

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

June 2024

Version: 1.0 Final



2 4 6 X F M 0 4 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from [oxfordaqa.com](https://www.oxfordaqa.com)

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2024 OxfordAQA International Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$H_0: \mu_A = \mu_B$ $H_1: \mu_A > \mu_B$ $z = \frac{8.2 - 6.5}{\sqrt{\frac{2.1^2}{7} + \frac{2.2^2}{9}}}$ $= 1.57$ $z(95\%) = 1.6449$ $1.57 < 1.65$ Do not reject H_0 Insufficient evidence to suggest that the mean number of days a patient takes to recover from the illness is less for medicine B than medicine A	B1 M1 A1 B1 A1ft E1	Both hypotheses Applies formula AWRT 1.57 Finds correct critical value AWRT 1.645 or 1.64 or finds correct probability AWRT 0.058 Correctly compares their z or t test statistic and their critical value or their p value and 0.05 (0.025 if a two-tailed test is attempted) and concludes that the null hypothesis is not rejected Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value oe Condones definite conclusion

	Question 1 Total	6	
--	------------------	---	--

Q	Answer	Marks	Comments
2(a)	$E(R) = \frac{E(X_1) + 2E(X_2) + 3E(X_3)}{6}$ $= \frac{\lambda + 2\lambda + 3\lambda}{6}$ $= \lambda \quad \text{therefore unbiased estimator}$	M1 A1	Finds $E(R) = \frac{\lambda + 2\lambda + 3\lambda}{6}$ or unsimplified equivalent Must see conclusion and no errors
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\text{Var}(R) = \frac{\text{Var}(X_1)}{6^2} + \frac{\text{Var}(X_2)}{3^2} + \frac{\text{Var}(X_3)}{2^2}$ $= \frac{\lambda}{36} + \frac{\lambda}{9} + \frac{\lambda}{4} = \frac{7\lambda}{18}$ $\text{Relative Efficiency} = \frac{\frac{1}{\text{Var}(R)}}{\frac{1}{\text{Var}(X_1)}} = \frac{18/7\lambda}{1/\lambda}$ $= \frac{18}{7}$	M1 A1 M1 A1	Finds $\text{Var}(R)$ in terms of $\text{Var}(X_1)$, $\text{Var}(X_2)$ and $\text{Var}(X_3)$ PI oe Applies relative efficiency formula either way round with their $\text{Var}(R)$ and $\text{Var}(X_1) = \lambda$ AWRT 2.57
		4	

Q	Answer	Marks	Comments
2(b)(ii)	Estimator R is more efficient than estimator X_1 as $\frac{18}{7} > 1$	E1ft	Compares their relative efficiency with 1 and makes correct conclusion
		1	

	Question 2 Total	7	
--	-------------------------	----------	--

Q	Answer			Marks	Comments																								
3	<table><tr><th>Outcomes</th><th>Mode</th><th>Probability</th></tr><tr><td>(0, 0, 0)</td><td>0</td><td>0.8^3 [= 0.512]</td></tr><tr><td>(0, 0, 20) (0, 20, 0) (20, 0, 0)</td><td>0</td><td>$3 \times 0.8^2 \times 0.2$ [= 0.384]</td></tr><tr><td>(0, 20, 20) (20, 0, 20) (20, 20, 0)</td><td>20</td><td>$3 \times 0.8 \times 0.2^2$ [= 0.096]</td></tr><tr><td>(20, 20, 20)</td><td>20</td><td>0.2^3 [= 0.008]</td></tr><tr><td colspan="3"></td></tr><tr><td>m</td><td>0</td><td>20</td></tr><tr><td>$P(M = m)$</td><td>0.896</td><td>0.104</td></tr></table>			Outcomes	Mode	Probability	(0, 0, 0)	0	0.8^3 [= 0.512]	(0, 0, 20) (0, 20, 0) (20, 0, 0)	0	$3 \times 0.8^2 \times 0.2$ [= 0.384]	(0, 20, 20) (20, 0, 20) (20, 20, 0)	20	$3 \times 0.8 \times 0.2^2$ [= 0.096]	(20, 20, 20)	20	0.2^3 [= 0.008]				m	0	20	$P(M = m)$	0.896	0.104	M1	Identifies a possible outcome PI
	Outcomes	Mode	Probability																										
	(0, 0, 0)	0	0.8^3 [= 0.512]																										
	(0, 0, 20) (0, 20, 0) (20, 0, 0)	0	$3 \times 0.8^2 \times 0.2$ [= 0.384]																										
	(0, 20, 20) (20, 0, 20) (20, 20, 0)	20	$3 \times 0.8 \times 0.2^2$ [= 0.096]																										
	(20, 20, 20)	20	0.2^3 [= 0.008]																										
	m	0	20																										
$P(M = m)$	0.896	0.104																											
A1	Identifies all the outcomes for an associated mode PI																												
M1	Finds at least one probability from the table PI																												
A1	Finds all probabilities in the table PI																												
A1	oe																												

