

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

June 2024

Version: 1.0 Final



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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$0^2 = u^2 + 2 \times (-9.8) \times 0.4$	M1	Uses constant acceleration equation or two term energy equation
	$u = \sqrt{7.84} = \frac{14}{5} = 2.8 \text{ [m s}^{-1}\text{]}$	A1	Correct rebound speed
		2	

Q	Answer	Marks	Comments
1(b)	$3.24 = 0.3 \times 2.8 - 0.3 \times -u$	M1	Uses impulse formula to create equation, PI by 8 m s^{-1}
	$u = 8$	A1	Correct impact speed Accept -8
	Impact Speed $\times e = 2.8$	M1	Uses their impact speed to form an equation for e PI by $e = 0.35$
	$e = \frac{2.8}{8} = \frac{7}{20} = 0.35$	A1	Correct value for e
		4	

	Question 1 Total	6	
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Q	Answer	Marks	Comments
2(a)	Resultant force in vertical direction $6a = \frac{300}{2} \times 0.5 - 6 \times 9.8$ $a = 2.7 \text{ [m s}^{-2}\text{]}$ Direction is vertically upwards	M1 A1 B1	Three term equation of motion Correct magnitude AWRT 2.7 Correct direction
		3	

Q	Answer	Marks	Comments
2(b)	Maximum speed when $6 \times 9.8 = \frac{300}{2} \times e$ $e = 0.392$ $\frac{1}{2} \times \frac{300}{2} \times 0.5^2 = \frac{1}{2} \times \frac{300}{2} \times 0.392^2 + 6 \times 9.8 \times 0.108 + \frac{1}{2} \times 6 \times v^2$ $v = 0.54 \text{ [m s}^{-1}\text{]}$	M1 A1 M1 A1 A1	Equation to find extension for max speed Correct extension M1: Four term energy equation A1: Correct equation Correct speed AWRT 0.54
		5	

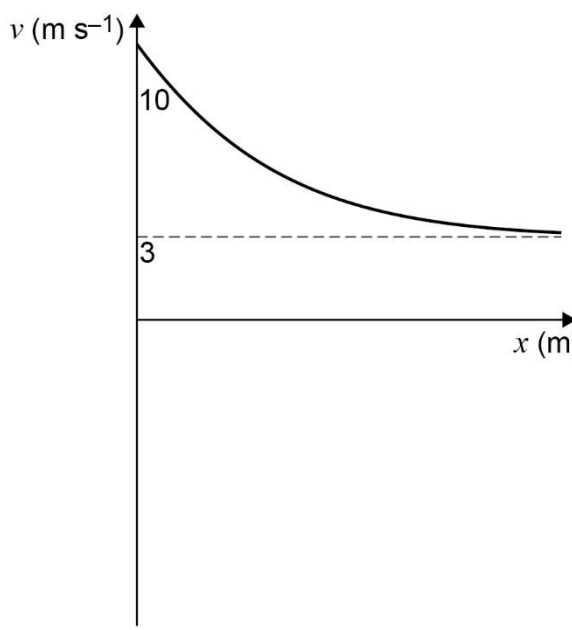
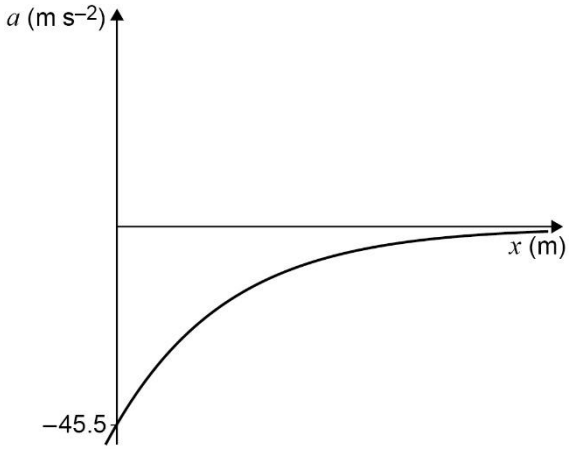
	Question 2 Total	8	
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Q	Answer	Marks	Comments
3(a)	$\int_0^5 10e^{-0.1x} dx$ $= \left[-100e^{-0.1x} \right]_0^5$ $= 100 - 100e^{-0.5}$ $= 39.346\dots$ $= 39 \text{ [J to 2 sf]}$	M1 A1 A1	Uses formula for work done by a variable force Condone missing or incorrect limits Correct integration Condone missing or incorrect limits Correct work done AWRT 39
		3	

