OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Thursday 13 June 2024 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 A ball of mass 0.3 kg falls vertically from rest.

It hits a horizontal surface and rebounds to a height of 0.4 metres above the surface.

1 (a) Calculate the speed at which the ball leaves the horizontal surface.

[2 marks]

Answer _____

1 (b) The impulse exerted on the ball by the surface has magnitude 3.24 N s

Find the coefficient of restitution between the ball and the surface.

[4 marks]

Answer _____

6

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2 A light elastic string has modulus of elasticity 300 newtons and natural length 2 metres.

One end of the string is attached to a fixed point.

The other end of the string is attached to a particle of mass 6 kg

The particle is released from rest 2.5 metres vertically below the fixed point.

2 (a) Find the magnitude and direction of the initial acceleration of the particle.

[3 marks]

Magnitude _____

Direction _____

2 (b) Use an energy method to find the maximum speed of the particle in the subsequent motion.

[5 marks]

Answer _____

8

Turn over ►

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3 A particle of mass 2 kg is initially at rest at the point *O* on a rough horizontal surface.

A horizontal force of magnitude F newtons and constant direction starts to act on the particle.

When the displacement of the particle from *O* is x metres, F is given by

$$F = 10e^{-0.1x}$$

The particle passes through the point *A* which is 5 metres from *O*

3 (a) Find the work done by the force F as the particle moves from *O* to the point *A*

[3 marks]

Answer _____

3 (b) The coefficient of friction between the particle and the surface is 0.2

3 (b) (i) Find the speed of the particle at *A*

[3 marks]

Answer _____

Do not write outside the box

3 (b) (ii) The particle comes to rest when the displacement from O is d metres.

Use a change of sign numerical method to verify that $d = 23$ correct to the nearest integer.

[4 marks]

10

Turn over for the next question

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4 A particle of mass 2 kg moves on a straight horizontal line.

As the particle moves it experiences a resistance force of magnitude $\left(v^2 - 9\right)$ newtons, where v m s^{-1} is the velocity of the particle.

At the point O the particle has a velocity of 10 m s^{-1}

4 (a) Find the velocity of the particle when its displacement from O is x metres.

Give your answer in the form $v = \sqrt{a + b\mathrm{e}^{-x}}$ where a and b are integers.

[6 marks]

Answer _____

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4 (b) On the axes below sketch a graph to show how the velocity of the particle varies with its displacement from *O*

[2 marks]

4 (c) On the axes below sketch a graph to show how the acceleration of the particle varies with its displacement from *O*

[3 marks]

11

Turn over ►

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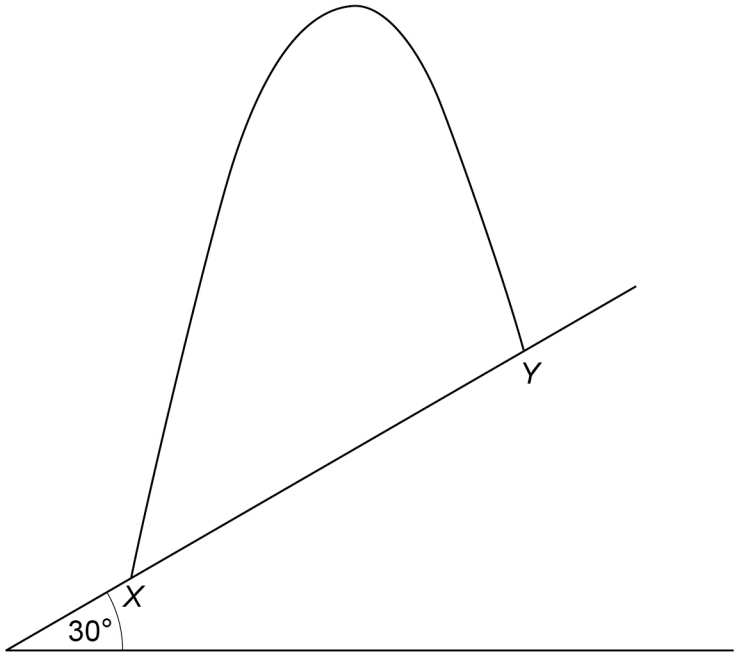
5 A plane is inclined at an angle of $30°$ to the horizontal.

A ball is projected up the plane from a point X on the plane.

The ball hits the plane for the first time at a point Y as shown in the diagram.



The line XY is a line of greatest slope of the plane.

The distance between the points X and Y is 4 metres.

