

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL A-LEVEL

MATHEMATICS

(9660/MA03) Unit P2 Pure Mathematics

Friday 31 May 2024

07:00 GMT

Time allowed: 2 hours 30 minutes

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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10	
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12	
13	
14	
TOTAL	

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ANSWER IN THE SPACES PROVIDED**



Answer **all** questions in the spaces provided.

- 1 (a)** Express $2\cos\theta + \sqrt{5}\sin\theta$ in the form $R\cos(\theta - \alpha)$
where $R > 0$ and $0^\circ < \alpha < 90^\circ$

Give the value of α to the nearest degree.

[3 marks]

Answer _____

- 1 (b) (i)** Hence write down the minimum value of $2\cos\theta + \sqrt{5}\sin\theta$

[1 mark]

Answer _____

- 1 (b) (ii)** Find the smallest possible positive value of θ at which the minimum value in **part (b)(i)** occurs.

Give your answer to the nearest degree.

[1 mark]

Answer _____

Turn over ►



2 The functions f and g are defined by

$$f(x) = \sqrt{18 - 2x^2} \quad \text{for } 0 \leq x \leq 3$$

$$g(x) = \frac{3}{2x-1} \quad \text{for } x > 0.5$$

2 (a) Determine the range of the function f

[1 mark]

Answer _____

2 (b) (i) Find $fg(x)$

[1 mark]

Answer _____

2 (b) (ii) Find the domain of the function fg

[2 marks]

Answer _____



2 (b) (iii) Solve the equation $fg(x) = 4$

[3 marks]

Answer _____

2 (c) The inverse of the function fg is h

Find $h(x)$

[3 marks]

Answer _____

10

Turn over ►



Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 + 12x + 9$

[illegible]

5



- Show that $\frac{dX}{dt} = -kX$

- Give your answer correct to one decimal place.

[illegible]

Answer

6

5 (a)

$$\int_0^2 3 - \tan^{-1}(2x - 0.5) \, dx$$

Give your answer to three decimal places.

[4 marks]

[illegible]

Answer



5 (b) A curve has equation

$$y = 0.5 \tan(3 - x) + 0.25$$

5 (b) (i) In the interval $\frac{\pi}{2} < x < \pi$, the curve intersects the line $y = x$ at a single point where $x = \alpha$

Show that α lies between 1.7 and 1.8

[2 marks]

5 (b) (ii) Show that the equation $0.5 \tan(3 - x) + 0.25 - x = 0$ can be rearranged into the form

$$x = 3 - \tan^{-1}(2x - 0.5)$$

[2 marks]

Question 5 continues on the next page

Turn over ►



5 (b) (iii) Use the iterative formula

$$x_{n+1} = 3 - \tan^{-1}(2x_n - 0.5)$$

with $x_1 = 1.7$ to find the value of x_2 and the value of x_3

Give your answers to three decimal places.

[2 marks]

