# OXFORD AQA
INTERNATIONAL QUALIFICATIONS

# INTERNATIONAL AS
# FURTHER MATHEMATICS
# FM01

(9665/FM01)  Unit FP1  Pure Mathematics

Mark scheme

January 2025

Version: 1.0 Final

251XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from www.oxfordaqa.com

**Key to mark scheme abbreviations**

| | |
|---|---|
| **M** | Mark is for method |
| **m** | Mark is dependent on one or more M marks and is for method |
| **A** | Mark is dependent on M or m marks and is for accuracy |
| **B** | Mark is independent of M or m marks and is for method and accuracy |
| **E** | Mark is for explanation |
| **√ or ft** | Follow through from previous incorrect result |
| **CAO** | Correct answer only |
| **CSO** | Correct solution only |
| **AWFW** | Anything which falls within |
| **AWRT** | Anything which rounds to |
| **ACF** | Any correct form |
| **AG** | Answer given |
| **SC** | Special case |
| **oe** | Or equivalent |
| **A2, 1** | 2 or 1 (or 0) accuracy marks |
| **−$x$ EE** | Deduct $x$ marks for each error |
| **NMS** | No method shown |
| **PI** | Possibly implied |
| **SCA** | Substantially correct approach |
| **sf** | Significant figure(s) |
| **dp** | Decimal place(s) |
| **ISW** | Ignore subsequent working |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **1(a)** | $8 + 5i$ | **B1** | oe |
|  |  | **1** |  |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **1(b)** | $-b = 8 - 5i + 8 + 5i$ <br> $c = (8 - 5i)(8 + 5i)$ | **M1** | Forms an equation in $b$ (or $c$) only <br> eg $-\dfrac{b}{2} = 8$ <br> or <br> Writes the sum of the roots or the product of the roots <br> May be seen in an expansion of <br> $(z - (8 - 5i))(z - (8 + 5i))$ |
|  | $b = -16$ | **A1** |  |
|  | $c = 89$ | **A1** |  |
|  |  | **3** |  |

| | | Marks | |
|---|---|---|---|
| | **Question 1 Total** | **4** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 2(a) | When $x = a$, $y = 5a^2 - 4a$ <br><br> When $x = a+h$, $y = 5(a+h)^2 - 4(a+h)$ <br><br> $y = 5a^2 + 10ah + 5h^2 - 4a - 4h$ <br><br> Gradient <br> $= \dfrac{5a^2 + 10ah + 5h^2 - 4a - 4h - \left(5a^2 - 4a\right)}{a+h-a}$ <br><br> $= \dfrac{10ah + 5h^2 - 4h}{h}$ <br><br> $= 10a + 5h - 4$ | M1 <br><br><br> M1 <br><br><br><br> A1 | Writes a correct expression for the $y$-coordinate when $x = a+h$ <br><br> Writes a correct expression for the gradient of the line <br><br><br> **oe** |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| 2(b) | Let $a = 3$ and $h \to 0$ <br><br><br><br> Gradient $= 10 \times 3 + 5 \times 0 - 4$ <br> $= 26$ | M1 <br><br><br><br> A1 | Considers $h \to 0$ <br> Condone $h = 0$ seen <br> **PI** by a correctly evaluated part **(a)** using $h = 0$ <br><br> Must see **M1** <br> Condone poor notation and poor bracket use <br> **A0** if $h = 0$ seen |
| | | **2** | |

| | | | |
|---|---|---|---|
| | **Question 2 Total** | **5** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 3 | $\dfrac{5+i}{7-3i} = \dfrac{(5+i)(7+3i)}{(7-3i)(7+3i)}$ | M1 | Multiplies numerator and denominator by the conjugate of $7-3i$ |
| | $= \dfrac{35+15i+7i-3}{49+21i-21i+9}$ | M1 | Correct numerator **or** denominator with an $i^2$ replaced with $-1$<br>May be simplified or unsimplified |
| | $= \dfrac{32+22i}{58}$ | | |
| | $= \dfrac{16+11i}{29}$ | A1 | Must include at least one intermediate fraction after the first **M1** |

