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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

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241XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	$(a+3\mathrm{i})(2-\mathrm{i}) = 2a - a\mathrm{i} + 6\mathrm{i} + 3$	**M1**	Expands the brackets and replaces i^2 with -1
	$= 2a+3 + \mathrm{i}(6-a)$	**A1**	**oe** with real and imaginary parts collected and i written as a factor of the imaginary part Accept $u = 2a+3$ and $v = 6-a$ Condone $2a+3 - \mathrm{i}(a-6)$ **ISW**
		2	

Q	Answer	Marks	Comments
1(a)(ii)	$\dfrac{a+3\mathrm{i}}{2+\mathrm{i}} = \dfrac{(a+3\mathrm{i})(2-\mathrm{i})}{(2+\mathrm{i})(2-\mathrm{i})}$	**M1**	multiplies both the numerator and the denominator by $2-\mathrm{i}$ **or** writes the fraction equivalent to $x+\mathrm{i}y$, multiplies by the denominator and equates real and imaginary parts
	$= \dfrac{2a+3 + (6-a)\mathrm{i}}{5}$		
	$= \dfrac{2a+3}{5} + \mathrm{i}\dfrac{6-a}{5}$	**A1ft**	**ft** their part **(a)(i)** **oe** with real and imaginary parts collected and i written as a factor of the imaginary part Accept $x = \dfrac{2a+3}{5}$ and $y = \dfrac{6-a}{5}$ Condone $\dfrac{2a+3}{5} - \mathrm{i}\dfrac{a-6}{5}$ **ISW**
		2	

Q	Answer	Marks	Comments
1(b)	Let $z = c + \mathrm{i}d$ where $c, d \in \mathbb{R}$	M1	Uses a suitable method
	$3(c - \mathrm{i}d) + \mathrm{i}(c + \mathrm{i}d) = 23 + 13\mathrm{i}$ $3c - 3\mathrm{i}d + c\mathrm{i} - d = 23 + 13\mathrm{i}$	B1	Uses the relationship between z and z^*
	$3c - d = 23$ and $c - 3d = 13$	M1	Compares real and imaginary parts to form simultaneous equations in c and d where $z = c + \mathrm{i}d$ **PI**
	$c = 7$ or $d = -2$	A1	Finds the value of c or d **PI**
	$z = 7 - 2\mathrm{i}$	A1	**oe**
		5	

	Question 1 Total	**9**	

Q	Answer	Marks	Comments
2(a)	$(1+h)^5 = 1+5h+10h^2+10h^3+5h^4+h^5$	B1	**oe** expanded and simplified polynomial
		1	

Q	Answer	Marks	Comments
2(b)(i)	$(1+h)^5 - 1^5 = 5h+10h^2+10h^3+5h^4+h^5$	M1	Subtracts 1 from their part **(a)** polynomial
	gradient of line $= \dfrac{(1+h)^5 - 1^5}{(1+h)-1}$	M1	Obtains an expression for the gradient of the line Accept any correct form **ft** their part **(a)**
	$= \dfrac{5h+10h^2+10h^3+5h^4+h^5}{h}$		
	$= 5+10h+10h^2+5h^3+h^4$	A1	**oe** polynomial Accept $a=5,\ b=10,\ c=10,\ d=5$
		3	

Q	Answer	Marks	Comments
2(b)(ii)	gradient of curve $= \lim_{h \to 0}\left(5+10h+10h^2+5h^3+h^4\right)$	M1	Considers their part **(b)(i)** as $h \to 0$
	$= 5$	A1ft	Obtains the correct limit of their part **(b)(i)** as $h \to 0$ **ft** their $a+bh+ch^2+dh^3+h^4$ **SC1** for 5 following $h=0$
		2	

