OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM05) Unit FM2 Mechanics

Monday 20 January 2025 07:00 GMT Time allowed: 1 hour 30 minutes

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to two significant figures, unless stated otherwise.
- Unless stated otherwise, the acceleration due to gravity, g, should be taken as 9.8 m s^{-2}

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 A particle of mass 0.5 kg moves on a straight line on a smooth horizontal surface.

The point O is on the line.

When the particle is at O its velocity is 50 m s^{-1}

When the displacement of the particle from O is x metres the velocity of the particle is v m s^{-1}

As the particle moves on the line it experiences a resistance force of magnitude $5v^2$ newtons.

The particle does not experience any other forces in the horizontal direction.

1 (a) Find in terms of x an expression for v

[5 marks]

Answer _____

1 (b) Find the displacement of the particle from O when its velocity is 25 m s^{-1}

Give your answer in an exact form.

[2 marks]

Answer _____

7

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2 Two particles A and B are moving on a smooth horizontal surface when they collide.

Particle A has mass 3 kg and before the collision has velocity $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ m s^{-1}

Particle B has mass 5 kg and before the collision has velocity $\begin{bmatrix} 1.6 \\ -1 \end{bmatrix}$ m s^{-1}

After the collision B has velocity $\begin{bmatrix} 1 \\ -0.4 \end{bmatrix}$ m s^{-1}

2 (a) Find the velocity of A after the collision.

[3 marks]

Answer _____

2 (b) Find the magnitude of the impulse exerted on B by A during the collision.

Give your answer in an exact form.

[3 marks]

Answer _____

6

Turn over ►

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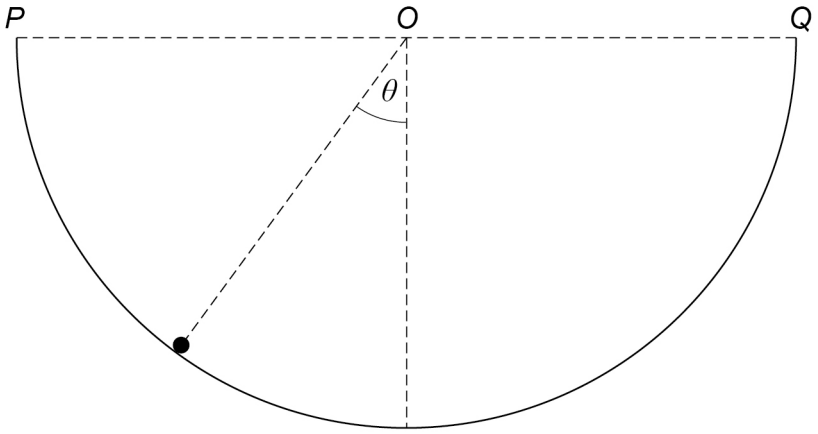
3 A smooth semi-circular track with radius a metres is fixed in a vertical plane.

The points P and Q are at opposite ends of a horizontal diameter and O is the midpoint of this diameter.

A particle of mass m kg is released from rest at the point P and slides on the inside of the track.

The radius from O to the particle is at angle θ to the vertical, as shown in the diagram.



3 (a) Show that the magnitude of the normal reaction force R newtons exerted by the track on the particle is given by

$$R = 3mg\cos\theta$$

[5 marks]

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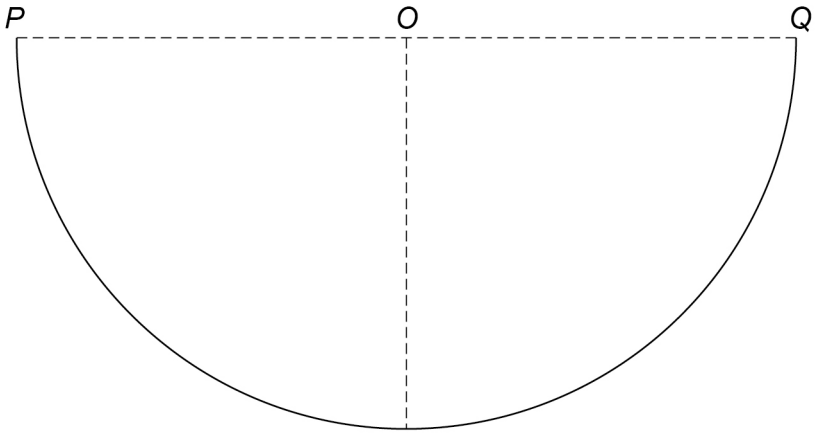
3 (b) On the diagram below, show the positions of the particle at which the normal reaction force has magnitude mg newtons.

Give any angles you calculate to the nearest degree.

[2 marks]



7

Turn over for the next question

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4 A particle moves with simple harmonic motion on a straight line.

The particle has its maximum speed at the point O

The points A and B lie on the line.

At A the displacement of the particle from O is -4 metres and its speed is $3\ \text{m s}^{-1}$

At B the displacement of the particle from O is 2 metres and its speed is $5\ \text{m s}^{-1}$

4 (a) Find the amplitude of the motion.

Give your answer in an exact form.

[5 marks]

Answer _____

Do not write outside the box

4 (b) Find the period of the motion.

Give your answer in terms of π in an exact form.

[3 marks]

Answer _____

4 (c) Find the shortest time that it takes for the particle to move from A to B

Give your answer to three significant figures.