	Question 3 Total	5	
--	------------------	---	--

Q	Answer	Marks	Comments									
4	H_0 : There is not an association between region lived in and holiday location H_1 : There is an association between region lived in and holiday location <table><tr><td>Expected</td><td>A</td><td>NA</td></tr><tr><td>N</td><td>53.148</td><td>49.852</td></tr><tr><td>S</td><td>75.852</td><td>71.148</td></tr></table> $\sum \frac{(O - E - 0.5)^2}{E} = \frac{(44 - 53.148 - 0.5)^2}{53.148} + \frac{(85 - 75.852 - 0.5)^2}{75.852} + \frac{(59 - 49.852 - 0.5)^2}{49.852} + \frac{(62 - 71.148 - 0.5)^2}{71.148}$ = 4.94 $\chi^2_1(95\%) = 3.841$ 4.94 > 3.841 Reject H_0 Sufficient evidence to suggest that there is an association between region lived in and holiday location	Expected	A	NA	N	53.148	49.852	S	75.852	71.148	B1 M1 A1 M1 A1 B1 A1ft E1	Both hypotheses, variables must be stated in at least the null hypothesis oe At least two expected values correct to at least 2 d.p. All correct to at least 2 d.p. Attempts to calculate test statistic Condone use of $\sum \frac{(O - E)^2}{E}$ AWRT 4.9 Finds correct critical value AWRT 3.8 or finds correct probability AWRT 0.026 Correctly compares their χ^2 test statistic and their critical value or their p value and 0.05 and rejects null hypothesis Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value oe Condone definite conclusion
Expected	A	NA										
N	53.148	49.852										
S	75.852	71.148										
	Question 4 Total	8										

Q	Answer	Marks	Comments
5(a)	X and Y are normally distributed	E1	States that X and Y are normally distributed or X and Y are independent
		1	

Q	Answer	Marks	Comments
5(b)	$H_0: \sigma_X = \sigma_Y$ $H_1: \sigma_X \neq \sigma_Y$ $s_X^2 = 0.009995$ $s_Y^2 = 0.004096$ $\frac{s_X^2}{s_Y^2} = \frac{0.009995}{0.004096}$ $= 2.44$ $F_{40,25}(99\%) = 2.453$ $2.44 < 2.453$ Do not reject H_0 Insufficient evidence to suggest that the population standard deviation of male reaction times is different from the population standard deviation of female reaction times	B1 B1 B1 M1 A1 B1 A1ft E1ft	Both hypotheses, oe Either way round PI AWR 2.44 or AWR 0.41 Finds correct critical value corresponding to their test statistic AWR 2.45 or AWR 0.408 or finds correct probability AWR 0.0103 Correctly compares their F test statistic and their critical value or their probability and 0.01 (0.02 if a one tailed test is attempted) and does not reject the null hypothesis Gives a conclusion in context based on a comparison of their F test statistic and their corresponding critical value oe Must not be definite (eg use of "suggest", "support")
		8	

