

**INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS**

FM04

(9665/FM04) Unit FS2 Statistics

Mark scheme

June 2025

Version: 1.0 Final



2 5 6 X F M 0 4 / M S

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$H_0 : \sigma = 0.5$ $H_1 : \sigma < 0.5$ $\frac{(37-1) \times 0.35^2}{0.5^2}$ $= 17.64$ $\chi_{36}^2(0.01) = 19.223$ $17.64 < 19.223$ <p>Reject H_0</p> <p>Sufficient evidence to suggest that the standard deviation of delivery times is less than 0.5 days</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses, oe Allow $H_0 : \sigma \geq 0.5$</p> <p>PI Condone not squaring 0.35 and 0.5</p> <p>oe or 17.6</p> <p>Finds correct critical value AWRT 19.2 or correct probability, AWRT 0.0044 Allow 19.233</p> <p>Correctly compares their chi squared test statistic and their critical value or their probability and 0.01 and makes the correct ft conclusion as to whether to reject the null hypothesis</p> <p>Gives a conclusion in context by referring to standard deviation of delivery times or the company's claim based on a comparison of the correct test statistic and the correct critical value oe Condone definite conclusions</p>

	Question 1 Total	6	
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Q	Answer	Marks	Comments
2(b)	$M_{X+Y}(t) = \frac{1-5e^{4t}+4e^{5t}}{10t^2} \times \frac{e^{5t}-1}{5t}$ $= \frac{e^{5t}-1-5e^{9t}+5e^{4t}+4e^{10t}-4e^{5t}}{50t^3}$ $= \frac{4e^{10t}-5e^{9t}-3e^{5t}+5e^{4t}-1}{50t^3}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>Identifies $M_{X+Y}(t)$ as equal to the mgfs of X and Y multiplied together</p> <p>Multiplies mgfs together to achieve a correct unsimplified form</p> <p>CAO</p>
		3	
	Question 2 Total	9	

Q	Answer	Marks	Comments
3	$H_0: \mu_C = \mu_H$ $H_1: \mu_C < \mu_H$ <p>Differences: 8, -7, 6, 2, 11</p> $\bar{d} = \frac{20}{5} = 4$ $s_d^2 = \frac{1}{4} \left(274 - \frac{20^2}{5} \right)$ $= 48.5$ $t = \frac{4 - 0}{\sqrt{\frac{48.5}{5}}}$ $= 1.28$ $t_4(0.95) = 2.132$ $1.28 < 2.132$ <p>Do not reject H_0</p> <p>Insufficient evidence to suggest that on average, the car travels further on a complete charge on a hot day than a cold day</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>B1</p> <p>A1ft</p> <p>E1</p>	<p>Both hypotheses oe Allow $H_0: \mu_C \geq \mu_H$ Accept $H_1: \mu_D > 0$ or $H_1: \mu_D < 0$ only if consistent with their differences If different subscripts are used they must be defined</p> <p>Calculates correct differences, either way round</p> <p>4 or -4 oe but must be consistent with their differences</p> <p>oe PI ft their differences but condone one error with substituting</p> <p>$\frac{97}{2}$ oe AWRT $s_d = 6.96$</p> <p>Applies formula with their values Condone $t = \frac{4 - 0}{\sqrt{s_p^2 \left(\frac{1}{5} + \frac{1}{5} \right)}}$</p> <p>AWRT 1.3 ft their values for either t-statistic</p> <p>Finds critical value AWRT 2.1 or finds p-value AWRT 0.13</p> <p>Correctly compares their t test statistic and their critical value or their p-value and 0.05 and makes the correct conclusion as to whether to reject the null hypothesis</p> <p>Gives a conclusion in context by referring to car travel on hot and cold days or the producer's claim based on a comparison of the correct test statistic and correct critical value oe Condone definite conclusion</p>
Question 3 Total		10	

Q	Answer	Marks	Comments
4(a)	$s^2 = \frac{0.0075}{3} = 0.0025$	B1	$\frac{0.0075}{3}$ oe eg $s = 0.05$
	$t_3(0.95) = 2.353$	B1	AWRT 2.35
	Width = $2 \times 2.353 \sqrt{\frac{0.0025}{4}}$	M1	Applies correct formula with their values Condone missing $2 \times$
	= 0.118	A1	AWRT 0.118
		4	