Q	Answer	Marks	Comments
3(b)(i)	Friction = $0.2 \times 2 \times 9.8$ $= 3.92$ $\frac{1}{2} \times 2 \times v^2 = 39.3 - 5 \times 3.92$ $v = 4.4 \text{ [m s}^{-1}\text{]}$	B1 M1 A1	Correct friction force Forms three term energy equation Correct speed
		3	

Q	Answer	Marks	Comments
3(b)(ii)	$\int_0^d 10e^{-0.1x} dx = 3.92d$ $f(d) = 100 - 100e^{-0.1d} - 3.92d = 0$ If $d = 22.5$ $f(d) = 1.26$ If $d = 23.5$ $f(d) = -1.66$ $\therefore d = 23$ to the nearest metre	M1 A1 M1 A1	Forms equation using the formula for the work done by a variable force Correct formula Uses numerical method to justify answer If $d = 22$ $f(d) = 2.67$ If $d = 24$ $f(d) = -3.15$ Correct conclusion from correct working
		4	

	Question 3 Total	10	
Q	Answer	Marks	Comments
4(a)	$2v \frac{dv}{dx} = -(v^2 - 9)$ $\int \frac{v}{v^2 - 9} dv = \int -\frac{1}{2} dx$ $\frac{1}{2} \ln(v^2 - 9) = -\frac{1}{2}x + c$ $x = 0, v = 10 \Rightarrow c = \frac{1}{2} \ln 91$ $\frac{1}{2} \ln(v^2 - 9) = -\frac{1}{2}x + \frac{1}{2} \ln 91$ $\frac{v^2 - 9}{91} = e^{-x}$ $v = \sqrt{9 + 91e^{-x}}$	B1 M1 M1 A1 M1 A1	Correct differential equation Separates variables M1: Integrates to get a ln term A1: Correct integration Condone missing constant of integration Finds constant of integration Correct expression for v
		6	

Q	Answer	Marks	Comments
4(b)		B1 B1	Correct shape Correct values of 10 on intercept and 3 for asymptote
		2	
Q	Answer	Marks	Comments
4(c)	 $2a = -91$ $a = -45.5$	B1 M1 A1	Correct shape with asymptote value of 0 M1: Calculates intercept, using either the differential equation or the derivative of the velocity A1: Correct intercept
		3	

	Question 4 Total	11	
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Q	Answer	Marks	Comments
5	$0 = 10 \sin \alpha t - 4.9 \cos(30^\circ) t^2$ $t = \frac{10 \sin \alpha}{4.9 \cos(30^\circ)}$ $4 = 10 \cos \alpha \left(\frac{10 \sin \alpha}{4.9 \cos(30^\circ)} \right) - 4.9 \sin(30^\circ) \left(\frac{10 \sin \alpha}{4.9 \cos(30^\circ)} \right)^2$ $\frac{4 \times 4.9}{100} = \frac{\cos \alpha \sin \alpha}{\cos(30^\circ)} - \frac{\sin(30^\circ) \sin^2 \alpha}{\cos^2(30^\circ)}$ $0.588 = 2\sqrt{3} \cos \alpha \sin \alpha - 2 \sin^2 \alpha$ $\frac{0.588}{\cos^2 \alpha} = 2\sqrt{3} \frac{\cos \alpha \sin \alpha}{\cos^2 \alpha} - 2 \frac{\sin^2 \alpha}{\cos^2 \alpha}$ $0.588(1 + \tan^2 \alpha) = 2\sqrt{3} \tan \alpha - 2 \tan^2 \alpha$ $2.588 \tan^2 \alpha - 2\sqrt{3} \tan \alpha + 0.588 = 0$ $\tan \alpha = 0.199 \text{ or } 1.139$ $\alpha = 11 \text{ or } 49 \text{ [to nearest degree]}$	M1 A1 M1 A1 M1 A1 M1 A1	Equation for the time of flight Correct time of flight Equation for motion parallel to plane using their time Correct equation Uses trig identity to create a quadratic in $\tan \alpha$ Correct quadratic equation Solves their quadratic to obtain two real roots Correct values for α

	Question 5 Total	8	
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Q	Answer	Marks	Comments
6(a)	$I = 2 \begin{bmatrix} 0.4 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $= \begin{bmatrix} -7.2 \\ 0 \end{bmatrix}$ $I = 7.2 \text{ [N s]}$	M1 A1 A1ft	Uses vector form of impulse formula Correct impulse as a vector Correct magnitude, must be positive ft their impulse vector of the form $\begin{bmatrix} k \\ 0 \end{bmatrix}$
		3	

