OXFORD AQA

INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

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I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Thursday 16 January 2025 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

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Answer **all** questions in the spaces provided.

1 A researcher asks a random sample of 500 customers to name their preferred brand of soft drink and their preferred brand of biscuits.

Some of the researcher's values are shown in the table below.

Preferred biscuits

Preferred soft drink		**Brand *X***	**Brand *Y***	**Total**
	Brand *A*	84		160
	Brand *B*	56	120	
	Brand *C*			
	Total	220		500

1 (a) Calculate the missing values and complete the table.

[1 mark]

1 (b) The researcher claims that there is an association between the customers' preferred brand of soft drink and the preferred brand of biscuits.

Test the researcher's claim using the 0.5% level of significance

[7 marks]

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8

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2 A random sample of size 12 is taken from a normal distribution with unknown population mean μ_1 and known population variance σ_1^2

This random sample has sample mean 90.2

A random sample also of size 12 is taken from a different normal distribution with unknown population mean μ_2 and known population variance σ_2^2

This random sample has sample mean 89.8

The two normal distributions are independent.

A hypothesis test is carried out with the hypotheses

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

The value of the test statistic is $\frac{4\sqrt{5}}{5}$

2 (a) Show that $\sigma_1^2 + \sigma_2^2 = \frac{3}{5}$

[3 marks]

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2 (b) It is given that $\sigma_1 = 4\sigma_2$

Find the value of σ_1^2 and the value of σ_2^2

[3 marks]

$\sigma_1^2 =$ _____ $\sigma_2^2 =$ _____

2 (c) The hypothesis test used the 10% level of significance.

Determine the outcome of the test.

[3 marks]

9

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3 A company employs 8 members of staff to sell its cars.

The company has recently changed the layout of its car showroom.

Before the layout change, a week is chosen at random and the total sales, in thousands of dollars, made by each member of staff is recorded.

After the layout change, a week is chosen at random and the total sales, in thousands of dollars, made by each member of staff is recorded.

The results are summarised in the following table.

	Total sales (thousands of dollars)	
Member of staff	**Before layout change**	**After layout change**
A	115	121
B	94	92
C	101	105
D	89	91
E	48	53
F	104	101
G	68	68
H	92	97

The company claims that total sales per member of staff have increased following the layout change.

3 (a) State a necessary assumption for a hypothesis test using the t-distribution to test the company's claim to be carried out.

[1 mark]

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3 (b) Test the company's claim using the 5% level of significance.

[9 marks]

10

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4 The score obtained by players in a game is known to have a normal distribution

A random sample of n players is taken where $5 < n < 10$

The unbiased estimator of the population variance is calculated using the sample and found to be $s^2 = 151.29$

A 95% confidence interval for the population mean score is calculated using the sample.

The width of the confidence interval is 20.6 correct to three significant figures.

4 (a) Find the value of n

Fully justify your answer.

[3 marks]

Answer _____

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4 (b) The sample mean score of the sample is 55.7 points.

Find the 95% confidence interval for the population mean score.

Give your values to three significant figures.

[1 mark]

Answer _____

4 (c) A player obtains a score of 40 in the game.

The player claims that their score is equal to the population mean score.

Explain whether the confidence interval found in **part (b)** supports the player's claim.

[1 mark]

5

Turn over for the next question

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5 An exam has a multiple-choice section.

The total marks that students obtain on the multiple-choice section are independent of each other and are known to have a normal distribution.

A random sample of 41 students is taken and their total marks on the multiple-choice section are recorded.

The unbiased estimator of the population variance for the total marks on the multiple-choice section is calculated using the sample and found to be 25.4

A hypothesis test is carried out to test the claim that the population standard deviation is different from k where k is a positive constant.

The hypothesis test is carried out at the 5% level of significance and the null hypothesis is not rejected.

5 (a) Find the range of possible values of k

Give your values to three significant figures.