$x_2 =$ _____ $x_3 =$ _____

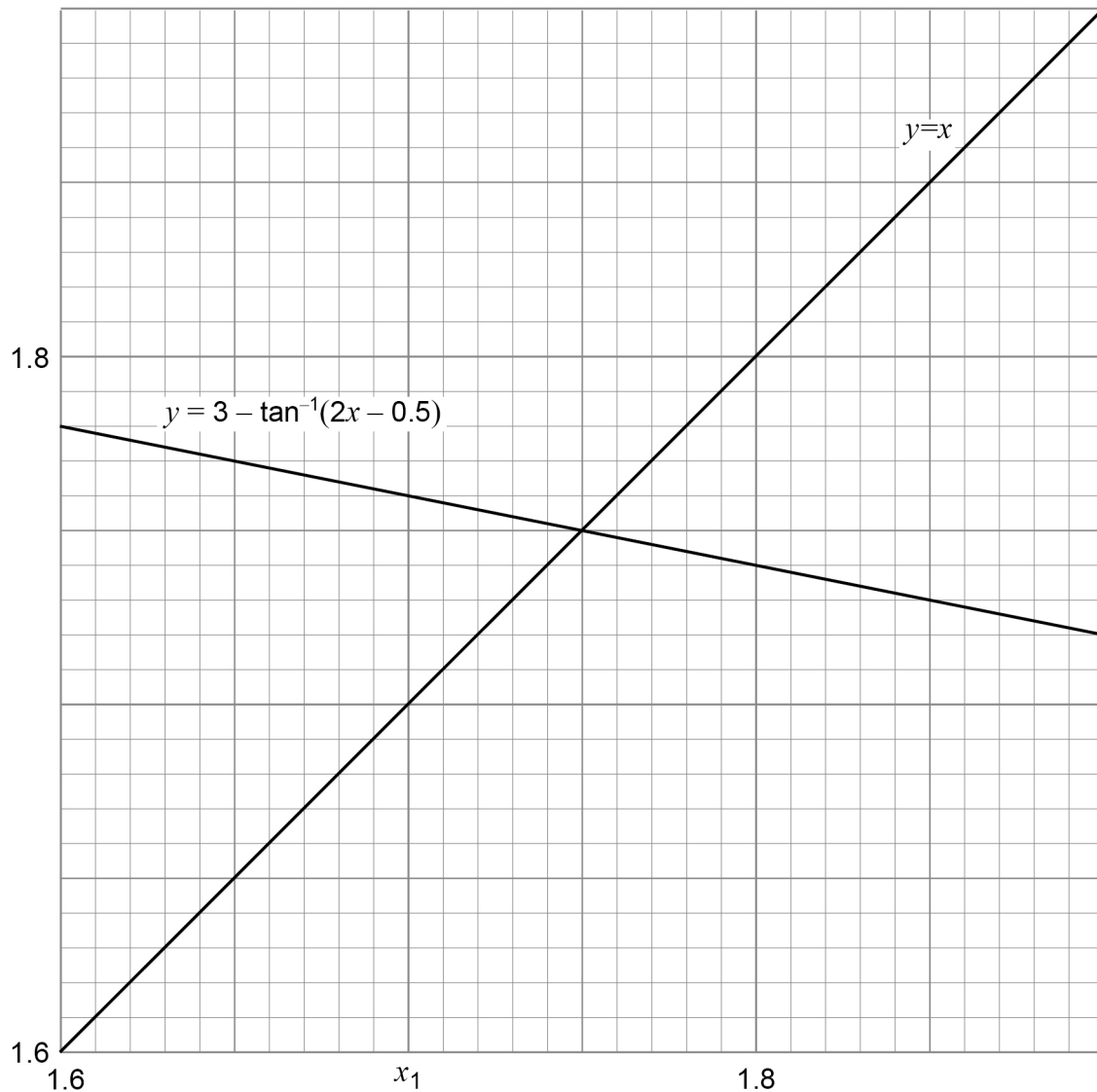


5 (b) (iv) The figure below shows parts of the graphs of

$$y = 3 - \tan^{-1}(2x - 0.5) \text{ and } y = x, \text{ and the position of } x_1$$

On the figure, draw a cobweb or staircase diagram to show how convergence takes place, indicating the position of x_2 and the positions of x_3

[2 marks]



Turn over for the next question

Turn over ►



$$(\sin\theta - \operatorname{cosec}\theta)(\cos\theta - \sec\theta) = 0.5\sin 2\theta$$
[illegible]

$$(\sin(1.5x + 0.1) - \operatorname{cosec}(1.5x + 0.1))(\cos(1.5x + 0.1) - \sec(1.5x + 0.1)) = 0.4$$

[3 marks]

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Answer _____

Turn over for the next question

6

Turn over ►



7 (a) Find $\int \frac{x-2}{2x^2-8x+3} dx$

[2 marks]

Answer _____

7 (b) Find $\int \frac{5x+1}{5x-1} dx$

[3 marks]

Answer _____



7 (c) Find $\int \sin^2 3x \, dx$

[3 marks]