[4 marks]

Answer _____

12

Do not write outside the box

5 A particle of mass 0.25 kg moves on a smooth straight horizontal line of infinite length.

The particle is initially at rest at the point O which lies on the line.

A force acts on the particle and is directed along the line.

When the displacement of the particle from O is x metres, the magnitude of the force is $2e^{-\frac{x}{10}}$ newtons.

The particle does not experience any other forces in the horizontal direction.

5 (a) Show that the work done by the force approaches a maximum value and state this maximum value.

Show the limiting process used.

[4 marks]

Maximum value _____

Do not write outside the box

5 (b) Find the range of values of the speed of the particle as it moves on the line.

Give your answer in an exact form.

[3 marks]

Answer _____

7

Turn over for the next question

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6 A light elastic string has natural length 0.5 metres and modulus of elasticity 14 newtons.

One end of the elastic string is attached to a fixed point P

The other end of the elastic string is attached to a sphere of mass 3.2 kg

The elastic string is stretched and the sphere released from rest at the point Q, which is 3 metres vertically below P

6 (a) Use an energy method to find the maximum speed of the sphere during the motion.

[5 marks]

Answer _____

6 (b) (i) Show that the elastic string becomes slack during the motion.

[2 marks]

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6 (b) (ii) Find the minimum distance between the sphere and the point P

[2 marks]

Answer _____

6 (c) A student carries out an experiment to test the answer obtained in **part (b)(ii)**

From their experiment they find that the minimum distance of the sphere from P is 0.41 metres.

The student claims that the result from the experiment is different to the calculated answer because of air resistance.

The student assumes that the magnitude of the air resistance force acting on the sphere is constant as it is rising.

Find the magnitude of this constant air resistance force.

[3 marks]

Answer _____

12

Turn over ►

Do not write outside the box

7 A simple pendulum consists of a light inextensible string of length d metres and a small sphere.

The sphere is released from rest with the string taut and at a small angle to the vertical.

7 (a) Show that the motion of the simple pendulum approximates to simple harmonic motion.

[4 marks]

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7 (b) In the case when $d = 0.8$ the sphere is released from rest with the string taut and at an angle of $\frac{\pi}{10}$ radians to the vertical.

Find the greatest distance that the sphere can move in an interval of time equal to one quarter of the period.

Give your answer to three significant figures.

[3 marks]

Answer _____

7

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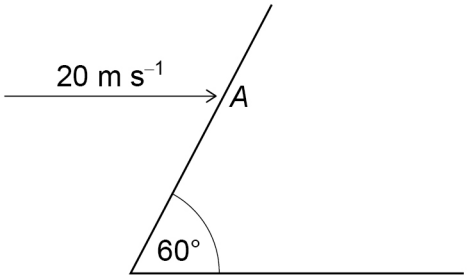
8 A smooth plane is inclined at an angle of $60°$ to the horizontal.

Two different balls are fired at different times so that they are travelling horizontally at $20\ \text{m s}^{-1}$ when they hit the plane at the point A, as shown in the diagram.



The velocities of the balls are in a plane that contains the line of greatest slope of the inclined plane.

8 (a) The coefficient of restitution between the first ball and the plane is e_1

It is given that the first ball moves vertically after it leaves the plane at A.

Find the value of e_1

Give your answer in an exact form.

[4 marks]

Answer _____

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8 (b) The coefficient of restitution between the second ball and the plane is e_2

The second ball leaves the inclined plane at A and returns to the plane for the first time at the point B

The point B is on the inclined plane further up than A

The distance between A and B is 2 metres.

Find the possible values of e_2

Give your answers to two decimal places.

[6 marks]

Answer _____

10

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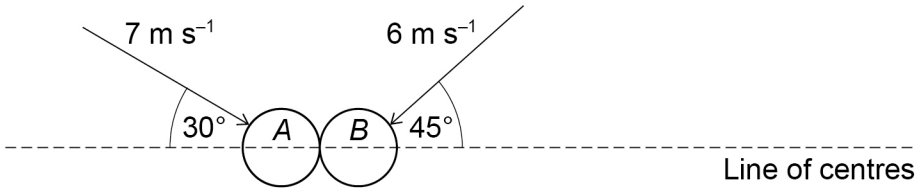
9 Two smooth spheres A and B of equal radii are moving on a horizontal surface when they collide.

The mass of A is 3 kg and the mass of B is 2 kg

Before the collision:

- A has a velocity of $7\ \text{m s}^{-1}$ at an angle of $30°$ to the line of centres
- B has a velocity of $6\ \text{m s}^{-1}$ at an angle of $45°$ to the line of centres

The velocities are shown in the diagram below.



The coefficient of restitution between the two spheres is e

It is given that $e \geq \frac{2}{3}$

9 (a) Find the range of possible speeds of A after the collision.

Give your answers to two decimal places.

[8 marks]

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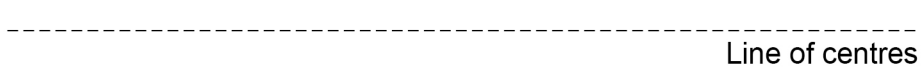
Answer _____

9 (b) On the diagram below show the range of possible directions of motion of *A* after the collision.

Give any angles you calculate to the nearest degree.

[4 marks]



12

END OF QUESTIONS

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