[5 marks]

Answer _____

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5 (b) The exam also has a written response section.

The total marks that students obtain on the written response section are independent of each other and are known to have a normal distribution.

The total marks that students obtain on the multiple choice and written response sections are independent.

A random sample of 11 students is taken and their total marks on the written response section are recorded.

The unbiased estimator of the population variance for the total marks on the written response section is calculated using the sample and found to be 66.1

It is claimed that the population variance of the total marks on the written response section is greater than the population variance of the total marks on the multiple-choice section.

Use both random samples to test the claim at the 1% level of significance.

[6 marks]

11

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6 Customers who have bought a particular product can use an online chat service if they need support.

The time T minutes between one customer starting to use the online chat service and the next customer is modelled by a random variable with mean $\frac{m}{2}$ and variance $\frac{m^2}{12}$

T_1, T_2, …, T_n are n independent observations of T where $n > 2$

The random variable X is given by $X = \sum_{i=1}^{n} T_i$

6 (a) Show that X is a biased estimator of m

[2 marks]

6 (b) It is given that kX is an unbiased estimator of m where k is a constant.

Find k in terms of n

[2 marks]

Answer _____

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6 (c) Use your expression for k in terms of n from **part (b)** to determine whether kX is a consistent estimator of m

[3 marks]

6 (d) Sana measures the time in minutes between one customer using the online chat service and the next customer on 5 random occasions.

Her results are

0.5 1.2 1.5 3.2 7.1

She uses her data and the unbiased estimator kX to calculate an estimate for m

Find Sana's estimate for m

[2 marks]

Answer _____

9

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7 The daily maximum height of river A has population mean μ_A cm and a known population standard deviation of 3.04 cm

The daily maximum height of river B has population mean μ_B cm and a known population standard deviation of 2.09 cm

Random samples of the daily maximum heights are taken for both rivers and the results are shown in the following table

River	Sample size	Sample mean (cm)
A	478	$\bar{x}_A = 18.3$
B	437	$\bar{x}_B = 17.9$

It is claimed that the population means of the daily maximum heights in cm of the two rivers μ_A and μ_B are different.

7 (a) Test the claim using the 2% level of significance.

[6 marks]

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7 (b) The daily maximum heights of the two rivers are not normally distributed.

Explain why the test carried out in **part (a)** is valid.

[1 mark]

7 (c) The hypothesis test in **part (a)** is repeated using random samples of the same sizes as in **part (a)**.

It is given that $\mu_A - \mu_B = 0.2$

7 (c) (i) Find the probability that a Type II error is made.

[5 marks]

Answer

7 (c) (ii) Find the power of the test.

[1 mark]

Answer

13

Turn over ►

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8 The random variable X has moment generating function $\mathrm{M}_X(t)$ where

$$\mathrm{M}_X(t) = \frac{\mathrm{e}^{pt}}{1-qt^2} \quad \text{for} \quad |t| < \frac{1}{\sqrt{q}}$$

where p and q are constants.

It is given that the mean of X is 0 and the variance of X is $\frac{1}{2}$

8 (a) (i) Show that the value of p is 0

[3 marks]

8 (a) (ii) Find the value of q

[4 marks]

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Answer _____

8 (b) The random variable A has an exponential distribution with parameter λ

8 (b) (i) Show that the moment generating function $\mathrm{M}_A(t)$ of A is given by

$$\mathrm{M}_A(t) = \frac{\lambda}{\lambda - t} \quad \text{for} \quad t < \lambda$$

[3 marks]

Question 8 continues on the next page

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8 (b) (ii) The random variable B has an exponential distribution with parameter λ

A and B are independent.

Write down the moment generating function $\mathrm{M}_{-B}(t)$ of $-B$

[1 mark]

Answer _____

8 (b) (iii) The random variable $A - B$ has the same distribution as X

Find the value of λ

[4 marks]

Answer _____

15

END OF QUESTIONS

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