	Question 2 Total	**6**	

Q	Answer	Marks	Comments
3(a)	$\left[x+\frac{\pi}{4} =\right] \frac{\pi}{6}$	B1	Finds one solution to $x+\frac{\pi}{4}$ Accept any correct angle in radians **PI** **oe**
	$x+\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{6}$ where $n \in \mathbb{Z}$	M1	Removes the trig function to form a general equation Condone n not defined **oe**
	$x = 2n\pi \pm \frac{\pi}{6} - \frac{\pi}{4}$	A1	Finds a general solution May be written in two parts **eg** $x = n\pi - \frac{\pi}{12}$ for even n and $x = n\pi + \frac{7\pi}{12}$ for odd n
		3	

Q	Answer	Marks	Comments
3(b)	Two solutions for every 2π radians of the interval	M1	**PI**
	Number of solutions $= 2m$	A1	
		2	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4	$\dfrac{dy}{dx} = \dfrac{1}{4}x^{-\frac{3}{4}}$	M1	Correctly differentiates $x^{\frac{1}{4}}$
	$\dfrac{dy}{dx} = \dfrac{1}{4} \times 81^{-\frac{3}{4}}$ $= \dfrac{1}{108}$	A1	Substitutes $x = 81$ into a correct derivative **PI** May be unsimplified
	$\delta y \approx \dfrac{dy}{dx} \times \delta x$	M1	Sight or use of $\delta y \approx \dfrac{dy}{dx} \times \delta x$ **PI** Condone use of $=$ sign
	$\delta y \approx \dfrac{1}{108} \times -6$	A1ft	Multiplies their $\dfrac{dy}{dx}$ (of the form $ax^{-\frac{3}{4}}$) by -6 (or 6) May be algebraic
	$\delta y \approx -\dfrac{1}{18}$		
	$y \approx 81^{\frac{1}{4}} - \dfrac{1}{18}$	M1	Full method for the required estimate from a derivative of the form $ax^{-\frac{3}{4}}$ **PI**
	$y \approx 2.944$	A1	**CAO** Do not condone $\dfrac{53}{18}$

	Question 4 Total	**6**	

Q	Answer	Marks	Comments
5(a)	$\sum_{r=1}^{n}\left(6r^2-4r+1\right) = 6\sum_{r=1}^{n}r^2 - 4\sum_{r=1}^{n}r + \sum_{r=1}^{n}1$	**M1**	Writes in terms of standard formulae Condone one error **PI**
	$= 6\times\frac{1}{6}n(n+1)(2n+1) - 4\times\frac{1}{2}n(n+1) + n$	**M1**	Replaces $\sum r^2$ with $\frac{1}{6}n(n+1)(2n+1)$ and $\sum r$ with $\frac{1}{2}n(n+1)$
		M1	Replaces $\sum 1$ with n
	$= n\left[(n+1)(2n+1) - 2(n+1) + 1\right]$ $= n\left(2n^2+3n+1-2n-2+1\right)$ $= n\left(2n^2+n\right)$		
	$= n^2(2n+1)$	**A1**	Correct expression following M1M1M1
		4	

Q	Answer	Marks	Comments
5(b)	$\sum_{r=p+1}^{2p}\left(6r^2-4r+1\right)$		
	$=\sum_{r=1}^{2p}\left(6r^2-4r+1\right)-\sum_{r=1}^{p}\left(6r^2-4r+1\right)$	**M1**	Correctly splits the required expression into the difference of two sums of the form $\sum_{r=1}$ **PI**
	$=(2p)^2(4p+1)-p^2(2p+1)$	**m1**	Writes the required expression in terms of p **ft** their part **(a)** in terms of n
	$=p^2(16p+4-2p-1)$		
	$=p^2(14p+3)$	**A1**	
		3	

	Question 5 Total	**7**	

Q	Answer	Marks	Comments
6(a)(i)	$b = -(\alpha + \beta)$	B1	
	$c = \alpha\beta$	B1	
		2	