| | | | |
|---|---|---|---|
| | **Question 3 Total** | **3** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 4(a)(i) | $\dfrac{7}{3}$ | B1 | oe |
| | | 1 | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 4(a)(ii) | $\dfrac{c}{3}$ | B1 | oe |
| | | 1 | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 4(b)(i) | $\left(\alpha + \dfrac{5}{\beta}\right)\left(\beta + \dfrac{5}{\alpha}\right) = \alpha\beta + 5 + 5 + \dfrac{25}{\alpha\beta}$ | M1 | Correctly writes $\left(\alpha + \dfrac{5}{\beta}\right)\left(\beta + \dfrac{5}{\alpha}\right)$ in terms of $\alpha\beta$ <br><br> **PI** by a correct substitution of their $\alpha\beta$ |
| | $\dfrac{c}{3} + 10 + \dfrac{75}{c}$ | A1 | **oe** expression with no fractions within the numerator or denominator |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 4(b)(ii) | $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$ <br><br> $= \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ <br><br> $= \dfrac{\left(\dfrac{7}{3}\right)^2 - 2\left(\dfrac{c}{3}\right)}{\dfrac{c}{3}}$ | M1 | Correctly writes $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ in terms of $\alpha + \beta$ and $\alpha\beta$ <br> **PI** by a correct substitution of their $\alpha + \beta$ and $\alpha\beta$ |
| | $= \dfrac{49 - 6c}{3c}$ | A1 | **oe** expression with no fractions within the numerator or denominator |
| | | 2 | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **4(b)(iii)** | $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ <br><br> $= \left(\dfrac{7}{3}\right)^3 - 3\left(\dfrac{c}{3}\right)\left(\dfrac{7}{3}\right)$ <br><br> $= \dfrac{343}{27} - \dfrac{7c}{3}$ | **M1** <br><br><br><br><br><br> **A1** | Correctly writes $\alpha^3 + \beta^3$ in terms of $\alpha + \beta$ and $\alpha\beta$ <br><br> **PI** by a correct substitution of their $\alpha + \beta$ and $\alpha\beta$ <br><br><br> **oe** expression |
| | | **2** | |

| | | | |
|---|---|---|---|
| | **Question 4 Total** | **8** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 5(a) | $\tan^{-1}(1) = \dfrac{\pi}{4}$ | M1 | Correctly calculates $\tan^{-1}(1)$<br>**PI** by a correct solution eg $-\dfrac{\pi}{12}$ |
| | $x + \dfrac{\pi}{3} = n\pi + \dfrac{\pi}{4}$   where  $n \in \mathbb{Z}$ | M1 | Correct non-trigonometric general equation in $x$<br>Condone $n$ (or eg $k$) not defined |
| | $[x =] n\pi - \dfrac{\pi}{12}$ | A1 | **oe** eg $x = n\pi + \dfrac{11\pi}{12}$<br>Accept $\pm n\pi - \dfrac{\pi}{12}$ |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 5(b) | $\left(-\dfrac{13\pi}{12}\right) + \left(-\dfrac{\pi}{12}\right) + \left(\dfrac{11\pi}{12}\right) + \left(\dfrac{23\pi}{12}\right)$ | M1 | Identifies three correct roots<br>May be unsimplified |
| | $= \dfrac{10\pi}{12} \times 2$ | | |
| | $= \dfrac{5\pi}{3}$ | A1 | |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 5(c) | $\dfrac{10\pi}{12} \times m = 5\pi$ | M1 | Forms correct equation in $m$<br>**ft** their part **(b)** |
| | $m = 6$ | A1ft | **ft** their part **(b)** if $m > 0$ |
| | | **2** | |

