

INTERNATIONAL AS FURTHER MATHEMATICS FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

June 2024

Version: 1.0 Final



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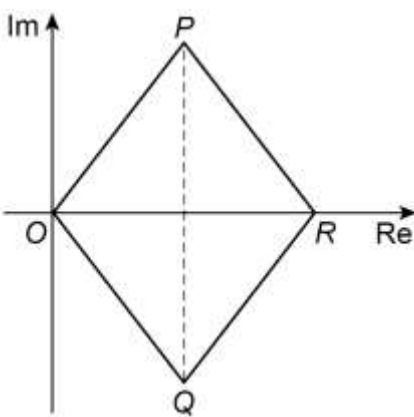
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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$r = \sqrt{2^2 + (\sqrt{5})^2} \quad [= 3]$ $\theta = \tan^{-1}\left(\frac{\sqrt{5}}{2}\right) \quad [= 0.841]$ $z = 3(\cos(0.841) + i \sin(0.841))$	M1 A1	Correct modulus and/or argument May be unsimplified
		2	

Q	Answer	Marks	Comments
1(b)	$z^* = 3(\cos(-0.841) + i \sin(-0.841))$	M1 A1ft	M1: Accept the correct conjugate written in any form. eg $2 - i\sqrt{5}$ or $3(\cos(0.841) - i \sin(0.841))$ A1ft: Answer in the correct form, ft their r and $-\theta$ from part (a)
		2	

Q	Answer	Marks	Comments
1(c)(i)		M1 A1	Points P and Q drawn as reflections of each other in the real axis – mark intention Condone P and Q swapped or to the left of the imaginary axis Correct rhombus
		2	

Q	Answer	Marks	Comments
1(c)(ii)	4	B1	Accept $4 + 0i$
		1	

Q	Answer	Marks	Comments
1(c)(iii)	$\text{area} = \frac{1}{2} \times 4 \times 2\sqrt{5}$ $= 4\sqrt{5}$	M1 A1ft	Full method for the area of $OPRQ$ ft Their part (c)(ii)
		2	

	Question 1 Total	9	
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Q	Answer	Marks	Comments
2(a)	$(4+h)^3 = 4^3 + 3 \times 4^2 \times h + 3 \times 4 \times h^2 + h^3$	M1	At least 3 correct terms May be unsimplified
	$= 64 + 48h + 12h^2 + h^3$	A1	
		2	

Q	Answer	Marks	Comments
2(b)(i)	$(4+h)^3 + 7(4+h)$ $= 64 + 48h + 12h^2 + h^3 + 28 + 7h$ $= 92 + 55h + 12h^2 + h^3$ Gradient of line $= \frac{92 + 55h + 12h^2 + h^3 - (4^3 + 7 \times 4)}{4 + h - 4}$ $= \frac{55h + 12h^2 + h^3}{h}$ $= 55 + 12h + h^2$	M1	Substitutes $4 + h$ into $x^3 + 7x$ and expands May be unsimplified PI
		M1	May be unsimplified
		A1	
		3	

Q	Answer	Marks	Comments
2(b)(ii)	Gradient of curve $= \lim_{h \rightarrow 0} (55 + 12h + h^2)$ $[= 55 + 0 + 0] = 55$	M1 A1ft	Considers their part (b)(i) as $h \rightarrow 0$ Obtains the correct limit of their part (b)(i) as $h \rightarrow 0$ ft their $a + bh + h^2$ SC1 for their 55 following $h = 0$
		2	
	Question 2 Total	7	

Q	Answer	Marks	Comments
3	$\sum_{r=1}^n (ar^3 + br^2) = \frac{a}{4}n^2(n+1)^2 + \frac{b}{6}n(n+1)(2n+1)$ Comparing coefficients of n^4: $\frac{a}{4} = \frac{15}{12}$ Comparing coefficients of n: $\frac{b}{6} = \frac{4}{12}$ $a = 5$ $b = 2$	M1 m1 A1 A1	Correct expression in a, b and n May be unsimplified PI or forms a correct equation in a and/or b independent of n Forms a correct equation in a (or b) only PI or forms a second correct equation in a and b independent of n
	Question 3 Total	4	

[illegible]

Q	Answer	Marks	Comments
6(a)(i)	$\frac{1}{r} - \frac{1}{r+1} = \frac{r+1-r}{r(r+1)}$ $= \frac{1}{r(r+1)}$	B1	Must include at least one intermediate line of working leading to AG
		1	

Q	Answer	Marks	Comments
6(a)(ii)	$\sum_{r=1}^n \frac{1}{r(r+1)} = \sum_{r=1}^n \left(\frac{1}{r} - \frac{1}{r+1} \right)$ $= \frac{1}{1} - \frac{1}{2}$ $+ \frac{1}{2} - \frac{1}{3}$ $+ \dots$ $+ \frac{1}{n-1} - \frac{1}{n}$ $+ \frac{1}{n} - \frac{1}{n+1}$ $= \frac{1}{1} - \frac{1}{n+1}$ $= \frac{n+1-1}{n+1}$ $= \frac{n}{n+1}$	B1 M1 M1 A1	Writes as a sum of $\frac{1}{r} - \frac{1}{r+1}$ Condone omission of brackets PI by correct use of the method of differences Writes at least two pairs of fractions of the form $\frac{1}{r} - \frac{1}{r+1}$ Writes at least three pairs of fractions of the form $\frac{1}{r} - \frac{1}{r+1}$ including the correct first pair, the correct last pair, and at least one other pair ISW
		4	

