

Please write clearly in block capitals.

Centre number

Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

I declare this is my own work.

# INTERNATIONAL A-LEVEL MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 2 June 2023

07:00 GMT

Time allowed: 2 hours 30 minutes

## Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
<b>TOTAL</b>	



Answer **all** questions in the spaces provided.

1 (a) For each of the following find  $\frac{dy}{dx}$

1 (a) (i)  $y = (3x^5 - 4)^8$

[2 marks]

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Answer \_\_\_\_\_

1 (a) (ii)  $y = \frac{3x^4}{7x-5}$

[2 marks]

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Answer \_\_\_\_\_

1 (a) (iii)  $\ln(x^2) + y^2 = xy$

[3 marks]

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Answer \_\_\_\_\_





2 (a) The curve with equation  $y = 2^{-x}$  intersects the line  $y = 4 - 2x$  at a point where  $x = \alpha$  and  $\alpha > 0$

2 (a) (i) Show that  $\alpha$  lies between 1.8 and 1.9

[2 marks]

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2 (a) (ii) Show that the equation  $4 - 2x = 2^{-x}$  can be rearranged into the form  $x = 2 - 2^{-(x+1)}$

[1 mark]

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2 (a) (iii) Use the iterative formula

$$x_{n+1} = 2 - 2^{-(x_n+1)}$$

with  $x_1 = 1.8$  to find the values of  $x_2$  and  $x_3$  giving your answers to three decimal places.

[2 marks]

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$$x_2 = \underline{\hspace{2cm}} \quad x_3 = \underline{\hspace{2cm}}$$





- 3 (a)** Describe a sequence of **two** geometrical transformations that maps the graph of  $y = \sec \theta$  onto the graph of  $y = 0.5\sec(\theta+1)$

[4 marks]

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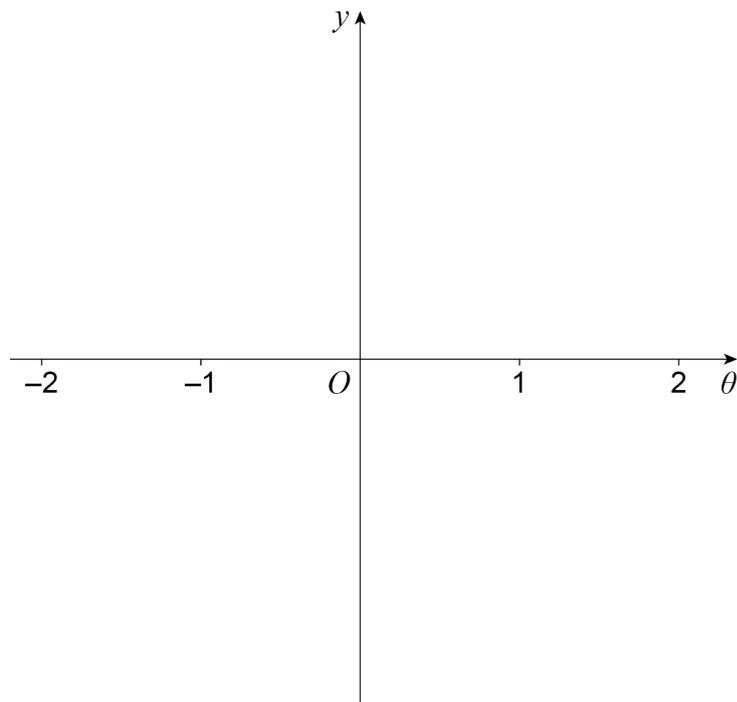
- 3 (b)** Sketch on the axes below the curve with equation

$$y = 0.5\sec(\theta+1) \quad \text{for} \quad -2 < \theta < 2$$

where  $\theta$  is in radians.

State the coordinates of the stationary point and the coordinates of any intercept with the axes.

[3 marks]




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3 (c) It is given that  $x = \frac{1}{2\cos y}$

3 (c) (i) Find  $\frac{dx}{dy}$

[1 mark]

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Answer \_\_\_\_\_

3 (c) (ii) Hence find  $\frac{dy}{dx}$  in terms of  $\sin y$

[2 marks]

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Answer \_\_\_\_\_

10

Turn over ►



4 The functions  $f$  and  $g$  are defined by

$$f(x) = \sqrt{1 - 0.25x} \quad \text{for } x \leq 4$$

$$g(x) = \frac{8}{x-1} \quad \text{for all real values of } x, x \neq 1$$

4 (a) State the range of the function  $f$

[1 mark]

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Answer \_\_\_\_\_

4 (b) (i) Find  $fg(x)$

[1 mark]

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Answer \_\_\_\_\_

4 (b) (ii) Find the domain of the function  $fg$

[2 marks]

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Answer \_\_\_\_\_

4 (b) (iii) Solve the equation  $fg(x) = 3$

[2 marks]

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Answer \_\_\_\_\_





- 5 (a)** Express  $7\cos\theta + 24\sin\theta$  in the form  $R\cos(\theta - \alpha)$  where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .  
Give the value of  $\alpha$  in radians to three significant figures.

**[3 marks]**


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Answer \_\_\_\_\_

- 5 (b)** Show that

$$2\operatorname{cosec} 4x + 2\cot 4x = \cot x - \tan x$$

for all  $x$  where  $x \neq \frac{n\pi}{4}$

**[4 marks]**


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Answer \_\_\_\_\_

- 7 (b) (ii) Hence write down a Cartesian equation of the curve defined by the parametric equations

$$x = e^{kn} + e^{-kn} \quad \text{and} \quad y = e^{kn} - e^{-kn}$$

where  $n$  is the parameter and  $k$  is a positive integer.

[1 mark]

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Answer \_\_\_\_\_

- 7 (c) A third curve is defined by the parametric equations

$$x = a \sin \theta + b \cos \theta \quad \text{and} \quad y = a \cos \theta - b \sin \theta$$

where  $\theta$  is the parameter and  $a$  and  $b$  are constants.

Find a Cartesian equation of this curve.

[3 marks]

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Answer \_\_\_\_\_



- 8 (a)** The rate at which the population of a colony of ants is increasing is directly proportional to the population  $P$  of the colony.

Currently the population is 1 000 000 and is increasing at a rate of 3000 per month.

Construct a differential equation for the population of the colony.

**[2 marks]**

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Answer \_\_\_\_\_

- 8 (b)** It is given that  $y = f(x)$  satisfies the differential equation

$$2x \frac{dy}{dx} + y^2 = 4$$

Find the solution of the differential equation for which  $y = 1$  when  $x = 1$

Give your answer in the form  $y = f(x)$

**[6 marks]**

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- 12 (a)** Find a vector equation of the line  $l_1$  which passes through the points  $A(2, -1, 3)$  and  $B(5, -2, -1)$

[1 mark]

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Answer \_\_\_\_\_

- 12 (b)** The line  $l_2$  has equation  $\mathbf{r} = \begin{bmatrix} -2 \\ 5 \\ 7 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ -2 \\ k \end{bmatrix}$

The lines  $l_1$  and  $l_2$  intersect.

- 12 (b) (i)** Find the value of  $k$

[3 marks]

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Answer \_\_\_\_\_

- 12 (b) (ii)** Find the coordinates of the point of intersection.

[1 mark]

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Answer \_\_\_\_\_





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