# OXFORD AQA
## INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number ☐☐☐☐☐    Candidate number ☐☐☐☐

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I declare this is my own work.

# INTERNATIONAL A-LEVEL
# MATHEMATICS

(9660/MA03)  Unit P2  Pure Mathematics

Friday 12 January 2024        07:00 GMT        Time allowed: 2 hours 30 minutes

## Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions
- Use black ink or black ball-point pen.  Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided.  Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book.  Write the question number against your answer(s).
- Do all rough work in this book.  Cross through any work you do not want to be marked.

## Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

## Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| TOTAL | |

Answer **all** questions in the spaces provided.

**1** The polynomial $f(x)$ is defined by

$$f(x) = 16x^3 + bx^2 + cx + 12$$

where $b$ and $c$ are constants.

When $f(x)$ is divided by $(4x + 1)$ the remainder is $11.5$

When $f(x)$ is divided by $(2x - 1)$ the remainder is $17.5$

Find the value of $b$ and the value of $c$

**[4 marks]**
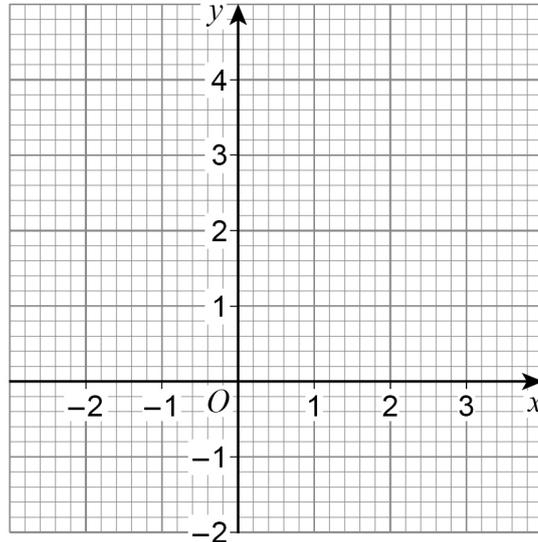
$b =$ _____     $c =$ _____

4

**2 (a)** On the axes below, draw the graph of

$$y = |2x - 1| - 1 \quad \text{for} \quad -2 \leq x \leq 3$$

**[2 marks]**



**2 (b)** Solve the inequality

$$|2x - 1| - 1 \geq |x|$$

**[2 marks]**

Answer _____

4

**Turn over ▶**

**3 (a)** For each of the following find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

**3 (a) (i)** $y = \mathrm{e}^{-0.5x}\sin 3x$

**[2 marks]**

$$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \underline{\hspace{6cm}}$$

**3 (a) (ii)** $y = \dfrac{\left(1-2x\right)^3}{3+\tan 5x}$

**[3 marks]**

$$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \underline{\hspace{6cm}}$$

**3 (a) (iii)** $x + \ln(xy) = x^3 + y^2$

**[4 marks]**

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\hspace{8cm}}$$

**Question 3 continues on the next page**

**Turn over ▶**

**3 (b) (i)** Find $\int \frac{x}{4x^2+5}\,dx$

[2 marks]

Answer

**3 (b) (ii)** Find $\displaystyle\int_0^{\frac{\pi}{2}} x\cos x\ \mathrm{d}x$

Give your answer in an exact form.

**[4 marks]**

Answer _____

15

**Turn over for the next question**

**Turn over ▶**

**4 (a)** Use the mid-ordinate rule with 5 strips, to find an estimate for

$$\int_0^{0.6} \left( 4^{-x} - \frac{1}{4} \right) \, dx$$

Give your answer to four decimal places.

[4 marks]

Answer _____

**4 (b)** The function $f$ is defined by

$$f(x) = 4^{-x} - \frac{1}{4} \qquad \text{for } x \geq 0$$

The inverse of $f$ is $f^{-1}$

**4 (b) (i)** Find $f^{-1}(x)$

[3 marks]

Answer _____

**4 (b) (ii)** Find the domain of $f^{-1}$

[2 marks]

Answer _____

**4 (c)** Describe the **single** geometrical transformation that maps the graph of $y = f(x)$ onto the graph of $y = f^{-1}(x)$

[1 mark]

10

**Turn over ▶**

**5 (a) (i)** Find $R$ and $\alpha$ such that $10\sin\theta - 24\cos\theta = R\sin(\theta - \alpha)$ where $R > 0$ and

$0 < \alpha < \dfrac{\pi}{2}$

Give your value of $\alpha$ in radians to three significant figures.

