OXFORD
INTERNATIONAL AQA EXAMINATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Wednesday 19 January 2022 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

1 A disc, of mass 0.3 kg, is sliding on a smooth horizontal surface when it collides with a smooth fixed vertical wall.

When the disc collides with the wall it has velocity 5 m s^{-1} at an angle of 50° to the wall.

When the disc leaves the wall it has velocity $v \text{ m s}^{-1}$ at an angle of α degrees to the wall.

The velocities of the disc are shown in the diagram below.

The coefficient of restitution between the disc and the wall is 0.8

1 (a) Find the value of v giving your answer correct to two decimal places.

[5 marks]

Answer _____

1 (b) Find the value of α giving your answer to the nearest integer.

[2 marks]

Answer _____

1 (c) Find the magnitude of the impulse exerted on the disc by the wall.

Give your answer to two significant figures and clearly state the units of your answer.

[3 marks]

Answer _____ Units _____

10

Turn over ▶

2 A particle of mass 4 kg is released from rest at a distance of 0.02 metres from the centre of a fixed sphere of radius 0.01 metres.

A force, F newtons, acts on the particle and is directed towards the centre of the sphere.

The magnitude of F is $\dfrac{8}{x^2}$ when the particle is x metres from the centre of the sphere.

Assume that F is the only force that acts on the particle.

2 (a) Show that the work done by the force F newtons as the particle moves from its initial position to the surface of the sphere is 400 joules.

[4 marks]

2 (b) Find the speed of the particle when it reaches the surface of the sphere, giving your answer to three significant figures.

[2 marks]

_____

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Answer _____

6

Turn over for the next question

Turn over ▶

6

3 A particle moves with simple harmonic motion on a straight line between the points *P* and *Q* which are 3 metres apart.

The period of the motion is 5 seconds.

3 (a) Find the maximum speed of the particle, giving your answer in terms of π

[2 marks]

Answer

3 (b) Find the speed of the particle when it is 1 metre from *P* giving your answer in exact form in terms of π

[3 marks]

Answer

3 (c) The point *O* is the mid-point of *PQ*

The particle starts from rest at *P*

3 (c) (i) Write down an expression for the displacement of the particle from *O* at time t seconds after leaving *P*

[1 mark]

Answer _____

3 (c) (ii) By using calculus with your answer to **part (c)(i)**, find the maximum acceleration of the particle giving your answer in terms of π

[3 marks]

Answer _____

9

Turn over for the next question

Turn over ▶

4   A simple pendulum consists of a string of length 0.7 metres and a small sphere.

4   **(a) (i)**   Show that, for small oscillations, the pendulum moves with approximately simple harmonic motion.

[5 marks]

4 (a) (ii) State two assumptions that you have made about the properties of the string.

[2 marks]

Assumption 1

Assumption 2

4 (b) Find the period of the motion of the pendulum for small oscillations, giving your answer in terms of π

[1 mark]

Answer

8

Turn over ▶

5 A particle of mass 2.5 kg moves in a straight line on a smooth horizontal surface.

The speed of the particle is v m s^{-1}

A resistance force of magnitude $10v^2$ newtons acts on the particle.

The speed of the particle is 20 m s^{-1} when it is at the origin O

Find the distance the particle has travelled from O when its speed is 10 m s^{-1}

Give your answer in the form $\dfrac{1}{b}\ln a$ where a and b are integers.

[5 marks]

Answer _____

5

Turn over for the next question

Turn over ▶

6 A particle of mass 2 kg is attached to one end of a spring of stiffness k Nm^{-1}

The other end of the spring is attached to a fixed point O

6 (a) Find, in terms of g and k, the extension of the spring when the particle hangs in equilibrium below O

[2 marks]

Answer

6 (b) The particle is pulled down and released from rest at a point vertically below the equilibrium position.

6 (b) (i) Show that the subsequent motion of the particle is simple harmonic motion.

[5 marks]

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_____
_____
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6 (b) (ii) The particle is released from rest at a point which is a distance $2a$ metres vertically below the equilibrium position.

The particle then takes $\dfrac{\pi}{8}$ seconds to move directly to a point $\dfrac{3a}{2}$ metres vertically below the equilibrium position.

Find the value of k

[4 marks]

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Answer _____

11

Turn over ▶

7 A particle of mass m kg is attached to one end of a light inextensible string of length $2a$ metres.

The other end of the string is attached to a fixed point O

The points A, B and O lie in the same vertical plane and are the vertices of an equilateral triangle.

The particle is released from rest at the point A where the string is taut.

The particle falls vertically until the string becomes taut again at the point B

Assume that the motion of the particle changes instantaneously at B from vertical motion to circular motion.

