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INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

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251XFM04/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	<table><tr><td>Observed</td><td>X</td><td>Y</td><td>Total</td></tr><tr><td>A</td><td>84</td><td>**76**</td><td>160</td></tr><tr><td>B</td><td>56</td><td>120</td><td>**176**</td></tr><tr><td>C</td><td>**80**</td><td>**84**</td><td>**164**</td></tr><tr><td>Total</td><td>220</td><td>**280**</td><td>500</td></tr></table>	**B1**	Completed observed values table
		1	

Q	Answer	Marks	Comments
1(b)	H_0 : There is not an association between preferred brand of soft drink and preferred brand of biscuits H_1 : There is an association between preferred brand of soft drink and preferred brand of biscuits	B1	Both hypotheses, variables must be stated in at least the null hypothesis **oe**
	<table><tr><td>Expected</td><td>*X*</td><td>*Y*</td></tr><tr><td>*A*</td><td>70.4</td><td>89.6</td></tr><tr><td>*B*</td><td>77.44</td><td>98.56</td></tr><tr><td>*C*</td><td>72.16</td><td>91.84</td></tr></table>	M1	At least two expected values correct
	$\sum\frac{(O-E)^2}{E}=\frac{(84-70.4)^2}{70.4}+\frac{(76-89.6)^2}{89.6}$ $+\frac{(56-77.44)^2}{77.44}+\frac{(120-98.56)^2}{98.56}+\frac{(80-72.16)^2}{72.16}$ $+\frac{(84-91.84)^2}{91.84}$	M1	Attempts to calculate test statistic **PI**
	$=16.81$	A1	**AWRT** 16.8
	$\chi^2_2(0.995)=10.597$	B1	Finds critical value **AWRT** 10.6 or p-value **AWRT** 0.0002
	$16.81 > 10.597$ Reject H_0	A1ft	Correctly compares their χ^2 test statistic and their critical value or their p-value and 0.005 and makes a correct **ft** decision whether to reject the null hypothesis
	Sufficient evidence to suggest that there is an association between preferred brand of soft drink and preferred brand of biscuits	E1ft	Gives a conclusion in context, by referring to association between preferred soft drink and preferred biscuits or the researcher's claim, based on a comparison of their test statistic and their critical value **oe** Conclusion must not be definite (eg use of suggest/support)
		7	

	Question 1 Total	**8**	

Q	Answer	Marks	Comments
2(a)	$\dfrac{90.2-89.8}{\sqrt{\dfrac{\sigma_1^2+\sigma_2^2}{12}}}$	**M1**	Applies test statistic formula either way round
	$\dfrac{90.2-89.8}{\sqrt{\dfrac{\sigma_1^2+\sigma_2^2}{12}}}=\dfrac{4\sqrt{5}}{5}$	**M1**	Sets their test statistic equal to $\dfrac{4\sqrt{5}}{5}$
	$\sqrt{\dfrac{\sigma_1^2+\sigma_2^2}{12}}=\dfrac{\sqrt{5}}{10}$		
	$\dfrac{\sigma_1^2+\sigma_2^2}{12}=\dfrac{1}{20}$		
	$\sigma_1^2+\sigma_2^2=\dfrac{3}{5}$	**A1**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
2(b)	$\sigma_1^2+\dfrac{1}{16}\sigma_1^2=\dfrac{3}{5}$ or $16\sigma_2^2+\sigma_2^2=\dfrac{3}{5}$	**M1**	Forms a correct equation in σ_1^2 or σ_2^2 **oe**
	$\sigma_1^2=\dfrac{48}{85}$	**A1**	Correct value of σ_1^2 **AWRT** 0.565
	$\sigma_2^2=\dfrac{3}{85}$	**A1**	Correct value of σ_2^2 **AWRT** 0.035
		3	

Q	Answer	Marks	Comments
2(c)	z critical value = 1.6449	**B1**	Finds critical value **AWRT** 1.64 or 1.645 or finds p-value **AWRT** 0.037
	$\frac{4\sqrt{5}}{5} = 1.788\ldots > 1.6449$	**M1**	Compares correct test statistic with their critical value or their p-value and 0.05
	Null hypothesis is rejected	**A1ft**	Correct conclusion from comparing the correct test statistic and their critical value **oe**
		3	