The initial velocity of the ball is 10 m s^{-1} at an angle $\alpha°$ above the plane, where $\alpha < 60$

By using the components of the ball's motion parallel and perpendicular to the plane, find the possible values of α

Give your values to the nearest integer.

[8 marks]

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Answer _____

8

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6 Two spheres A and B are moving on a smooth horizontal surface.

The two spheres collide.

Sphere A has mass 2 kg and velocity $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ m s^{-1} before the collision.

Sphere B has mass 3 kg and velocity $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$ m s^{-1} before the collision.

After the collision sphere A has velocity $\begin{bmatrix} 0.4 \\ 3 \end{bmatrix}$ m s^{-1}

6 (a) Find the magnitude of the impulse exerted on A during the collision.

[3 marks]

Answer _____

6 (b) Find the velocity of sphere B after the collision.

[3 marks]

Do not write outside the box

Answer _____

6 (c) Explain why the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is parallel to the line of centres during the collision.

[2 marks]

6 (d) Find the coefficient of restitution between the two spheres.

[2 marks]

Answer _____

10

Turn over ►

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7 Two springs are attached to a sphere of mass m kg which rests on a smooth horizontal surface.

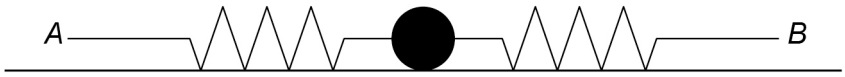
The other end of one spring is attached to the fixed point *A* and the other end of the second string is attached to a fixed point *B*

The points *A* and *B* are a distance $3a$ metres apart.

Both springs have natural length a metres.

The spring attached to *A* has stiffness k N m^{-1} and the spring attached to *B* has stiffness $2k$ N m^{-1}

The diagram shows the springs and the sphere.



7 (a) Show that the extension of the spring attached to *A* is $\frac{2a}{3}$ metres when the system is in equilibrium.

[3 marks]

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7 (b) The sphere is pulled a distance of $\frac{a}{10}$ metres from the equilibrium position towards B and released from rest.

7 (b) (i) Show that the sphere moves with simple harmonic motion.

[5 marks]

7 (b) (ii) The period of the motion is π seconds.

Express k in terms of m

[2 marks]

Answer _____

Question 7 continues on the next page

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7 (b) (iii) The point C is $\frac{a}{20}$ metres from the equilibrium position.

Hence, find the speed of the sphere at C

Give your answer in an exact form in terms of a

[3 marks]

Answer _____

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7 (b) (iv) Find the shortest time that it takes for the sphere to move from C and return back to C

[4 marks]

Answer _____

17

Turn over for the next question

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8 A smooth hemisphere with centre O has its base fixed to a horizontal surface.

A particle is released from rest at a point A on the curved surface of the hemisphere.

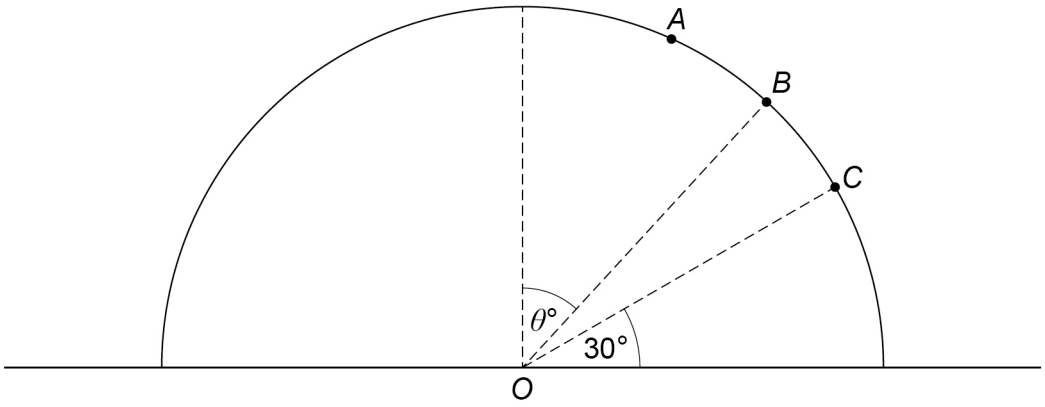
The particle passes through the point B and leaves the hemisphere at the point C

At B the angle between the radius OB and the vertical is θ°

At B the magnitude of the normal reaction force exerted on the particle is half of the maximum experienced by the particle as it moves from A to C

At C the angle between the radius OC and the horizontal is 30°

The diagram shows the hemisphere and the points A, B and C



Find $\cos\theta^\circ$

Give your answer as a fraction.

[10 marks]

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Answer _____

10

END OF QUESTIONS

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