| | | | |
|---|---|---|---|
| | **Question 5 Total** | **7** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 6(a) | $x^2 + 2xy\mathrm{i} - y^2 = 45 - 28\mathrm{i}$<br><br>Comparing imaginary parts:<br>$2xy = -28$ | M1 | Expands LHS and compares imaginary parts.<br>Ignore sign errors.<br>Accept $xy = -14$ (or $xy + 14 = 0$) for this mark. |
|  | $y = -\dfrac{14}{x}$ | A1 | **AG** Must be convincingly shown.<br>If $xy = -14$ (or $xy + 14 = 0$) is seen instead of $2xy = -28$ (or $2xy + 28 = 0$), then either $x^2 - y^2 = 45$ must also be seen or a statement that imaginary parts have been compared. |
|  |  | **2** |  |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 6(b) | Comparing real parts:<br>$x^2 - y^2 = 45$<br><br>$k = 45$ | B1 |  |
|  |  | **1** |  |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 6(c) | $x^2 - \left(-\dfrac{14}{x}\right)^2 = 45$ | M1 | Forms a correct equation in $x$ (or $y$) only<br>**ft** their part **(b)** |
|  | $x^4 - 45x^2 - 196 = 0$<br><br>$x^2 = 49$ | A1 | Correct value for $x$ or $y$ or $x^2$ or $y^2$<br>**PI** by a correct root |
|  | $y^2 = 4$ | M1 | Substitutes their value of $x$ or $x^2$ to find a value of $y$ or $y^2$ (or vice versa) |
|  | $z = 7 - 2\mathrm{i}$ , $z = -7 + 2\mathrm{i}$ | A1 | **oe**<br>Accept $\pm 7 \mp 2\mathrm{i}$ but not $\pm 7 \pm 2\mathrm{i}$ |
|  |  | **4** |  |

|  |  |  |  |
|---|--------|-------|----------|
|  | **Question 6 Total** | **7** |  |

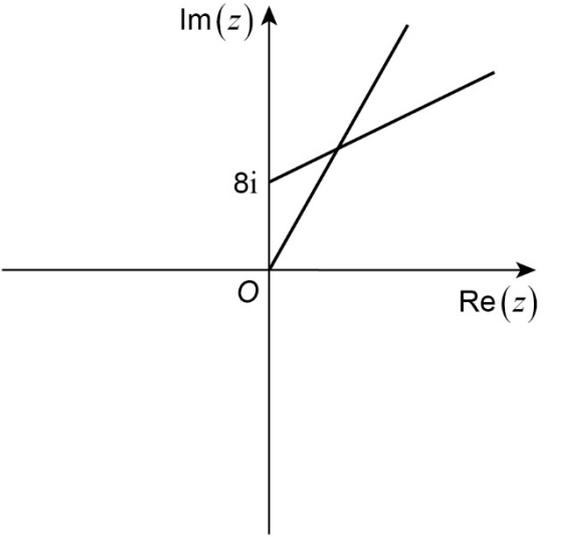| Q | Answer | Marks | Comments |
|---|---|---|---|
| **7(a)** | $\log\left(1+\dfrac{2}{r}\right) = \log\left(\dfrac{r+2}{r}\right)$ $= \log(r+2) - \log(r)$ | **B1** | Must include at least one intermediate stage eg $\log\left(\dfrac{r+2}{r}\right)$ or $\log\left(\dfrac{Ar+B}{r}\right)$ Accept RHS shown equal to $\log\left(A+\dfrac{B}{r}\right)$ with $A=1$ and $B=2$ given |
| | | **1** | |

