OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Forename(s) _____

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I declare this is my own work.

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Wednesday 20 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 Two particles, *A* and *B*, are moving on a smooth horizontal surface when they collide and coalesce, to form a single combined particle.

Particle *A* has mass 2 kg and before the collision has velocity $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ m s^{-1}

Particle *B* has mass m kg and before the collision has velocity $\begin{bmatrix} 2 \\ U \end{bmatrix}$ m s^{-1}

After the collision the single combined particle has velocity $\begin{bmatrix} 2.8 \\ -1 \end{bmatrix}$ m s^{-1}

1 (a) Find the value of m

[2 marks]

Answer _____

1 (b) Find the value of U

[3 marks]

Answer _____

Do not write outside the box

1 (c) Find the magnitude of the impulse exerted on *A* during the collision.

[3 marks]

Answer _____

8

Turn over for the next question

Do not write outside the box

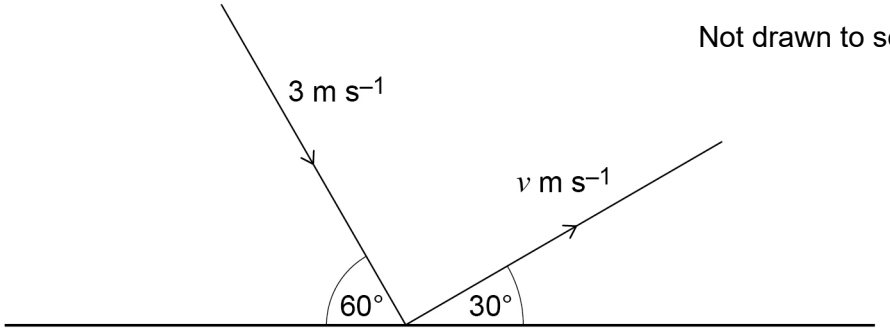
2 A ball of mass 80 grams is moving on a smooth horizontal surface when it hits a smooth fixed vertical wall.

When the ball hits the wall, it is moving with velocity 3 m s^{-1} at an angle of 60° to the wall.

After the ball leaves the wall, its velocity is v m s^{-1} at an angle of 30° to the wall.

The velocities of the ball are shown in the diagram below.

Not drawn to scale



The coefficient of restitution between the ball and the wall is e

2 (a) Find the value of e

[5 marks]

Answer _____

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2 (b) Find the magnitude of the impulse that the wall exerts on the ball, giving your answer in exact form.

[3 marks]

Answer _____

8

Turn over for the next question

Do not write outside the box

3 A particle moves with simple harmonic motion.

The period of the motion is 4 seconds.

The maximum speed of the particle is 6 m s^{-1}

3 (a) Find the amplitude of the motion, giving your answer in terms of π

[3 marks]

Answer _____

3 (b) Find the possible values of the displacement of the particle from its equilibrium position when the speed of the particle is 5 m s^{-1}, giving your answers in terms of π

[3 marks]

Answer _____

6

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4 A body of mass 2 kg moves in a straight line on a rough horizontal surface.

The coefficient of friction between the surface and the body is 0.1

At time t seconds the body has velocity v m s^{-1}

An air resistance force of magnitude $0.49v^2$ newtons acts on the body as it moves.

When $t = 0$, $v = 20$

Find the time that it takes for the body to come to rest.

[9 marks]

Answer _____

9

Turn over ►

Do not write outside the box

5 A spring has stiffness k N m^{-1} and natural length 40 cm.

One end of the spring is attached to a fixed point *O*

A sphere of mass 2.5 kg is attached to the other end of the spring.

5 (a) When the sphere is in equilibrium it remains at rest 45 cm vertically below *O*

Find the value of k

[2 marks]

Answer _____

5 (b) The sphere is set in motion so that it moves at 2 m s^{-1} vertically downwards away from its equilibrium position.

5 (b) (i) Show that the sphere moves with simple harmonic motion.

[5 marks]

Do not write outside the box

5 (b) (ii) Find the amplitude of the motion, giving your answer in exact form.

[2 marks]

Answer _____

5 (b) (iii) Find the percentage of the period for which the sphere is within 10 cm of its equilibrium position.