Q	Answer	Marks	Comments
6(a)(ii)	$[\beta =]\ x - \mathrm{i}y$	B1	
		1	

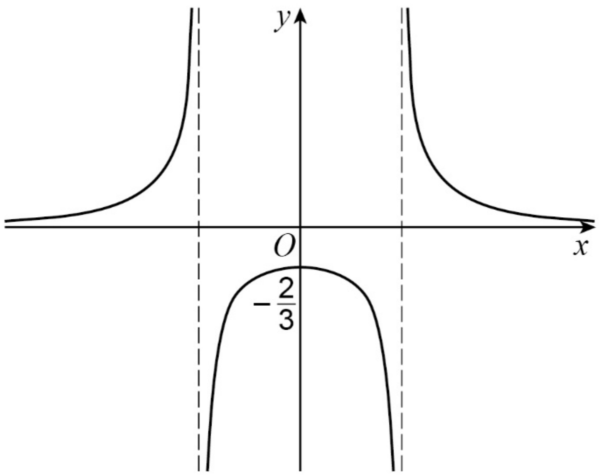
Q	Answer	Marks	Comments
6(b)(i)	Let α and β be $x \pm \mathrm{i}y$ $x + \mathrm{i}y + x - \mathrm{i}y = -6$ $2x = -6$ $x = -3$	M1	Forms a correct equation in x
	Area $= \frac{1}{2} \times 2y \times (8 - x) = 11\sqrt{3}$	M1	Forms a correct equation in y **ft** their x
	$y = \sqrt{3}$	A1	
	$[\alpha =]\ -3 + \mathrm{i}\sqrt{3}$ **and** $[\beta =]\ -3 - \mathrm{i}\sqrt{3}$	A1	
		4	

Q	Answer	Marks	Comments
6(b)(ii)	$\frac{c}{1} = \left(-3 + \mathrm{i}\sqrt{3}\right)\left(-3 - \mathrm{i}\sqrt{3}\right)$	M1	Forms an equation in c **ft** a conjugate pair
	$[c =]\,12$	A1ft	**ft** a conjugate pair
		2	

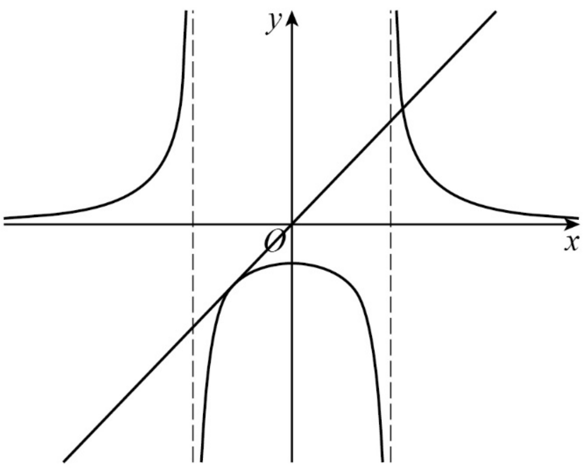
Q	Answer	Marks	Comments
6(b)(iii)	$[r =]\ \sqrt{(-3)^2 + (\sqrt{3})^2}\quad [= \sqrt{12}]$	M1	Correct calculation of the modulus of at least one of their roots
	$[\theta_\alpha =]\ \tan^{-1}\left(\frac{\sqrt{3}}{-3}\right) + \pi\quad \left[= \frac{5\pi}{6}\right]$	M1	Full method for the argument of one of their roots Accept a correct equivalent radian angle outside the interval $-\pi < \theta \le \pi$
	$2\sqrt{3}\left(\cos\left(\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(\frac{5\pi}{6}\right)\right)$	A1	Accept $\sqrt{12}$ for $2\sqrt{3}$
	$2\sqrt{3}\left(\cos\left(-\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(-\frac{5\pi}{6}\right)\right)$	A1	
		4	