Q	Answer	Marks	Comments
6(b)	$\sum_{r=1}^{\infty} \frac{1}{r(r+1)} = 1$	B1ft	ft Their part (a)(ii)
		1	

Q	Answer	Marks	Comments
6(c)	$\sum_{r=1001}^{2000} \frac{1}{r(r+1)} = \sum_{r=1}^{2000} \frac{1}{r(r+1)} - \sum_{r=1}^{1000} \frac{1}{r(r+1)}$ $= \frac{2000}{2001} - \frac{1000}{1001}$ $= \frac{1000}{2003001}$	M1 PI A1	Correctly splits the required sum into two sums of the form $\sum_{r=1}^n \frac{1}{r(r+1)}$
		2	

	Question 6 Total	8	
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Q	Answer	Marks	Comments
7(a)	$y = 1$	B1	
		1	

Q	Answer	Marks	Comments
7(b)	$a^2 - 4 \times 1 \times 3 < 0$ $a^2 < 12$	M1 A1	Considers the discriminant of the denominator Inequality not required for this mark
		2	

Q	Answer	Marks	Comments
7(c)	$k = \frac{x^2}{x^2 + ax + 3}$ $kx^2 + kax + 3k = x^2$ $(k-1)x^2 + kax + 3k = 0$ No intersection points, so $(ka)^2 - 4(k-1)3k < 0$ $k^2a^2 - 12k^2 + 12k < 0$ $k^2(12 - a^2) - 12k > 0$	M1 M1 M1 A1	Forms a quadratic equation in x in terms of k Correctly applies the discriminant to their quadratic in x Inequality not required for this mark Sets their discriminant < 0 AG Must be convincingly shown
		4	

Q	Answer	Marks	Comments
7(d)	Stationary points occur when $y^2(12-5)-12y = 0$ $y(7y-12) = 0$ $y = 0 \quad \text{or} \quad y = \frac{12}{7}$	M1	Forms an equation in y Accept k instead of y
		A1	At least one correct y -coordinate Condone k instead of y
		A1	Both y -coordinates correct and no incorrect y -coordinates
		3	
	Question 7 Total	10	

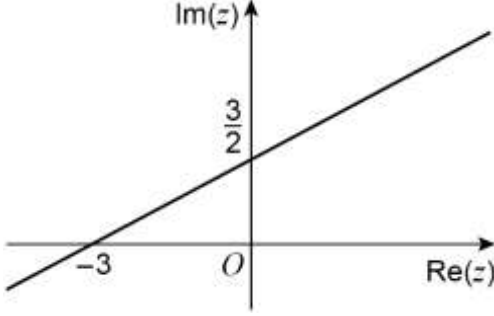
Q	Answer	Marks	Comments
8(a)	$(\alpha + \beta)^4 = \alpha^4 + 4\alpha^3\beta + 6\alpha^2\beta^2 + 4\alpha\beta^3 + \beta^4$	M1 A1	M1: At least three correct terms May be unsimplified A1: All correct
		2	

Q	Answer	Marks	Comments
8(b)	$(\alpha + \beta)^4 - 4\alpha^3\beta - 6\alpha^2\beta^2 - 4\alpha\beta^3 = \alpha^4 + \beta^4$ $\alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6(\alpha\beta)^2$ $= (\alpha + \beta)^4 - 4\alpha\beta((\alpha + \beta)^2 - 2\alpha\beta) - 6(\alpha\beta)^2$ $\alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha + \beta)^2 + 2(\alpha\beta)^2$	M1 M1 A1	Rearranges to write $\alpha^4 + \beta^4$ in terms of $\alpha + \beta$ and $\alpha\beta$ and $\alpha^2 + \beta^2$ PI Replaces $\alpha^2 + \beta^2$ with $(\alpha + \beta)^2 - 2\alpha\beta$ PI ACF eg $\left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$
		3	

Q	Answer	Marks	Comments
8(c)(i)	$\alpha + \beta = \frac{1}{2}$	B1	
	$\alpha\beta = 3$	B1	
		2	

