mark]

Answer _____

8 (a) (ii) State $\mathrm{Var}(X)$ in terms of p

[1 mark]

Answer _____

8 (b) The discrete random variables X_1, X_2, ..., X_n are independent and identically distributed Bernoulli distributions with parameter p

It is given that $\mathrm{E}\left(\sum_{i=1}^{n} X_i\right) = 6.25$ and $\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = 4.6875$

Find the value of n and the value of p

[5 marks]

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$n =$ _____ $p =$ _____

7

Turn over for the next question

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 A sphere of mass 2 kg moves with speed $5\,\text{m}\,\text{s}^{-1}$ in a straight line on a smooth horizontal surface towards a fixed vertical wall.

The wall is perpendicular to the path of the sphere.

The sphere collides with the wall and rebounds with speed $1.5\,\text{m}\,\text{s}^{-1}$

9 (a) Find the magnitude of the momentum of the sphere before the collision.

[1 mark]

Answer _____

9 (b) Find the magnitude of the impulse exerted on the wall by the sphere during the collision.

[2 marks]

Answer _____

3

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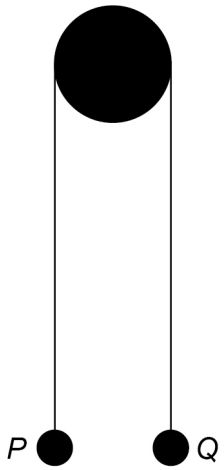
10 **The acceleration due to gravity, g, should be taken as $9.8\,\text{m s}^{-2}$**

Two masses P and Q are connected by a light inextensible string that passes over a smooth fixed peg, as shown below.



The mass of P is 0.2 kg and the mass of Q is m kg

P and Q are released from rest with the string taut.

Immediately after release Q accelerates vertically downwards at $1.25\,\text{m s}^{-2}$

Find the value of m

[4 marks]

$m =$ _____

4

Turn over ►

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11 A particle moves along a straight line.

The velocity v m s^{-1} of the particle at time t seconds after it starts its motion is given by

$$v = \frac{t^3}{2} - 3t^2 + 5t$$

11 (a) Find the acceleration of the particle at $t = 2$

[3 marks]

Answer _____

11 (b) Show that the particle never changes direction.

[3 marks]

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11 (c) Find the change in displacement for the particle between $t = 0$ and $t = 4$

[3 marks]

Answer _____

9

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12 A firework is launched from rest vertically upwards from ground level and travels vertically with constant acceleration for 3.4 seconds and then explodes.

When it explodes the firework is at a height of 150 metres above the ground.

12 (a) Find the acceleration of the firework.

[3 marks]

Answer _____

12 (b) After the explosion parts from the firework fall to the ground.

A method to find the speed v m s^{-1} at which one of the parts collides with the ground is shown below.

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2 \times 9.8 \times 150$$
$$v = 54 \quad \text{(to two significant figures)}$$

Other than air resistance not being considered, give a reason why it may **not** be appropriate to use this method to calculate the value of v

[1 mark]

4

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