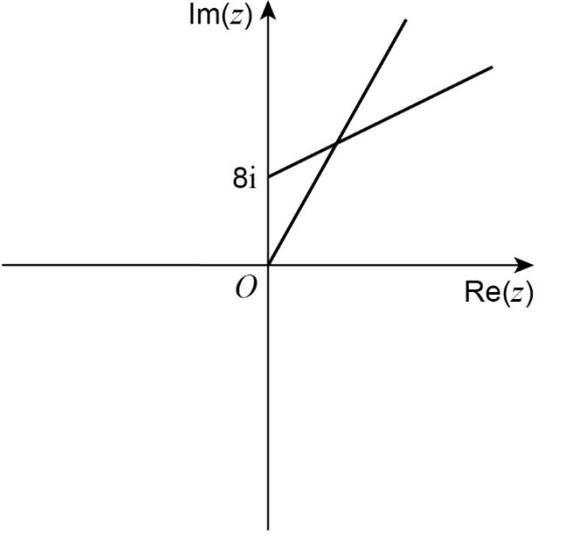| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **7(b)** | $\displaystyle\sum_{r=1}^{2n}\log\left(1+\frac{2}{r}\right)=\sum_{r=1}^{2n}\left(\log(r+2)-\log(r)\right)$ | **M1** | Writes at least 3 pairs of logs of the form $\log(r+2)-\log(r)$ |
| | $=\log(3)-\log(1)$ $+\log(4)-\log(2)$ $+\log(5)-\log(3)$ $+\ldots$ $+\log(2n)-\log(2n-2)$ $+\log(2n+1)-\log(2n-1)$ $+\log(2n+2)-\log(2n)$ | **M1** | Includes at least one pair of cancelling logs, eg $\log(3)$ and $-\log(3)$ **PI** by all four terms of $\log(2n+2)+\log(2n+1)-\log2-\log1$ |
| | $=\log(2n+2)+\log(2n+1)-\log(2)-\log(1)$ $=\log\left(\dfrac{2n+2}{2}\right)+\log(2n+1)$ | **A1** | Correctly expresses the required sum in terms of no more than 4 logs |
| | $=\log(n+1)+\log(2n+1)$ | **A1** | **CAO** with M1M1 awarded **SC1** $\log\left(\dfrac{3}{1}\times\dfrac{4}{2}\times\dfrac{5}{3}\times\ldots\times\dfrac{2n}{2n-2}\times\dfrac{2n+1}{2n-1}\times\dfrac{2n+2}{2n}\right)$ **SC2** $\log\left(\dfrac{3}{1}\times\dfrac{4}{2}\times\dfrac{5}{3}\times\ldots\times\dfrac{2n}{2n-2}\times\dfrac{2n+1}{2n-1}\times\dfrac{2n+2}{2n}\right)$ $=\log\left(\dfrac{2n+1}{1}\times\dfrac{2n+2}{2}\right)$ **SC3** $\log\left(\dfrac{3}{1}\times\dfrac{4}{2}\times\dfrac{5}{3}\times\ldots\times\dfrac{2n}{2n-2}\times\dfrac{2n+1}{2n-1}\times\dfrac{2n+2}{2n}\right)$ $=\log\left(\dfrac{2n+1}{1}\times\dfrac{2n+2}{2}\right)$ $=\log(2n+1)+\log(n+1)$ |
| | | **4** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **7(c)** | $$\sum_{r=1}^{1200} \log\left(1+\frac{2}{r}\right) = \log(600+1) + \log(2\times600+1)$$ | M1 | Substitutes $n = 600$ into their part **(b)** of the correct form<br>Allow $\log(Cn+D) + \log(En+\mathrm{F})$ for non-integer $C, D, E, F$ |
| | $= \log(601\times1201)$ | A1 | **ACF**<br>**PI** by correct answer |
| | $= \log(721801)$ | A1 | May follow any correct expression in part **(b)** eg $\log(2n+2) + \log\left(n+\frac{1}{2}\right)$ |
| | | 3 | |