[6 marks]

Answer _____

15

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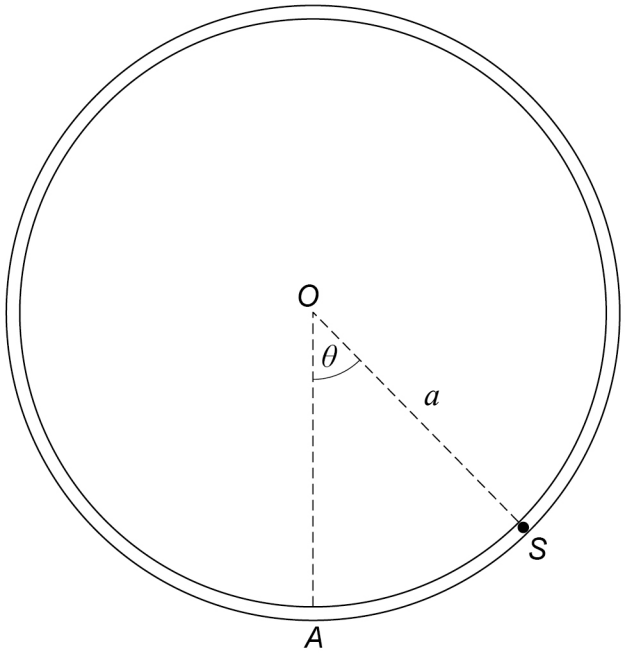
6 A smooth hollow tube is bent into a circle with centre *O* and is fixed in a vertical plane.

A small smooth sphere *S* has mass m kg and is set in motion inside the tube.

The sphere moves on the arc of a circle with centre *O* and radius a

The lowest point of the tube is *A*

The angle between *OA* and *OS* is θ as shown in the diagram below.



The speed of the sphere *S* at *A* is U m s^{-1}

The magnitude of the normal reaction force exerted on the sphere by the tube is R newtons.

6 (a) Show that

$$R = m\left(\frac{U^2}{a} - 2g + 3g\cos\theta\right)$$

[5 marks]

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Answer _____

Question 6 continues on the next page

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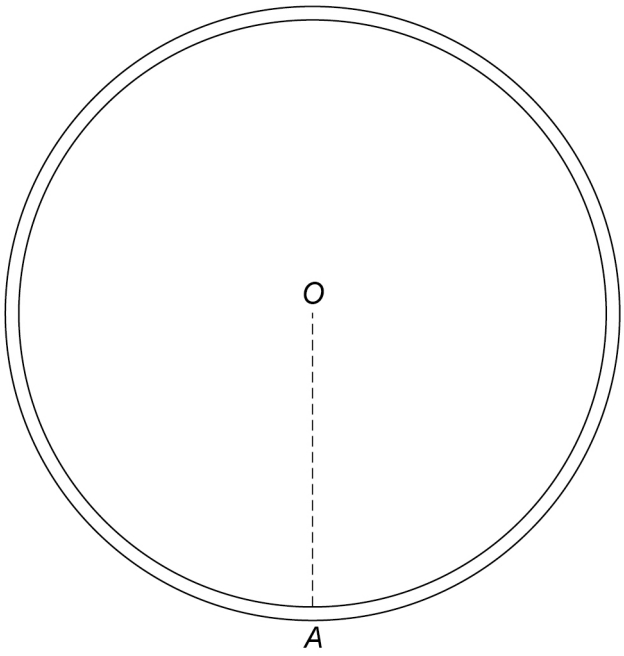
6 (b) It is given that $U = \sqrt{\frac{7ag}{2}}$

6 (b) (i) Find θ for the positions where the normal reaction force on the sphere is zero.