Q	Answer	Marks	Comments
6(b)	$2 \begin{bmatrix} 4 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 5 \end{bmatrix} = 2 \begin{bmatrix} 0.4 \\ 3 \end{bmatrix} + 3v_B$ $v_B = \frac{1}{3} \begin{bmatrix} 4.2 \\ 15 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 5 \end{bmatrix} \text{ [m s}^{-1}\text{]}$	M1 A1 A1	M1: Uses conservation of momentum or impulse to form a vector equation for the velocity of B A1: Correct equation Correct velocity
		3	

Q	Answer	Marks	Comments
6(c)	Velocity of A does not change in the direction $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, so line of centres must be perpendicular to this vector, which is in the direction $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	M1 A1	States that the velocity of A or B does not change in one direction or that the impulse is in the direction $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Uses this to create a valid argument
		2	

Q	Answer	Marks	Comments
6(d)	$0.4 - 1.4 = -e(4 - (-1))$	M1	Uses coefficient of restitution formula
	$e = 0.2$	A1	Correct value
		2	

	Question 6 Total	10	
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Q	Answer	Marks	Comments
7(a)	$e_B = 3a - (a + e_A) - a$ $e_B = a - e_A$ $ke_A = 2k(a - e_A)$ $e_A = 2a - 2e_A$ $e_A = \frac{2a}{3} \text{ [m]}$	M1 M1 A1	Finds extension of second spring Uses Hooke's Law to form an equation AG Must be convincingly shown
		3	

Q	Answer	Marks	Comments
7(b)(i)	$T_A = k\left(x + \frac{2a}{3}\right)$ $\left[T_B = 2k\left(3a - \left(x + \frac{5a}{3}\right) - a\right)\right]$ $T_B = 2k\left(\frac{a}{3} - x\right)$ $m\frac{d^2x}{dt^2} = 2k\left(\frac{a}{3} - x\right) - k\left(x + \frac{2a}{3}\right)$ $= -3kx$ $\frac{d^2x}{dt^2} = -3\frac{kx}{m}$ SHM as the acceleration is proportional to the displacement and in the opposite direction	B1 B1 M1 A1 B1	Correct tension for one spring Correct tension in the other spring Forms correct differential equation Correct simplified differential equation Correct explanation
		5	

Q	Answer	Marks	Comments
7(b)(ii)	$\pi = 2\pi\sqrt{\frac{m}{3k}}$	M1	Uses formula for the period with their ω
	$k = \frac{4m}{3}$	A1ft	Correct expression Follow through their ω
		2	

Q	Answer	Marks	Comments
7(b)(iii)	$\omega = 2$	B1	Correct value for ω
	$v^2 = 2^2 \left(\left(\frac{a}{10} \right)^2 - \left(\frac{a}{20} \right)^2 \right)$	M1	Uses SHM speed formula with their ω
	$v = \frac{\sqrt{3}a}{10} \text{ [m s}^{-1}\text{]}$	A1ft	Correct speed Follow through their ω
		3	

Q	Answer	Marks	Comments
7(b)(iv)	$x = \frac{a}{10} \cos(2t)$	B1	Correct expression for displacement PI
	$\frac{a}{20} = \frac{a}{10} \cos(2t)$	M1	Forms an equation to find a time PI
	$t = \frac{\pi}{6}$	A1	Correct time, PI
	Required time = $2 \times \frac{\pi}{6}$ $= \frac{\pi}{3} \text{ [seconds]}$	A1	Correct time to move from C to C CSO AWRT 1.05
		4	

	Question 7 Total	17	
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Q	Answer	Marks	Comments
8	Let the angle between OA and the vertical = α Let r = radius At C $\frac{mv^2}{r} = mg \cos 60^\circ$ $v^2 = \frac{gr}{2}$ $mgr(\cos \alpha - \cos 60^\circ) = \frac{1}{2}mv^2 = \frac{mgr}{4}$ $\cos \alpha - \frac{1}{2} = \frac{1}{4}$ $\cos \alpha = \frac{3}{4}$ At A $R = mg \cos \alpha = \frac{3mg}{4}$ At B $R = \frac{3mg}{8}$ $\frac{mv^2}{r} = mg \cos \theta - R$ $v^2 = gr \left(\cos \theta - \frac{3}{8} \right)$ $mgr(\cos \alpha - \cos \theta) = \frac{1}{2}mv^2$ $\frac{3}{4} - \cos \theta = \frac{\cos \theta}{2} - \frac{3}{16}$ $\frac{3}{2} \cos \theta = \frac{15}{16}$ $\cos \theta = \frac{5}{8}$	M1 A1 M1 A1 M1 A1 M1 M1 A1 A1	Resolves radially at C Correct velocity at C Energy equation at A Correct value for $\cos \alpha$ Reaction force at A Correct reaction force at B Resolves radially at B Energy equation Correct equation Correct value for $\cos \theta$
	Question 8 Total	10	