**[3 marks]**

Answer

**5 (a) (ii)** Write down the minimum value of $10\sin\theta - 24\cos\theta$

**[1 mark]**

Answer

**5 (a) (iii)** Find the value of $\theta$ in the interval $2\pi < \theta < 4\pi$ at which the minimum value of $10\sin\theta - 24\cos\theta$ occurs.

Give your answer to two decimal places.

**[1 mark]**

Answer

**5 (a) (iv)** Solve the equation

$$10\sin(x-0.6)-24\cos(x-0.6)=6.5 \quad \text{for} \quad -\pi < x < \pi$$

Give all values of $x$ to two decimal places.

**[3 marks]**

Answer

**5 (b)** Solve the equation

$$16\tan^2(2y-10°)-14=4\sec(2y-10°) \quad \text{for} \quad -90° < y < 90°$$

Give all values of $y$ to the nearest degree.
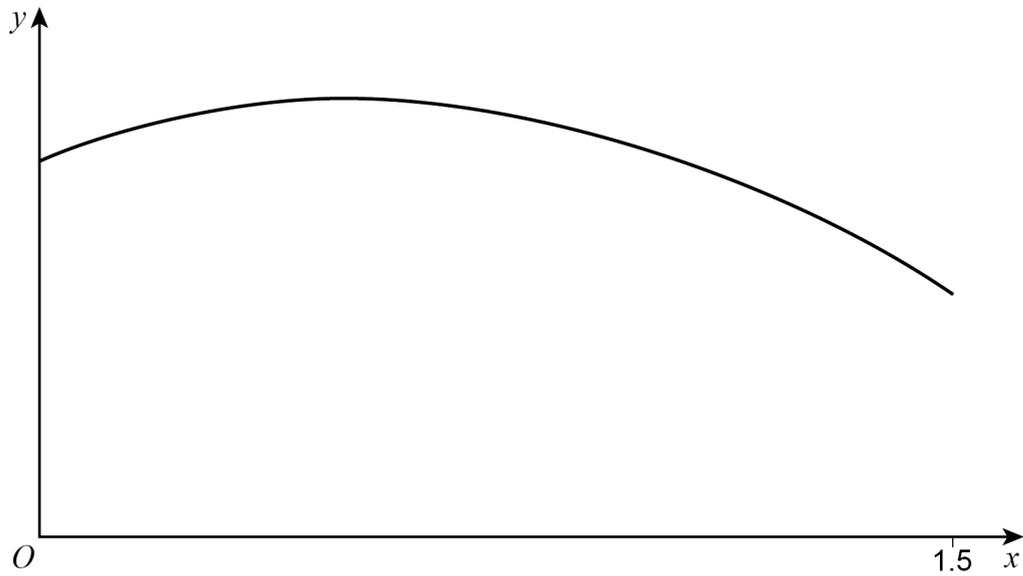
**[5 marks]**

Answer

**Turn over ▶**

13

**6**     The function $f$ is defined by

$$f(x) = -x^2 + \ln(12 + 24x) \quad \text{for} \quad 0 \le x \le 1.5$$

The graph of $y = f(x)$ is shown below.



**6 (a)**     Find the range of $f$

Give your answer in an exact form.

**[5 marks]**

Answer _____

**Question 6 continues on the next page**

**6 (b)** The graph of $y = f(x)$ intersects the graph of $y = 2x$ at the point where $x = \alpha$

**6 (b) (i)** Show that $\alpha$ lies between 1.1 and 1.2

**[2 marks]**

**6 (b) (ii)** Show that the equation $-x^2 + \ln(12 + 24x) = 2x$ can be rearranged into the form

$$x = -1 + \sqrt{1 + \ln(12 + 24x)}$$

**[1 mark]**

**6 (b) (iii)** Use the iterative formula

$$x_{n+1} = -1 + \sqrt{1 + \ln(12 + 24x_n)}$$

with $x_1 = 1.1$ to find the value of $x_2$ and the value of $x_3$

Give your answers to three decimal places.

**[2 marks]**

$x_2 = $ _____     $x_3 = $ _____

**6 (c) (i)** Describe the **single** geometrical transformation that maps the graph of $y = -x^2 + \ln(12 + 24x)$ onto the graph of $y = -x^2 + \ln(1 + 2x)$

[2 marks]

**6 (c) (ii)** It is given that

$$\int_0^{1.5} \left( -x^2 + \ln(12 + 24x) \right) \, dx = A$$

where $A$ is a constant.