	Question 2 Total	**9**	

Q	Answer	Marks	Comments
3(a)	Differences [in total sales between the two time periods] are normally distributed	**E1**	**oe**
		1	

Q	Answer	Marks	Comments
3(b)	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A > \mu_B$	**B1**	Both hypotheses **oe**
	Differences: $6, -2, 4, 2, 5, -3, 0, 5$	**M1**	Calculates at least 5 correct differences, either way round
	$\bar{d} = \frac{17}{8} = 2.125$	**A1**	2.125 or -2.125 **oe** but must be consistent with their differences
	$s_d^2 = \frac{1}{7}\left(119 - \frac{17^2}{8}\right) = \frac{663}{56}$	**A1**	$\frac{663}{56}$ **oe** **AWRT** $s_d^2 = 11.8$ or $s_d = 3.44$
	$t = \frac{2.125 - 0}{\sqrt{\frac{\left(\frac{663}{56}\right)}{8}}}$	**M1**	Applies formula with their values
	$t = 1.75$	**A1ft**	**AWRT** 1.75 **ft** their values
	$t_7(95\%) = 1.895$	**B1**	Finds critical value **AWRT** 1.90 or finds p-value **AWRT** 0.062
	$1.75 < 1.895$ Do not reject H_0	**A1ft**	Correctly compares their t test statistic and their critical value or their p-value and 0.05 and makes a correct **ft** decision whether to reject the null hypothesis
	Insufficient evidence to suggest that the total sales per member of staff have increased as a result of the layout change	**E1**	Gives a conclusion in context, by referring to total sales per member of staff or the company's claim, based on a comparison of the correct test statistic and correct critical value **oe** Condone definite conclusion
		9	

	Question 3 Total	**10**	

<table>
<tr><th>Q</th><th>Answer</th><th>Marks</th><th>Comments</th></tr>
<tr><td>4(a)</td><td>
<table>
<tr><th>n</th><th>Calculation</th><th>Width</th></tr>
<tr><td>6</td><td>$2\times 2.571\times\sqrt{\frac{151.29}{6}}$</td><td>25.8</td></tr>
<tr><td>7</td><td>$2\times 2.447\times\sqrt{\frac{151.29}{7}}$</td><td>22.8</td></tr>
<tr><td>8</td><td>$2\times 2.365\times\sqrt{\frac{151.29}{8}}$</td><td>20.6</td></tr>
<tr><td>9</td><td>$2\times 2.306\times\sqrt{\frac{151.29}{9}}$</td><td>18.9</td></tr>
</table>
$n = 8$
</td><td>M1

m1

A1</td><td>Applies confidence interval formula with a value of n where $5 < n < 10$
or
Finds the width of a confidence interval for a value of n where $5 < n < 8$ or $n = 9$
PI, oe

Applies confidence interval formula with $n = 8$
PI, oe

Correct working leading to $n = 8$</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Q	Answer	Marks	Comments
4(b)	$[55.7 \pm 0.5\times 20.6]$ $(45.4, 66.0)$	**B1**	**AWRT** 45.4 and 66.0 Condone 66 for 66.0
		1	

Q	Answer	Marks	Comments
4(c)	40 is outside the confidence interval so the confidence interval does not support the claim	**B1ft**	Correct reason and conclusion **ft** their confidence interval
		1	

	Question 4 Total	**5**	

Q	Answer	Marks	Comments
5(a)	Critical values = 24.433 and 59.342	**M1 A1**	**M1**: At least one value **A1**: Both values
	$24.433 < \frac{40 \times 25.4}{k^2} < 59.342$	**M1**	Obtains correct inequality for their critical values or **AWRT** 4.14 and 6.45 seen Condone σ for k
	$17.1 < k^2 < 41.6$	**A1**	Obtains correct inequality for k^2 **AWRT** 17.1 and 41.6
	$4.14 < k < 6.45$	**A1ft**	Obtains correct inequality for k **AWRT** 4.14 and 6.45 **ft** their values of k^2 provided an inequality of the form $a < k^2 < b$ is obtained
		5	