	Question 6 Total	13	

Q	Answer	Marks	Comments
7(a)	$x=\sqrt{3}$	**B1**	
	$x=-\sqrt{3}$	**B1**	Award a maximum of **B2** if any errors are included, **eg** an extra asymptote
	$y=0$	**B1**	
		3	

Q	Answer	Marks	Comments
7(b)		**M1**	Any two of: $y\to 0^{+}$ as $x\to\infty$ **or** $y\to 0^{+}$ as $x\to-\infty$ **or** $y\to\infty$ as $x\to\sqrt{3}^{+}$ **or** $y\to\infty$ as $x\to-\sqrt{3}^{-}$ Can be drawn or written Accept implied asymptotic behaviour Accept feathering
		A1	Both sections drawn correctly Do not accept feathering – must be one continuous curve for each of the two sections Must clearly show correct asymptotic behaviour for all four of $x\to\infty$, $x\to-\infty$, $x\to\sqrt{3}^{+}$, $x\to-\sqrt{3}^{-}$
		2	

Q	Answer	Marks	Comments
7(c)	$x=\frac{2}{x^2-3}$		
	$x^3-3x-2=0$	M1	Forms a cubic equation **PI**
	$(x-2)(x^2+2x+1)=0$		
	The other intersection point(s) occur when $x^2+2x+1=0$		
	$x=-1$ is a repeated root	M1	Correct solution (other than $x=2$) of their cubic equation **PI**
	$(-1, -1)$	A1	Correct intersection point Accept $x=-1$ and $y=-1$ unambiguously written as the required coordinates Ignore $(2, 2)$ included
		3	

Q	Answer	Marks	Comments
7(d)		B1	Draws a tangent through the origin, intersecting the curve in the 1st and 3rd quadrants Coordinates of intersection not needed for this mark
		1	

Q	Answer	Marks	Comments
7(e)	$x < -\sqrt{3}$ $x = -1$ $\sqrt{3} < x \leq 2$	B1 B1 B1	Condone $\sqrt{3} < x < 2$ for $\sqrt{3} < x \leq 2$ if only one (or neither) of the previous **B1**s has been awarded Award a maximum of **B2** if any errors are included, **eg** an extra region
		3	

	Question 7 Total	**12**	

Q	Answer	Marks	Comments
8(a)	The integrand is not defined for $x = 0$ [when $n < 0$]	E1	**oe** Condone 'infinite' for 'not defined' Accept 0^n is not defined (or infinite)
		1	

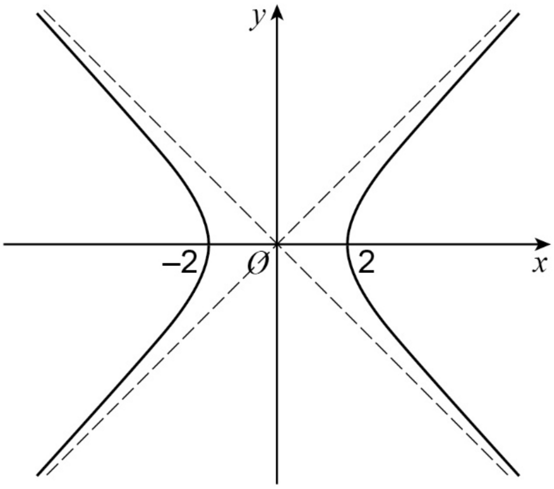
Q	Answer	Marks	Comments
8(b)	$I_{-\frac{3}{4}} = \int_0^4 x^{-\frac{3}{4}} \, dx$		
	$= \lim_{a \to 0^+} \left(\int_a^4 x^{-\frac{3}{4}} \, dx \right)$	M1	Replaces lower limit with a variable
	$= \lim_{a \to 0^+} \left[4x^{\frac{1}{4}} \right]_a^4$	m1	Correct integration and limiting process shown
	$= \lim_{a \to 0^+} \left(4 \times 4^{\frac{1}{4}} - 4 \times a^{\frac{1}{4}} \right)$ $= 4 \times 4^{\frac{1}{4}} - 0$		
	$= 4\sqrt{2}$	A1	**oe** eg $4\sqrt[4]{4}$ **NMS** scores 1 mark out of 3 **SC2** for correct integration and correct answer without use of limiting process Condone $a \to 0$ throughout
		3	