Answer _____

8

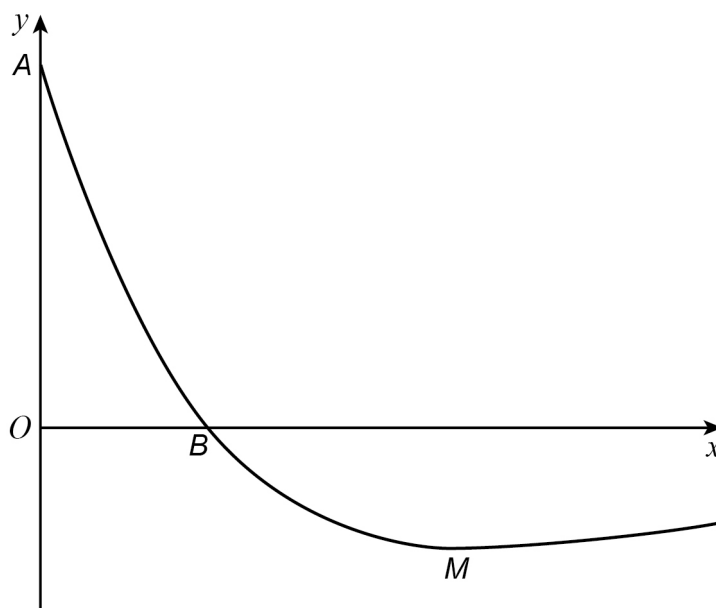
Turn over for the next question

Turn over ►



8 The diagram shows part of the curve

$$y = (5 - 4x)e^{-0.5x}$$



The curve intersects the axes at A and B and has a stationary point at M

8 (a) Write down the coordinates of A and B

[1 mark]

A _____ B _____

8 (b) (i) Find $\frac{dy}{dx}$

Give your answer in the form $(ax + b)e^{-0.5x}$ where a and b are constants.

[3 marks]

$\frac{dy}{dx} =$ _____



Give your answer in an exact form.

[3 marks]

Answer

Give your answer in an exact form.

[6 marks]

[illegible]

Answer



9

$$x = a(2\theta - \sin 2\theta) \quad \text{and} \quad y = a(1 - \cos 2\theta) \quad \text{for} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

where a is a non-zero constant.

9 (a) Find an equation of the tangent to the curve at the point P where $\theta = \frac{\pi}{4}$

[6 marks]

[illegible]

Answer _____



- 9 (b)** Find the equation of the normal to the curve at P

Give your answer in the form $y = mx + c$

[2 marks]

Answer _____

- 9 (c)** The tangent to the curve at P intersects the y -axis at the point Q

The normal to the curve at P intersects the y -axis at the point R

Show that the area of the triangle PQR is $\frac{1}{4}a^2(\pi - 2)^2$

[2 marks]

Turn over for the next question

Turn over ►



10 A curve C has equation

$$y = 4x - (x - y)^2$$

10 (a) The line $x = 4$ intersects the curve C at the points A and B

Find the length of the line segment AB

[2 marks]

Answer _____



Give your answer in an exact form.

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Answer _____

6

Turn over ►



[4 marks]

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Answer

[2 marks]

Answer



11 (c)

for small values of x , where D , E and F are constants.

[illegible]

Answer

Turn over ►



It is given that $y = f(x)$ satisfies the differential equation

Find the solution of the differential equation given that $y=1$ when $x=0$

Give your answer in the form $y = f(x)$

[illegible]

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Answer _____

9

Turn over ►



13 The point A has coordinates $(2, -2, 6)$

The point B has coordinates $(8, 6, -4)$

13 (a) Find a vector equation of the line AB

[1 mark]

Answer _____

13 (b) The line l has equation $\mathbf{r} = \begin{bmatrix} -2 \\ 3 \\ c \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$

13 (b) (i) It is given that the line l intersects the line AB

Show that $c = 3$

[3 marks]



[5 marks]

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9

Use the quotient rule to show that

$$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

[2 marks]

14 (a) (ii) Use the substitution $u = 1 + \cot x$ to find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\operatorname{cosec}^2 x}{(1 + \cot x)^2} dx$

Give your answer in the form $p+q\sqrt{3}$ where p and q are rational numbers.

[5 marks]

Answer



$$\int \frac{1}{\sqrt{(2x+1)} + \sqrt{(2x-1)}} dx$$

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Answer

10



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[illegible]

[illegible]