On the diagram below, clearly mark each of the positions with an **X**

[4 marks]



Answer _____

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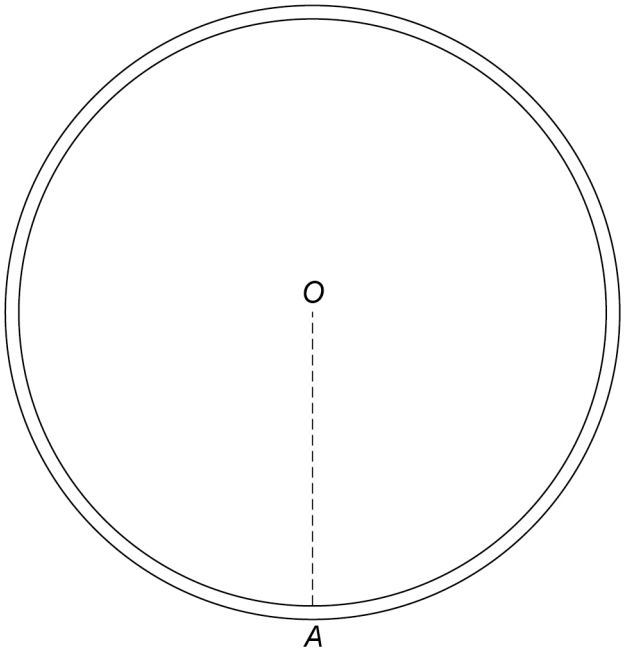
6 (b) (ii) Find θ for the positions where the speed of the sphere is zero.

On the diagram below, clearly mark each of the positions with an **X**

[4 marks]



Answer _____

13

Turn over ►

7 A uniform metal rod PQ has midpoint M and mass m kg

Three elastic strings are attached to the rod such that the rod and the strings are all in a vertical plane.

The fixed points B and C are at the same level and the fixed point A is a height $4d$ above the level of B and C

The string AM is attached to A and M

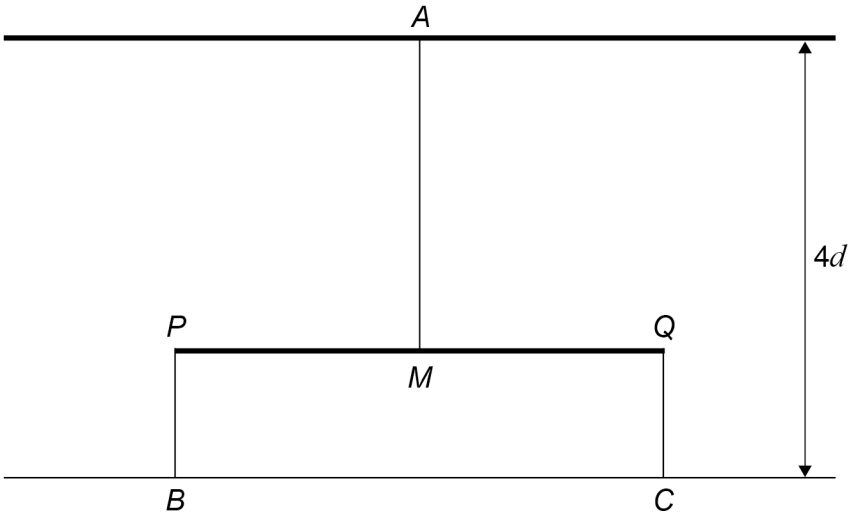
The string BP is attached to B and P

The string CQ is attached to C and Q

The table below shows the natural length and modulus of elasticity for each of the three strings.

String	Natural length (metres)	Modulus of elasticity (newtons)
AM	d	$4mg$
BP	d	$3mg$
CQ	d	$3mg$

The diagram shows the rod, strings and fixed points.



The rod is released from rest at a height d above the level of B and C

Initially the rod is horizontal and as it moves it remains horizontal.

Assume that there is no air resistance acting on the rod.

Find, in terms of m, g and d, the maximum kinetic energy of the rod.

[10 marks]

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Answer _____

10

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8 A plane is inclined at an angle of 20° to the horizontal.

The points *C* and *D* are on the inclined plane and in the same vertical plane of greatest slope.

The distance between *C* and *D* is 10 metres with *D* lower than *C*

A ball is projected down the slope, from *C*, with an initial velocity of 25 m s^{-1} at 30° above the plane.

At the same time, a second ball is projected down the slope from *D*, with an initial velocity of U m s^{-1} at 60° above the plane so that the balls collide.

The motion of both balls takes place in the same plane of greatest slope of the plane.

8 (a) Find the time for which the balls are moving before they collide, giving your answer in exact form.

[9 marks]

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Answer _____

8 (b) Hence find the distance of the balls from the plane when they collide.

[2 marks]

Answer _____

11

END OF QUESTIONS

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