Find, in terms of $A$, the exact value of $\int_0^{1.5} \left( -x^2 + \ln(1 + 2x) \right) \, dx$

[1 mark]

Answer

13

**Turn over ▶**

**7 (a)** The curve $C_1$ has Cartesian equation

$$x^2 - y^2 = 6y - 2x + 20$$

Find an equation of the tangent to $C_1$ at the point $(3, -1)$

**[4 marks]**

Answer _____

**7 (b)** The curve $C_2$ has parametric equations

$$x = \frac{1 + \sqrt{17}\cos\theta}{2} \qquad \text{and} \qquad y = -1 + \sqrt{17}\sin\theta \qquad \text{for} \ \ 0 \le \theta \le 2\pi$$

**7 (b) (i)** Find a Cartesian equation of $C_2$

**[2 marks]**

Answer _____

**7 (b) (ii)** Find an equation of the normal to $C_2$ at the point where $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{17}}\right)$

**[4 marks]**

Answer _____

**7 (c)** The tangent found in **part (a)** intersects the coordinate axes at the points $A$ and $B$

The normal found in **part (b)(ii)** intersects the coordinate axes at the points $P$ and $Q$

Find the ratio

Area of triangle $OAB$ : Area of triangle $OPQ$

where $O$ is the origin of the coordinate axes.

**[1 mark]**

Answer _____

**11**

**Turn over ▶**

**8**     The region bounded by the curve $y = \dfrac{1}{10 - 2x}$, the line $y = 1$ and the $y$-axis

from $y = 0.1$ to $y = 1$ is rotated through $2\pi$ radians about the **$y$-axis** to form a solid.

Find the volume of the solid generated.

Give your answer in an exact form.

[**7 marks**]

Answer _____

**7**

**Turn over for the next question**

**Turn over ▶**

**9 (a)** The gradient of a curve at the point $(x, y)$ is directly proportional to the sum of 3 times the $x$-coordinate and 4 times the $y$-coordinate.

Construct a differential equation for the curve.

**[2 marks]**

Answer _____

**9 (b) (i)** Solve the differential equation

$$\frac{dy}{dx} + e^{2y} = 4xe^{2y} \quad \text{for} \quad -0.6 < x < 1.1$$

such that $y = 0$ when $x = 1$

Give your answer in the form $y = f(x)$

**[6 marks]**

Answer _____

**9 (b) (ii)** Find a second value for $x$ when $y = 0$ for the solution $y = f(x)$ found in **part (b)(i)**.

**[2 marks]**

Answer _____

**Turn over ▶**

**10 (a)** Find the values of $A$, $B$ and $C$ such that

$$\frac{x^2}{(3-x)(3+2x)(3-2x)} = \frac{A}{3-x} + \frac{B}{3+2x} + \frac{C}{3-2x}$$

**[4 marks]**

$A =$ _____  $B =$ _____  $C =$ _____

**10 (b) (i)** Find the binomial expansion of $(3-x)^{-1}$ up to and including the term in $x^2$

**[2 marks]**

Answer _____

**10 (b) (ii)** State the values of $x$ for which the full binomial expansion of $(3-x)^{-1}$ is valid.

**[1 mark]**

Answer _____

**10 (c)** Use your answers to **parts (a)** and **(b)(i)** to show that for small values of $x$

$$\frac{x^2}{(3-x)(3+2x)(3-2x)} = Dx^2$$

where $D$ is a rational number.

**[5 marks]**

**12**

**Turn over ▶**

**11**     Use the substitution $u^2 = 9 - x^2$ to find

$$\int_0^1 \frac{x^3}{\sqrt{(9 - x^2)}} \, dx$$

Give your answer in the form $a + b\sqrt{2}$ where $a$ and $b$ are constants.

**[7 marks]**

Answer _____

**7**

**Turn over for the next question**

**12** The points *A*, *B* and *C* have coordinates $(4, 2, 3)$, $(-2, 6, 15)$ and $(-1, 10, 6)$ respectively.

**12 (a)** Show that for all values of $p$, the point $P(-3p - 2, \ 2p + 6, \ 6p + 15)$ lies on the line passing through *A* and *B*

[4 marks]

**12 (b) (i)** Find the value of $p$ for which *CP* is perpendicular to *AB*

[4 marks]

Answer _____

**12 (b) (ii)** Hence find the area of triangle *ABC*

**[4 marks]**

Answer _____

**12 (c)** Find the angle *BAC*

**[2 marks]**

Answer _____

**END OF QUESTIONS**

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