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**Pearson Edexcel International Advanced Level**

**Thursday 10 June 2025**

Morning (Time: 1 hour 30 minutes) **Paper reference** **WFM03/01**

**Mathematics**

**International Advanced Subsidiary/ Advanced Level**

**Further Pure Mathematics F3**

**You must have:**  
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions:

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information:

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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**1:** (a) Use the definition of  $\cosh x$  in terms of exponentials to show that

$$2 \cosh 5x \cosh x \equiv \cosh 6x + \cosh 4x \quad (2)$$

(b) Hence determine the exact values of  $x$  for which

$$\cosh 6x + \cosh 4x = 8 \cosh x$$

giving your answers in terms of natural logarithms in simplest form. (4)

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Question 1 continued

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(Total for Question 1 is 6 marks)



2: Determine

(i)  $\int \frac{1}{\sqrt{4x^2 + 8x + 9}} dx$

(3)

(ii)  $\int \operatorname{arcosh} 3x dx$

(4)



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Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 7 marks)



3:

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & a & b \\ a & 1 & b \end{pmatrix}$$

where  $a$  and  $b$  are constantsGiven that  $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  is an eigenvector of  $\mathbf{M}$ ,

(a) determine the corresponding eigenvalue.

(2)

(b) Hence determine the value of  $a$  and the value of  $b$ .

(2)

(c) Determine

(i) the other eigenvalues of  $\mathbf{M}$ ,

(3)

(ii) eigenvectors which correspond to these eigenvalues.

(3)



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Question 3 continued

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Question 3 continued

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Question 3 continued

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(Total for Question 3 is 10 marks)



4:

$$y = \operatorname{arsinh} x + \operatorname{arsinh} \left( \frac{1}{x} \right) \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{x-1}{x\sqrt{1+x^2}} \quad (2)$$

(b) Hence determine the exact value of  $y$  for which  $\frac{dy}{dx} = 0$ , giving your answer as a simplified natural logarithm.

(3)



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Question 4 continued

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(Total for Question 4 is 5 marks)



5:

$$I_n = \int_1^6 x^n (3x - 2)^{-\frac{1}{2}} dx \quad n \geq 0$$

(a) Show that, for  $n \geq 1$ 

$$(3 + 6n)I_n = 4nI_{n-1} + 8 \times 6^n - 2 \quad (5)$$

(b) Use the reduction formula in part (a) to determine the exact value of

$$\int_1^6 x^3 (3x - 2)^{-\frac{1}{2}} dx \quad (4)$$



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Question 5 continued

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Question 5 continued

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Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 9 marks)



6:

$$\mathbf{A} = \begin{pmatrix} 1 & k & 2 \\ 5 & 3 & -2 \\ 6 & -1 & 4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the value of  $k$  for which  $\mathbf{A}$  is singular.

(3)

Given that  $\mathbf{A}$  is non-singular,

(b) determine  $\mathbf{A}^{-1}$ , giving your answer in simplest form in terms of  $k$ .

(4)



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Question 6 continued

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Question 6 continued

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Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 7 marks)



7:

In this question you must show all stages of your working.  
Solutions relying entirely on calculator technology are not acceptable.

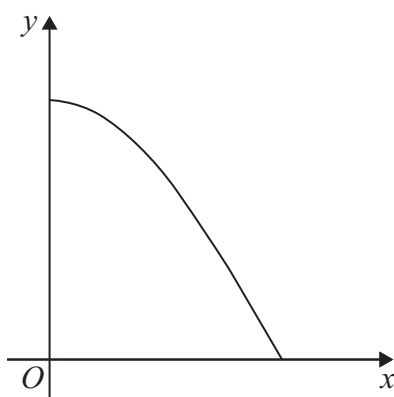


Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \cos 2x \quad 0 \leq x \leq \frac{\pi}{4}$$

The curve is rotated through  $2\pi$  radians about the  $x$ -axis.

(a) Show that the area of the curved surface generated is given by

$$S = 2\pi \int_0^{\frac{\pi}{4}} \cos 2x \sqrt{1 + 4 \sin^2 2x} \, dx \quad (2)$$

(b) Hence, using the substitution  $2 \sin 2x = \sinh \theta$ , show that

$$S = \frac{\pi}{4} \left( \ln(a + \sqrt{b}) + a\sqrt{b} \right)$$

where  $a$  and  $b$  are integers to be determined.

(7)



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Question 7 continued

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Question 7 continued

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Question 7 continued

Lined area for writing answers.

(Total for Question 7 is 9 marks)



**8:** The plane  $\Pi_1$  has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 5 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters.

(a) Determine  $(7\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$  (1)

(b) Hence show that the equation of  $\Pi_1$  can be written in the form

$$\mathbf{r} \cdot (8\mathbf{i} + 2\mathbf{j} + 11\mathbf{k}) = p$$

where  $p$  is a constant to be determined. (2)

Given that

- the plane  $\Pi_2$  has equation  $x - y + z = 7$
  - the planes  $\Pi_1$  and  $\Pi_2$  intersect in the line  $l_1$
- (c) determine an equation for  $l_1$  giving your answer in the form  $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$  where  $\mathbf{a}$  and  $\mathbf{b}$  are constant vectors.
- (3)

Given also that

- the point  $A$  has coordinates  $(2, 1, 3)$
  - the point  $B$  has coordinates  $(3, 0, 2)$
  - the line  $l_2$  passes through  $A$  and  $B$
- (d) determine the shortest distance between  $l_1$  and  $l_2$
- (4)**

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Question 8 continued

Lined area for writing the answer to Question 8.



Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



9: The hyperbola  $H$  has equation  $\frac{x^2}{64} - \frac{y^2}{49} = 1$  and eccentricity  $e$ .

(a) Show that  $e = \frac{\sqrt{113}}{8}$  (2)

The point  $(8 \sec \theta, 7 \tan \theta)$ , where  $0 < \theta < \frac{\pi}{2}$ , lies on  $H$ .

(b) Use calculus to show that the tangent to  $H$  at  $P$  has equation

$$\frac{x}{8} \sec \theta - \frac{y}{7} \tan \theta = 1 \quad (3)$$

The tangent to  $H$  at  $P$  meets the  $y$ -axis at the point  $Q$ .

(c) Write down the coordinates of  $Q$ . (1)

The normal to  $H$  at  $P$

- has equation  $8x \cos \theta + 7y \cot \theta = 113$
- meets the  $y$ -axis at the point  $R$

(d) Write down the coordinates of  $R$ . (1)

(e) Hence show that the circle with  $QR$  as a diameter passes through the foci of  $H$ . (5)



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Question 9 continued

Lined area for writing the answer to Question 9.



Question 9 continued

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Question 9 continued

Lined area for writing the answer to Question 9.



**Question 9 continued**

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**(Total for Question 9 is 12 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