From B the particle moves on the arc of a circle until the string becomes slack again at the point C as shown in the diagram below.
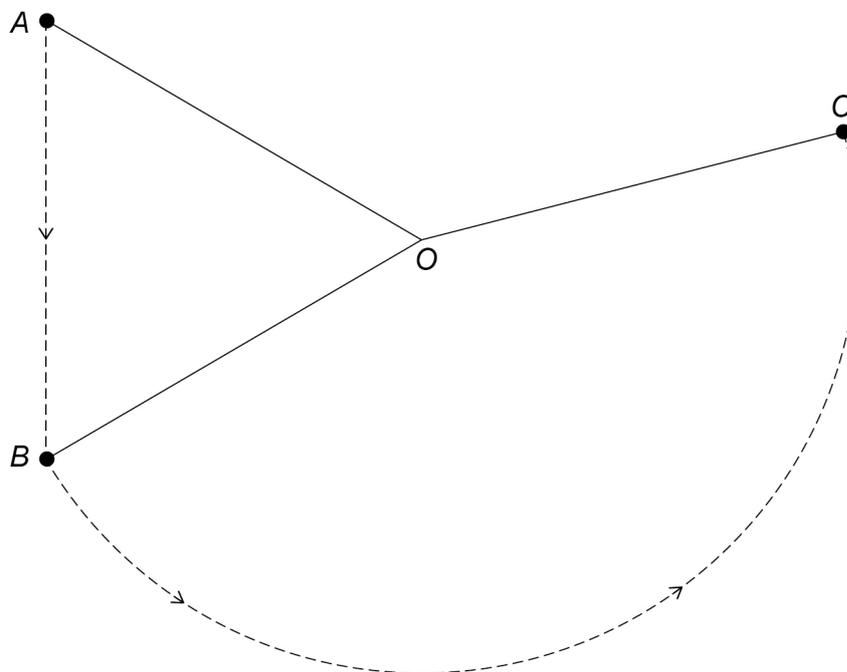
7 (a) Find, in terms of a and g, the speed of the particle at B
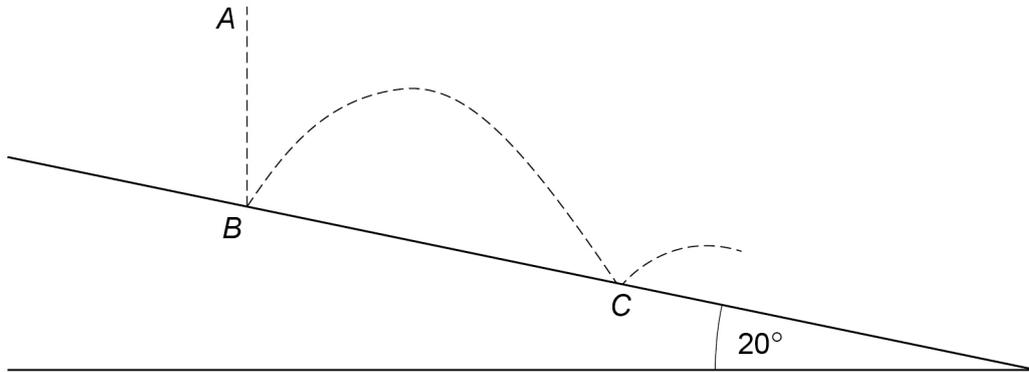
[2 marks]

Answer _____

7 (b) At *B* an impulse exerted on the particle by the string changes the motion of the particle, so that its velocity is perpendicular to *OB*

Show that this impulse changes the speed of the particle to $\sqrt{3ag}$ and find the magnitude of the impulse.

[5 marks]

Answer _____

Question 7 continues on the next page

Turn over ▶

7 (c) Find the acute angle between *OC* and the vertical, giving your answer to the nearest degree.

[6 marks]

Answer _____

13

Turn over for the next question

Turn over ▶

8 A smooth plane is inclined at 20° to the horizontal.

A ball is released from rest from the point *A* and falls a distance of 3 metres vertically before bouncing on the plane at the point *B*

The ball next bounces on the plane at the point *C*

The plane, the points and the motion of the ball are shown in the diagram below.

The coefficient of restitution between the ball and the plane is 0.6

The line *BC* is a line of greatest slope of the plane.

Find the distance *BC*

[9 marks]

Answer _____

9

Turn over ►

9 Two smooth spheres, *A* and *B*, with the same radius are moving on a horizontal surface when they collide.

The mass of *A* is 2 kg and the mass of *B* is 3 kg

As the spheres collide:

- sphere *A* is moving at 4 m s^{-1} at an angle of 60° to the line of centres.

- sphere *B* is moving at 1 m s^{-1} along the line of centres.

The velocities of the spheres as they collide are shown in the diagram below.
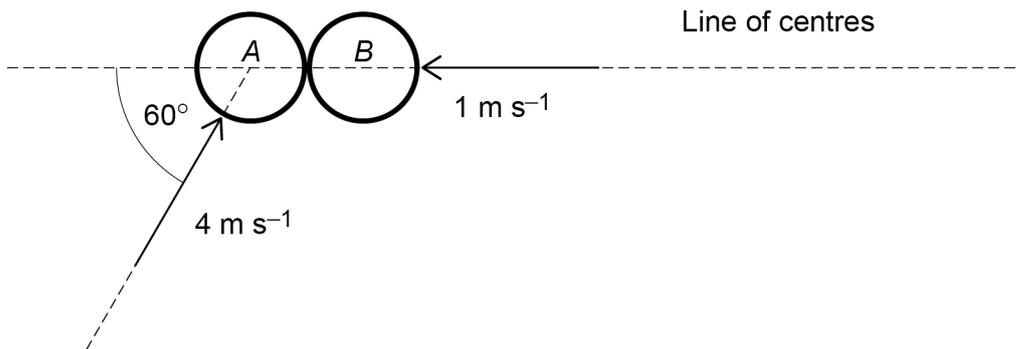
The coefficient of restitution between the two spheres is *e*

The direction of motion of *A* is changed by an angle α degrees as a result of the collision.

Find the range of possible values of α

[9 marks]

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Answer

END OF QUESTIONS

9

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ANSWER IN THE SPACES PROVIDED

Question number	Additional page, if required. Write the question numbers in the left-hand margin.

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