	Question 5 Total	9	
--	------------------	---	--

Q	Answer	Marks	Comments
6(a)(i)	$M_X(t) = e^{\mu t} M_Z(\sigma t)$	M1	Applies formula
	$M_X(t) = e^{\mu t} e^{\frac{1}{2}\sigma^2 t^2} = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$	A1	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$M_{X+Y}(t) = M_X(t) M_Y(t)$	M1	Applies formula
	$= e^{\mu t + \frac{1}{2}\sigma^2 t^2} e^{\lambda(e^t - 1)}$	A1	
	$M_{X+Y}(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2 + \lambda(e^t - 1)}$	A1	
		2	

Q	Answer	Marks	Comments
6(b)(i)	$M_W(t) = \int_0^{\ln 2} e^{tw} \times \frac{2}{3} e^{2w} dw$	M1	Applies mgf formula Condone missing or incorrect limits
	$M_W(t) = \int_0^{\ln 2} \frac{2}{3} e^{(t+2)w} dw$	A1	Correctly integrates Condone missing or incorrect limits
	$M_W(t) = \left[\frac{2e^{(t+2)w}}{3(t+2)} \right]_0^{\ln 2}$	A1	
	$M_W(t) = \frac{2(e^{(t+2)\ln 2} - 1)}{3(t+2)}$	A1	AG Must be convincingly shown
		3	

Q	Answer	Marks	Comments
6(b)(ii)	$M'_W(t) = \frac{6(t+2)e^{(t+2)\ln 2} \ln 2 - 6(e^{(t+2)\ln 2} - 1)}{9(t+2)^2}$	M1	Applies quotient rule oe Condone one sign error
	$M'_W(t) = \frac{2(t\ln 2 + 2\ln 2 - 1)e^{(t+2)\ln 2} + 2}{3(t+2)^2}$	A1	CSO ACF provided numerator is $3(t+2)^2$ ISW once a correct form is seen
		2	

Q	Answer	Marks	Comments
6(b)(iii)	$\text{Mean} = M'_W(0) = \frac{2(2\ln 2 - 1)e^{2\ln 2} + 2}{3(2)^2}$	M1	Attempts to find their $M'_W(0)$
	Mean = 0.424	A1ft	AWRT 0.424 ft their $M'_W(t)$ if full marks not awarded in part (b)(ii)
		2	

	Question 6 Total	11	
--	------------------	----	--

Q	Answer	Marks	Comments
7(a)	$z(96\%) = 1.7507$ $115 \pm 1.7507 \times \frac{18}{\sqrt{200}}$ (112.77, 117.23) Power = $P\left(Z < \frac{112.77 - 116}{\frac{18}{\sqrt{200}}}\right) + P\left(Z > \frac{117.23 - 116}{\frac{18}{\sqrt{200}}}\right)$ $= 0.0056 + 0.1673$ $= 0.173$	B1 M1 A1 M1 A1 A1	AWRT 1.75 Attempts to calculate one of the limits Correct upper and lower limits AWRT 112.77 and 117.23 Identifies correct standardised probabilities for power or Type II error corresponding to their limits PI Correctly calculates one of the probabilities AWRT 0.006 or 0.167 or calculates P(Type II error) AWFW [0.825, 0.828] AWFW [0.172, 0.175] May be calculated using 1 – P(Type II error)
		6	

Q	Answer	Marks	Comments
7(b)	The power of the test decreases	E1	oe
		1	

	Question 7 Total	7	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
8(a)	$z(99\%) = 2.3263$	B1	AWRT 2.33
	$z(96.5\%) = 1.8119$	B1	AWRT 1.81
	$\bar{x} + 2.3263 \frac{\sigma}{\sqrt{40}} = 5243.65$	M1 A1ft	M1: Forms at least one correct equation using their $z(99\%)$ or $z(96.5\%)$
	$\bar{x} - 1.8119 \frac{\sigma}{\sqrt{40}} = 4858.26$		A1ft: Forms both correct equations using their $z(99\%)$ and $z(96.5\%)$
	$\sigma = \$589$	A1	Condone omission of \$ sign
	$\bar{x} = \$5027$	B1	Condone omission of \$ sign
		6	