Q	Answer	Marks	Comments
8(c)(ii)	$\text{New sum} = \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\beta^2 \alpha^2}$ $= \frac{(\alpha + \beta)^4 - 4\alpha\beta(\alpha + \beta)^2 + 2(\alpha\beta)^2}{(\alpha\beta)^2}$ $= \frac{\left(\frac{1}{2}\right)^4 - 4 \times 3 \times \left(\frac{1}{2}\right)^2 + 2 \times 3^2}{3^2} = \frac{241}{144}$ $\text{New product} = \frac{\alpha^2}{\beta^2} \times \frac{\beta^2}{\alpha^2} = \frac{\alpha^2 \beta^2}{\beta^2 \alpha^2} = 1$ [New equation is] $x^2 - \frac{241}{144}x + 1 [= 0]$ $144x^2 - 241x + 144 = 0$	M1 Writes the new sum in terms of $\alpha + \beta$ and $\alpha\beta$ ft Their $\alpha^4 + \beta^4$ PI Accept $\frac{(\alpha + \beta)^2 - 2\alpha\beta - 12(\alpha + \beta) + 72}{\alpha\beta - 6(\alpha + \beta) + 36}$ for roots $\frac{\alpha - 6}{\beta - 6}$ and $\frac{\beta - 6}{\alpha - 6}$ A1 PI B1 PI M1 ft Their new sum and new product A1 Accept any integer multiple	5
	Question 8 Total	12	

Q	Answer	Marks	Comments
9(a)	$\frac{1}{2}(2 + 0i + 0 + 4i) = 1 + 2i$ $c = 2$	B1	
		1	

Q	Answer	Marks	Comments
9(b)	gradient of L is $-1 \div \left(-\frac{4}{2}\right) = \frac{1}{2}$ 	B1 B1 B1 B1	Correct calculation for the gradient of L PI by axis intercepts Straight line through the 1st, 2nd and 3rd quadrants Correct imaginary axis intercept Condone $\frac{3}{2}i$ Correct real axis intercept
		4	

Q	Answer	Marks	Comments
9(c)(i)	$a = -(-3 + 0i) = 3$	B1ft	ft Their real intercept
		1	

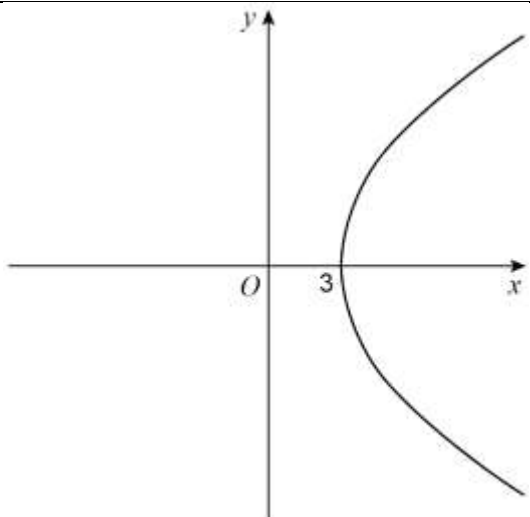
Q	Answer	Marks	Comments
9(c)(ii)	$b = \frac{1.5}{3} = \frac{1}{2}$	B1ft	ft Their axis intercepts or their gradient if stated
		1	

	Question 9 Total	7	
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Q	Answer	Marks	Comments
10(a)	$(x-0)^2 + (y-9)^2$ or $[\pm](y+3)$	B1	Writes a correct distance from P to $(0,9)$ or to $y = -3$ Seen or used
	$\sqrt{(x-0)^2 + (y-9)^2} = \pm(y+3)$	M1	Forms an equation in x and y using their distances PI
	$(x-0)^2 + (y-9)^2 = (y+3)^2$	M1	Removes the square root correctly
	$x^2 + y^2 - 18y + 81 = y^2 + 6y + 9$	A1	If B0M0M0 then award SC2 for $x^2 = 24y + b$ or $x^2 = ay - 72$ for non-zero a and b
	$x^2 = 24y - 72$		
		4	

Q	Answer	Marks	Comments
10(b)(i)	$y^2 = 24x - 72$	B1ft	oe ft Their $x^2 = ay + b$ for non-zero a and b
		1	

Q	Answer	Marks	Comments
10(b)(ii)		B1	Correct shape, symmetrical about the x-axis

		B1ft	Correct x-axis intercept ft Their $-\frac{b}{a}$
		2	
Q	Answer	Marks	Comments
10(b)(iii)	The line meets C_2 when $(mx)^2 = 24x - 72$ $m^2x^2 - 24x + 72 = 0$ Two intersection points, so $(-24)^2 - 4m^2 \times 72 > 0$ $576 > 288m^2$ $m^2 < 2$ $-\sqrt{2} < m < 0, \quad 0 < m < \sqrt{2}$	M1 A1 M1 A1 A1	Forms a quadratic equation in x (or y) in terms of m Correct 3-term quadratic equation in x in terms of m equal to zero [$= 0$ can be implied] Correctly substitutes their quadratic coefficients into $b^2 - 4ac > 0$ PI by correct final inequalities
		5	

Q	Answer	Marks	Comments
10(b)(iv)	$y = mx$ is a tangent when $m^2 = 2$ $y = x\sqrt{2}$ and $y = -x\sqrt{2}$	M1 A1ft	Replaces the inequality with equals PI By one correct tangent (allow ft) ft Their $m^2 = 2$
		2	

	Question 10 Total	14	
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