Q	Answer	Marks	Comments
4(b)	$2.7 - 0.5 \times 0.118$	M1	Correct calculation with their width
	= 2.64	A1ft	AWRT 2.64 ft their width
		2	

	Question 4 Total	6	
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Q	Answer	Marks	Comments
6	$P(M = 0) = (P(\text{White}))^3$ $P(\text{White}) = \sqrt[3]{0.042875} = 0.35$ <p>Number of white balls = $120 \times 0.35 = 42$</p> <p>Number of black balls = $120 - 42$</p> <p>Number of black balls = 78</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Recognises $P(M = 0) = (P(\text{White}))^3$ or sets up equation $Ap(1-p)^2 + Bp^2(1-p) + p^3 = 0.957125$ where $A > 0$ and $B > 0$ PI Finds $P(\text{White})$ or sets up equation $3p(1-p)^2 + 3p^2(1-p) + p^3 = 0.957125$ PI oe Multiplies their $P(\text{White})$ by 120 or finds $P(\text{Black})$ by subtracting their $P(\text{White})$ from 1 or solving their cubic equation in terms of $P(\text{Black})$ Subtract their number of white balls from 120 or multiplies their $P(\text{Black})$ by 120 CAO</p>
	Question 6 Total	5	

Q	Answer	Marks	Comments
7(a)	$H_0: \mu_X = \mu_Y$ $H_1: \mu_X > \mu_Y$ $\bar{x} = \frac{18000}{160} = 112.5 \text{ and } \bar{y} = \frac{15480}{150} = 103.2$ $s_x^2 = \frac{1}{159} \left(2502000 - \frac{18000^2}{160} \right)$ $s_y^2 = \frac{1}{149} \left(2033212 - \frac{15480^2}{150} \right)$ $s_x^2 = 3000$ $s_y^2 = 2924$ $z = \frac{112.5 - 103.2}{\sqrt{\frac{3000}{160} + \frac{2924}{150}}}$ $= 1.50$ $z \text{ critical value} = 1.6449$ $1.50 < 1.6449$ <p>Do not reject H_0</p> <p>Insufficient evidence to suggest that on average the length of films released in cinemas in country A are longer than those released in country B</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p> <p>E1ft</p> <p>10</p>	<p>Both hypotheses oe Allow $H_0: \mu_X \leq \mu_Y$ Allow μ_A for μ_X and μ_B for μ_Y If different subscripts are used they must be defined</p> <p>Finds both sample means oe</p> <p>Applies formula to find at least one of s_x^2 or s_y^2 oe</p> <p>Correct s_x^2 or AWRT $s_x = 54.8$</p> <p>Correct s_y^2 or AWRT $s_y = 54.1$</p> <p>Applies formula</p> <p>AWRT 1.50 Condone 1.5</p> <p>Finds critical value AWRT 1.64 or 1.645 or finds p-value AWRT 0.066</p> <p>Correctly compares their z or t test statistic and their critical value and makes the correct conclusion as to whether to reject the null hypothesis</p> <p>Gives a conclusion in context by referring to the length of films in countries A and B or Gan's claim based on a comparison of their test statistic and their critical value oe Conclusion must not be definite (eg use of suggest/support)</p>

Q	Answer	Marks	Comments
7(b)	$s_p^2 = \frac{(6-1) \times 3005 + (7-1) \times 2939}{6+7-2} = 2969$ $t = \frac{10.2}{\sqrt{2969 \left(\frac{1}{6} + \frac{1}{7} \right)}}$ $= 0.336$ $t_{11}(0.95) = 1.796$ $0.336 < 1.796$ <p>Jenna reaches the same conclusion as Gan</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1ft</p>	<p>Applies formula</p> <p>AWRT 0.34</p> <p>Finds critical value AWRT 1.80 or finds p-value AWRT 0.37</p> <p>Correctly compares their t test statistic and the correct critical value oe and makes the correct ft conclusion as to whether Jenna reaches the same conclusion as Gan</p>
		5	
	Question 7 Total	15	