Q	Answer	Marks	Comments
5(b)	$H_0 : \sigma_M^2 = \sigma_W^2$ $H_1 : \sigma_M^2 < \sigma_W^2$	**B1**	Both hypotheses, **oe**
	$\frac{s_W^2}{s_M^2} = \frac{66.1}{25.4}$	**M1**	Applies test statistic formula, either way round **PI**
	$= 2.60$	**A1**	**AWRT** 2.60 or 0.38 Allow 2.6 for 2.60
	$F_{10,40}(0.99) = 2.801$	**B1**	Finds correct critical value **AWRT** 2.80 or 0.36 Allow 2.8 for 2.80 or correct p-value, **AWRT** 0.016
	$2.60 < 2.801$ Do not reject H_0	**A1ft**	Correctly compares their F test statistic and their critical value or their p-value and 0.01 and makes a correct **ft** decision whether to reject the null hypothesis
	Insufficient evidence to suggest that the population variance of the total marks on the written response section is different from the population variance of total marks on the multiple-choice section	**E1**	Gives a conclusion in context, by referring to the variances of marks on the written section and the multiple-choice section, based on a comparison of the correct test statistic and the correct critical value **oe** Condone definite conclusions
		6	

	Question 5 Total	**11**	

Q	Answer	Marks	Comments
6(a)	$\mathrm{E}(X)=\sum_{i=1}^{n}\mathrm{E}(T_i)=\frac{nm}{2}$	**M1**	Finds $\mathrm{E}(X)$ in terms of m and n
	$\frac{nm}{2}\neq m$ therefore biased estimator	**A1**	Must see conclusion
		2	

Q	Answer	Marks	Comments
6(b)	$\frac{knm}{2}=m$	**M1**	Sets their $\mathrm{E}(kX)$ equal to m
	$k=\frac{2}{n}$	**A1**	**CAO**
		2	

Q	Answer	Marks	Comments
6(c)	$\mathrm{Var}(kX)=\left(\frac{2}{n}\right)^2\sum_{i=1}^{n}\mathrm{Var}(T_i)$	**M1**	Applies formula with k or their expression for k
	$\mathrm{Var}(kX)=\frac{4}{n^2}\left(\frac{nm^2}{12}\right)=\frac{m^2}{3n}$	**A1ft**	Correct expression for $\mathrm{Var}(kX)$ for their k
	As $n\to\infty$, $\mathrm{Var}(kX)\to 0$ so estimator is consistent	**A1ft**	**ft t**heir $\mathrm{Var}(kX)$
		3	

Q	Answer	Marks	Comments
6(d)	$0.5+1.2+1.5+3.2+7.1=13.5$	**M1**	Correctly sums results
	$\frac{2}{5}\times 13.5=5.4$ minutes	**A1ft**	**ft** their k **CSO**
		2	

	Question 6 Total	**9**	

Q	Answer	Marks	Comments
7(a)	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A \neq \mu_B$	**B1**	Both hypotheses, **oe**
	$z = \dfrac{18.3 - 17.9}{\sqrt{\dfrac{3.04^2}{478} + \dfrac{2.09^2}{437}}}$	**M1**	Applies formula Condone not squaring 3.04 **and** 2.09
	$= 2.34$	**A1**	**AWRT** 2.34
	z critical value $= 2.3263$	**B1**	Finds critical value **AWRT** 2.33 or finds p-value **AWRT** 0.0098
	$2.34 > 2.3263$ Reject H_0	**A1ft**	Correctly compares their z or t test statistic and their critical value or their p-value and 0.01 and makes a correct ft decision whether to reject the null hypothesis
	Sufficient evidence to suggest that the population mean daily maximum heights of the two rivers are different	**E1**	Gives a conclusion in context, by referring to the mean daily maximum heights of the two rivers, based on a comparison of the correct test statistic and correct critical value **oe** Condone definite conclusion
		6	

Q	Answer	Marks	Comments
7(b)	The sample sizes are large [and so the central limit theorem applies]	**E1**	**oe**
		1	

Q	Answer	Marks	Comments
7(c)(i)	$\pm 2.3263 \times \sqrt{\dfrac{3.04^2}{478} + \dfrac{2.09^2}{437}}$	**M1**	Attempts to calculate one of the limits with their critical value Condone attempt to calculate confidence interval limits Condone not squaring 3.04 **and** 2.09
	(–0.398, 0.398)	**A1**	Correct upper and lower limits **AWRT** ± 0.398