| | | | |
|---|---|---|---|
| | **Question 7 Total** | **8** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 8(a) | $y = -1$ | B1 | Correct horizontal asymptote |
| | $4 + 3x - x^2 = 0$ | M1 | Attempts to solve denominator = 0 **PI** by one correct vertical asymptote |
| | $\Rightarrow \quad x = -1 \ , \ x = 4$ | A1 | Correct vertical asymptotes Maximum 2 marks if any incorrect asymptotes |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 8(b) | $k = \dfrac{x^2}{4 + 3x - x^2}$ $k\left(4 + 3x - x^2\right) = x^2$ | | |
| | | M1 | Forms a quadratic equation of the form $ax^2 + bx + c = 0$ with coefficients in terms of $k$ |
| | $(k+1)x^2 - 3kx - 4k = 0$ | A1 | Correct quadratic equation in $x$ with coefficients in terms of $k$ Condone missing = 0 **PI** by correct further work |
| | | M1 | Correct discriminant in terms of $k$ for their quadratic equation **PI** by $\left(-3k\right)^2 \geq 4\left(k+1\right)\left(-4k\right)$ |
| | At least one real root, so $\left(-3k\right)^2 - 4\left(k+1\right)\left(-4k\right) \geq 0$ $25k^2 + 16k \geq 0$ | M1 | Sets their discriminant $\geq 0$ Accept $> 0$ **PI** by a correct range for their quadratic equation |
| | $k \leq -\dfrac{16}{25} \quad$ or $\quad k \geq 0$ | A1 | Accept $<$ for $\leq$ and $>$ for $\geq$ **ACF** eg $k \in \left(-\infty, -0.64\right] \cup \left[0, \infty\right)$ |
| | | **5** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **8(c)** | Let $k = -1$, so $3x + 4 = 0$ | **M1** | Equates $\dfrac{x^2}{4 + 3x - x^2}$ with their $y$-value from part **(a)** <br> **PI** by a correct **ft** $x$-value |
| | $\Rightarrow \quad x = -\dfrac{4}{3}$ <br><br> Required point is $\left(-\dfrac{4}{3}, -1\right)$ | **A1** | Accept $x = -\dfrac{4}{3}$ and $y = -1$ if unambiguous |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **8(d)** |  | **B1** | Correct RHS and correctly approaching the asymptotes |
| | | **M1** | Maximum point to the left of the left-hand vertical asymptote, above the horizontal asymptote and below the $x$-axis |
| | | **A1** | Correct LHS and correctly approaching the asymptotes |
| | | **3** | |

| | | | |
|---|---|---|---|
| | **Question 8 Total** | **13** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **9(a)(i)** |  | **M1** | Line from (or through) *O* into the 1st quadrant |
| | | **A1** | Half-line from *O* into 1st quadrant at a steeper gradient than their other line<br><br>If no other line is drawn then it must be at an angle of more than 45° from the real axis<br><br>Condone an unruled line. Mark intention. |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| **9(a)(ii)** |  | **M1** | Line from (or through) the positive imaginary axis into the 1st quadrant at a positive angle from the real axis |
| | | **A1** | Half-line from 8i into 1st quadrant at a shallower gradient than their other line<br><br>If no other line is drawn then it must be at an angle of less than 45° from the real axis<br>Accept the imaginary intercept labelled as 8. Condone $(0, 8)$<br><br>Condone an unruled line. Mark intention. |
| | | **2** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(b)** | $\dfrac{r}{\sin\left(\dfrac{2\pi}{3}\right)} = \dfrac{8}{\sin\left(\dfrac{\pi}{6}\right)}$ | **M1** | Forms a correct equation in the modulus (or the real or imaginary part) only |
| | $r = 8\sqrt{3}$ | **A1** | Calculates the correct value of the modulus (or the real or imaginary part) |
| | Required complex number is $8\sqrt{3}\left(\cos\left(\dfrac{\pi}{3}\right) + i\sin\left(\dfrac{\pi}{3}\right)\right)$ | | |
| | $= 4\sqrt{3} + 12i$ | **A1** | **oe** eg $\sqrt{48} + i12$ |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(c)(i)** | $8i - \left(4\sqrt{3} + 12i\right)$ | **M1** | Full method for $R$ or at least one correct part |
| | $= -4\sqrt{3} - 4i$ | **A1** | **oe** eg $-\sqrt{48} - i4$ |
| | | **2** | |