Q	Answer	Marks	Comments
8(b)	$z(98\%) = 2.0537$	B1	AWRT 2.05
	$2 \times 2.0537 \times \frac{589}{\sqrt{n}} = 300$	M1	Set up an equation of the form $k \times \text{their } z \times \frac{\text{their } \sigma}{\sqrt{n}} = 300$ oe where $k = 1$ or 2
	$n = 65$	A1	Allow $n = 66$
		3	

	Question 8 Total	9	
--	-------------------------	----------	--

Q	Answer	Marks	Comments														
9(a)	H_0 : N has a binomial distribution $B(5, 0.5)$ H_1 : N does not have a binomial distribution with $B(5, 0.5)$ <table><tr><th>Number of heads</th><th>Expected frequency</th></tr><tr><td>0</td><td>6.25</td></tr><tr><td>1</td><td>31.25</td></tr><tr><td>2</td><td>62.5</td></tr><tr><td>3</td><td>62.5</td></tr><tr><td>4</td><td>31.25</td></tr><tr><td>5</td><td>6.25</td></tr></table> $\sum \frac{(O-E)^2}{E} = \frac{(11-6.25)^2}{6.25} + \frac{(40-31.25)^2}{31.25} + \frac{(75-62.5)^2}{62.5} + \frac{(48-62.5)^2}{62.5} + \frac{(22-31.25)^2}{31.25} + \frac{(4-6.25)^2}{6.25}$ $= 15.472$ $\chi^2_5(99\%) = 15.086$ $15.472 > 15.086$ Reject H_0 Sufficient evidence to suggest that Aeryn's model is not a good fit	Number of heads	Expected frequency	0	6.25	1	31.25	2	62.5	3	62.5	4	31.25	5	6.25	B1 M1 A1 M1 A1 B1 A1ft E1	Both hypotheses $B(5, 0.5)$ oe must be stated M1: Finds one correct expected frequency A1: Finds all correct expected frequencies Attempts to calculate test statistic AWRT 15.5 from correct work Finds correct critical value AWRT 15.1 or correct probability AWRT 0.009 Correctly compares their χ^2 test statistic and their critical value or their probability and 0.01 and rejects the null hypothesis Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value oe Condone definite conclusion
Number of heads	Expected frequency																
0	6.25																
1	31.25																
2	62.5																
3	62.5																
4	31.25																
5	6.25																
		8															

Q	Answer	Marks	Comments
9(b)	4	B1ft	ft Their degrees of freedom –1
		1	

	Question 9 Total	9	
--	------------------	---	--

Q	Answer	Marks	Comments
10(a)	$\left[s_p^2 = \right] \frac{(n-1) \times 5.4^2 + (n+2) \times 5.2^2}{2n+1}$ $= 27.933$ $56.2n + 24.92 = 55.866n + 27.933$ $0.334n = 3.013$ $n = 9.02 \Rightarrow n = 9$	M1 m1 A1	Forms correct equation oe Rearranges equation to reach the form $an + b = cn + d$ PI By sight of 9.02 CSO
		3	

Q	Answer	Marks	Comments
10(b)	$H_0: \mu_A = \mu_B$ $H_1: \mu_A < \mu_B$ $t = \frac{43.2 - 39.4}{\sqrt{27.933 \left(\frac{1}{9} + \frac{1}{12} \right)}}$ $= 1.63$ $t_{19}(95\%) = 1.729$ $1.63 < 1.729$ Do not reject H_0 Insufficient evidence to suggest that the mean salaries in town B are higher than the mean salaries in town A	B1 M1 A1ft B1ft A1ft E1ft	Both hypotheses Applies formula AWRT 1.63 ft their positive integer n Finds critical value AWRT 1.73 or finds probability AWRT 0.0597 ft their positive integer n, if $n > 30$ allow use of normal Correctly compares their t or z test statistic and their critical value or their probability and 0.05 (0.025 if a two tailed test is attempted) and concludes that the null hypothesis is not rejected Gives a conclusion in context based on a comparison of the correct test statistic and correct critical value for their positive integer n Condone definite conclusion
		6	

	Question 10 Total	9	
--	-------------------	---	--