	P(Type II error) = $\mathrm{P}\left(\frac{-0.398-0.2}{\sqrt{\frac{3.04^2}{478}+\frac{2.09^2}{437}}} < Z < \frac{0.398-0.2}{\sqrt{\frac{3.04^2}{478}+\frac{2.09^2}{437}}}\right)$	**m1**	Identifies correct probabilities corresponding to their limits **PI**
	$\mathrm{P}(Z<1.16) - (1 - \mathrm{P}(Z<-3.49)$ $= 0.87698 - (1 - 0.99976)$	**A1**	Correctly calculates one of the probabilities **AWRT** 0.877 or 1.000
	$= 0.877$	**A1**	**CSO**, **AWFW** [0.876, 0.877]
		5	

Q	Answer	Marks	Comments
7(c)(ii)	$1 - 0.877 = 0.123$	**B1ft**	**ft** their Type II probability
		1	

	Question 7 Total	**13**	

Q	Answer	Marks	Comments
8(a)(i)	$\mathrm{M}'_X(t) = \dfrac{p\,\mathrm{e}^{pt}\left(1-qt^2\right)+2qt\,\mathrm{e}^{pt}}{\left(1-qt^2\right)^2}$	**M1 A1**	**M1**: $\dfrac{k_1\,\mathrm{e}^{pt}\left(1-qt^2\right)+k_2 t\,\mathrm{e}^{pt}}{\left(1-qt^2\right)^2}$ **oe** **A1**: Fully correct **oe**
	$\mathrm{M}'_X(0) = \dfrac{p(1)+2q\times 0}{(1)^2} = 0 \Rightarrow p = 0$	**A1**	**AG** Must be convincingly shown
		3	

Q	Answer	Marks	Comments
8(a)(ii)	$\mathrm{M}'_X(t) = \dfrac{2qt}{\left(1-qt^2\right)^2}$	**M1**	Correctly simplifies $\mathrm{M}'_X(t)$ **PI**
	$\mathrm{M}''_X(t) = \dfrac{2q\left(1-qt^2\right)^2+8q^2t^2\left(1-qt^2\right)}{\left(1-qt^2\right)^4}$	**A1**	**oe**
	$\mathrm{M}''_X(0) = 2q$	**M1**	Finds their $\mathrm{M}''_X(0)$
	$\dfrac{1}{2} = 2q \Rightarrow q = \dfrac{1}{4}$	**A1**	**CSO**
		4	

Q	Answer	Marks	Comments
8(b)(i)	$\mathrm{M}_A(t) = \int_0^\infty \mathrm{e}^{ta} \times \lambda \mathrm{e}^{-\lambda a}\,\mathrm{d}a = \int_0^\infty \lambda \mathrm{e}^{-(\lambda - t)a}\,\mathrm{d}a$	M1	Applies mgf formula and simplifies Condone missing or incorrect limits
	$\mathrm{M}_A(t) = \left[\dfrac{-\lambda \mathrm{e}^{-(\lambda-t)a}}{\lambda - t}\right]_0^\infty$	A1	Correctly integrates Condone missing or incorrect limits
	$\mathrm{M}_A(t) = \lim_{a\to\infty}\left(\dfrac{-\lambda \mathrm{e}^{-(\lambda-t)a}}{\lambda - t}\right) + \dfrac{\lambda}{\lambda - t}$		
	$= 0 + \dfrac{\lambda}{\lambda - t}$		
	$= \dfrac{\lambda}{\lambda - t}$	A1	**AG** Must be convincingly shown Condone not showing evidence of a limiting process
		3	

Q	Answer	Marks	Comments
8(b)(ii)	$\mathrm{M}_{-B}(t) = \mathrm{M}_B(-t) = \dfrac{\lambda}{\lambda + t}$	B1	**oe**
		1	

Q	Answer	Marks	Comments
8(b)(iii)	$\mathrm{M}_{A-B}(t) = \mathrm{M}_A(t) \times \mathrm{M}_{-B}(t) = \dfrac{\lambda}{\lambda - t} \times \dfrac{\lambda}{\lambda + t}$	M1	Applies formula
	$\mathrm{M}_{A-B}(t) = \dfrac{\lambda^2}{\lambda^2 - t^2} = \dfrac{1}{1 - \frac{1}{\lambda^2}t^2}$	A1	Rearranges to the required form **PI**
	$\dfrac{1}{\lambda^2} = \dfrac{1}{4}$	M1	Sets their $\dfrac{1}{\lambda^2}$ equal to their q **PI**
	$\lambda = 2$	A1	**CSO**
		4	

	Question 8 Total	**15**	