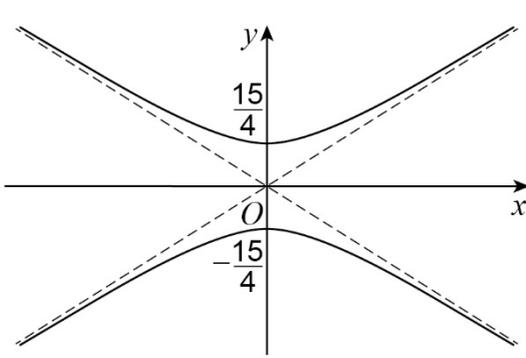| Q | Answer | Marks | Comments |
|---|---|---|---|
| **9(c)(ii)** | Area of triangle $OPQ$ $= \dfrac{1}{2} \times 8 \times 8\sqrt{3} \times \sin\left(\dfrac{\pi}{6}\right)$ | **M1** | Full method for area of parallelogram $OPQR$ or triangle $OPQ$ (or $ORQ$) |
| | $= 16\sqrt{3}$ | | |
| | Area of parallelogram $OPQR$ $= 32\sqrt{3}$ | **A1** | **oe** eg $\sqrt{3072}$ |
| | | **2** | |

| | | | |
|---|---|---|---|
| | **Question 9 Total** | **11** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 10(a) | $\sqrt{(x-0)^2 + (y-5)^2} = \pm 3(y+5)$ | M1 | Correct distance from $P$ to $(0,5)$ or correct distance from $P$ to $y = -5$ **PI** by $x^2 + (y-5)^2 = 3^2(y+5)^2$ |
| | | M1 | Forms a correct equation in terms of $x$ and $y$ |
| | $x^2 + y^2 - 10y + 25 = 9(y^2 + 10y + 25)$ | M1 | Correctly removes square root **PI** by $x^2 + (y-5)^2 = 3^2(y+5)^2$ |
| | $x^2 = 8y^2 + 100y + 200$ | A1 | **CAO** with M1M1M1 awarded Condone $y - -5$ in the place of $\pm(y - -5)$ |
| | | **4** | |

| 10(b)(i) | $x^2 = 8\left(y^2 + \dfrac{25}{2}y\right) + 200$ $x^2 = 8\left(\left(y + \dfrac{25}{4}\right)^2 - \dfrac{625}{16}\right) + 200$ | M1 | Calculates $\dfrac{100}{2 \times 8}$ Accept as part of an expression Accept $-\dfrac{100}{2 \times 8}$ |
|---|---|---|---|
| | Translation vector is $\begin{bmatrix} 0 \\ \dfrac{25}{4} \end{bmatrix}$ | M1 | Indicates the translation is parallel to the $y$-axis eg writes a vector of the form $\begin{bmatrix} 0 \\ y \end{bmatrix}$ for any non-zero $y$ |
| | | A1 | **CAO** |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|--------|-------|----------|
| 10(b)(ii) | $x^2 = 8\left(y + \dfrac{25}{4}\right)^2 - \dfrac{625}{2} + 200$ $b = -\dfrac{225}{2}$ | B1ft | **ft** Their $a$ value |
| | | **1** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **10(b)(iii)** | $\dfrac{225}{2} = 8y^2 - x^2$ | | |
| | $1 = \dfrac{16y^2}{225} - \dfrac{2x^2}{225}$ | **M1** | Writes the equation of $H_2$ in the form $py^2 - qx^2 = 1$ (or $px^2 - qy^2 = 1$ if their $b$ is positive) where $p$ and $q$ are positive <br> **PI** by a correct asymptote <br> **ft** their part **(b)(ii)** |
| | Asymptotes of $H_2$ are <br> $\dfrac{16y^2}{225} = \dfrac{2x^2}{225}$ | **M1** | Writes at least one asymptote equation for their $py^2 - qx^2 = 1$ (or $px^2 - qy^2 = 1$ if their $b$ is positive) <br> Award M1M1 for $x^2 = 8y^2$ oe |
| | $y = \pm \dfrac{x}{4}\sqrt{2}$ | **A1** | |
| | | **3** | |

| Q | Answer | Marks | Comments |
|---|---|---|---|
| **10(b)(iv)** |  | **B1ft** | Correct shape for their $b$ |
| | | **B1ft** | Two asymptotes drawn through the origin with correct curve behaviour relative to the asymptotes <br> **ft** Their $b$ |
| | | **B1** | Correct $y$-intercepts |
| | | **3** | |

| | | | |
|---|---|---|---|
| | **Question 10 Total** | **14** | |