Q	Answer	Marks	Comments
8(c)	Accept any value of n where $n \le -1$ eg $n = -1$ or $n = -\frac{5}{2}$ etc	**B1**	Writes down any number in the range $n \le -1$ Accept a range of numbers within the range $n \le -1$ Condone $-\infty$ or ∞
		1	

	Question 8 Total	**5**	

Q	Answer	Marks	Comments
9(a)	Let $P = (x, y)$ $\sqrt{(x-4)^2 + (y-0)^2}$ or $[\pm][2](x-1)$	B1	A correct distance from P to $(4,0)$ or to $x = 1$ seen or used May be the square of the distance
	$\sqrt{(x-4)^2 + (y-0)^2} = [\pm]2(x-1)$	M1	Forms an equation in x and y using at least one correct distance and a factor of 2 **PI**
	$(x-4)^2 + (y-0)^2 = 4(x-1)^2$	M1	Removes the square root correctly
	$x^2 - 8x + 16 + y^2 = 4(x^2 - 2x + 1)$ $x^2 - 8x + 16 + y^2 = 4x^2 - 8x + 4$ $12 = 3x^2 - y^2$		
	$\frac{x^2}{4} - \frac{y^2}{12} = 1$	A1	
		4	

Q	Answer	Marks	Comments
9(b)	$\frac{x}{2} = \pm \frac{y}{\sqrt{12}}$	M1	Accept one correct asymptote in any form
	$y = \pm x\sqrt{3}$	A1ft	**ft** their $\frac{x^2}{m} - \frac{y^2}{n} = 1$
		2	

Q	Answer	Marks	Comments
9(c)		B1 B1ft	Correct shape Must include the asymptotes Accept asymptotes drawn as continuous lines Correct axis intercepts with no incorrect axis intercepts **ft** their m from part **(a)**
		2	

Q	Answer	Marks	Comments
9(d)	 $y = \pm\, x\sqrt{3} + c$	M1 A1ft	Accept any vertical translation of one of the asymptotes Accept any correct form **ft** their part **(b)** of the form $y = \pm kx$
		2	

	Question 9 Total	**10**	

Q	Answer	Marks	Comments
10(a)	$\alpha\beta = \frac{m}{2}$ or $\alpha^2\beta\beta^2\alpha = \frac{m}{3}$	**B1**	Uses the product of roots rule on at least one equation
	$\left(\frac{m}{2}\right)^3 = \frac{m}{3}$	**M1**	Forms an equation in m only
	$3m^3 = 8m$		
	$m^2 = \frac{8}{3}$ or $m = 0$		
	but $m > 0$, so $m = \frac{2}{3}\sqrt{6}$	**A1**	**oe** eg $\sqrt{\frac{8}{3}}$
		3	

Q	Answer	Marks	Comments
10(b)	$\alpha + \beta = -\frac{1}{2}$ or $\alpha^2\beta + \beta^2\alpha = -\frac{n}{3}$	**B1**	Uses the sum of roots rule on at least one equation May be seen in part **(a)**
	$\alpha\beta(\alpha + \beta) = -\frac{n}{3}$		
	$\frac{m}{2} \times -\frac{1}{2} = -\frac{n}{3}$	**M1**	Forms a correct equation in m and n May be seen in part **(a)** **PI**
	$3m = 4n$		
	$3 \times \frac{2}{3}\sqrt{6} = 4n$	**m1**	Forms an equation in n only
	$n = \frac{1}{2}\sqrt{6}$	**A1**	**oe** eg $\sqrt{\frac{3}{2}}$
		4	

	Question 10 Total	**7**	