Q	Answer	Marks	Comments
8(a)	$\text{Relative Efficiency} = \frac{\frac{1}{\text{Var}(B)}}{\frac{1}{\text{Var}(A)}} = \frac{6\sigma^2}{c\sigma^2}$	M1	Applies relative efficiency formula either way round oe PI
	$\frac{6\sigma^2}{c\sigma^2} = 10 \Rightarrow \frac{6}{c} = 10$	M1	Sets their relative efficiency equal to 10 and eliminates σ^2 correctly oe
	$c = 0.6$	A1	oe
		3	

Q	Answer	Marks	Comments
8(b)	$E(Y) = 1^2 \times E(X_1) + 2^2 \times E(X_2) + \dots + n^2 \times E(X_n)$	M1	Finds $E(Y)$ in terms of $E(X_i)$ oe PI
	$E(Y) = (1^2 + 2^2 + \dots + n^2)\mu$	A1	Finds $E(Y)$ in terms of μ oe
	$\neq \mu \text{ so biased estimator}$	A1	AG Must be convincingly shown and must see conclusion
		3	

Q	Answer	Marks	Comments
8(c)	$E(kY) = kE(Y)$	M1	PI
	$= k(1^2 + 2^2 + \dots + n^2)\mu$	A1	Finds $E(kY)$ in terms of μ , k and n oe
	$= k\left(\frac{n(n+1)(2n+1)}{6}\right)\mu$	M1	Uses $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$ Must be seen in part (c)
	$k\left(\frac{n(n+1)(2n+1)}{6}\right)\mu = \mu$		
	$k = \frac{6}{n(n+1)(2n+1)}$	A1	oe
		4	

	Question 8 Total	10	
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Q	Answer	Marks	Comments
9(a)	Start with the number of categories and minus 1 [for limitation on frequency]	E1	oe
	Subtract 1 for estimating a [population] parameter so $4 - 1 - 1 = 2$	E1	oe Condone found from the data rather than estimated but do not condone estimating the sample mean
		2	

Q	Answer	Marks	Comments														
9(b)	H_0 : Number of online chat questions per hour follows a Poisson distribution H_1 : Number of online chat questions per hour does not follow a Poisson distribution	B1	Both hypotheses Variable must be mentioned in at least the null hypothesis Must not give a value for λ oe														
	<table border="1"> <thead> <tr> <th>Questions per hour</th> <th>Probability</th> <th>Expected frequency</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0.2231</td> <td>8.48</td> </tr> <tr> <td>1</td> <td>0.3347</td> <td>12.7</td> </tr> <tr> <td>2</td> <td>0.2510</td> <td>9.54</td> </tr> <tr> <td>3 or more</td> <td>0.1912</td> <td>7.26</td> </tr> </tbody> </table>	Questions per hour	Probability	Expected frequency	0	0.2231	8.48	1	0.3347	12.7	2	0.2510	9.54	3 or more	0.1912	7.26	M1 A1
Questions per hour	Probability	Expected frequency															
0	0.2231	8.48															
1	0.3347	12.7															
2	0.2510	9.54															
3 or more	0.1912	7.26															
	$\sum \frac{(O-E)^2}{E} = \frac{(11-8.48)^2}{8.48} + \frac{(7-12.7)^2}{12.7} + \frac{(8-9.54)^2}{9.54} + \frac{(12-7.26)^2}{7.26}$	M1	Attempts to calculate test statistic														
	= 6.7	A1	AWRT 6.7														
	$\chi^2(0.95) = 5.991$	B1	Finds critical value AWRT 6.0 or p -value AWRT 0.04														
	$6.7 > 5.991$ Reject H_0	A1ft	Correctly compares their χ^2 test statistic and their critical value or their p -value and 0.05 and makes a correct ft decision whether to reject the null hypothesis														

	Sufficient evidence to suggest that the number of on-line chat queries per hour does not follow a Poisson distribution	E1	Gives a conclusion in context by referring to number of online chat questions per hour following a Poisson distribution or the data manager's claim based on a comparison of the correct test statistic and the correct critical value oe Condone definite conclusion Condone giving a value for λ
		8	

Q	Answer	Marks	Comments
9(c)	$\chi_3^2(0.95) = 7.815$ $6.7 < 7.815$ The team member does not reject the null hypothesis	B1 M1 A1ft	Finds critical value AWRT 7.8 or p -value AWRT 0.08 Compares their test statistic with their critical value or their p -value and 0.05 Makes the correct ft conclusion based a comparison of their test statistic and the correct critical value
		3	

	Question 9 Total	13	
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