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FURTHER MATHEMATICS

FM01

(9665/FM01) Unit FP1 Pure Mathematics

Mark scheme

June 2025

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256XFM01/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$(5+a\mathrm{i})a = (7+b\mathrm{i})(1+\mathrm{i})$		
		M1	Rewrites the equation to a form allowing parts comparison
	$5a + a^2\mathrm{i} = 7 + 7\mathrm{i} + b\mathrm{i} - b$	B1	Replaces i^2 with -1
	$5a = 7 - b$ and $a^2 = 7 + b$	m1	Equates real **or** imaginary parts to form at least one correct equation in a and b
	$a^2 + 5a - 14 = 0$		
	$a = -7$ or $a = 2$ $b = 42$ or $b = -3$	A1	Obtains at least one correct value of a or b
	$a = -7$ and $b = 42$	A1	Obtains the correct values of a and b only

	Question 1 Total	**5**	

Q	Answer	Marks	Comments
2	$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1	Differentiates to find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$ or $\frac{\mathrm{d}r}{\mathrm{d}V}$ **PI** eg $\frac{\mathrm{d}V}{\mathrm{d}r} = 217$
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$	M1	Writes a correct chain rule linking volume, radius and time. **PI** by a correct equation linking $\frac{\mathrm{d}r}{\mathrm{d}t}$ and r
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi r^2} \times 12$	m1	Correctly substitutes into a correct chain rule
	$300 = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{225}{\pi}}$	M1	Rearranges $V = \frac{4}{3}\pi r^3$ to find the radius in terms of the volume, or the radius when $V = 300$, eg $r = 4.15$
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{\pi\left(\sqrt[3]{\frac{225}{\pi}}\right)^2}$		
	$\left[\frac{\mathrm{d}r}{\mathrm{d}t}\right] = 0.0554\ \text{cm s}^{-1}$ (3 sf)	A1	Obtains the correct value Condone missing units Accept **AWRT** 0.0554

	Question 2 Total	**5**	

Q	Answer	Marks	Comments
3(a)		B1	Obtains a correct angle for $2x+\frac{\pi}{3}$
	$2x+\frac{\pi}{3} = n\pi+(-1)^n\frac{\pi}{4} \qquad n\in\mathbf{Z}$	M1	Writes a non-trigonometric form of the given equation eg $\frac{\pi}{2}-\left(2x+\frac{\pi}{3}\right) = 2n\pi\pm\frac{\pi}{4}$ May be seen as a pair of equations
	$2x = n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{3}$	M1	Correctly rearranges their non-trigonometric equation to make x the subject **PI** by $n\pi-\frac{\pi}{24}$ or $n\pi+\frac{5\pi}{24}$
	$[x=]\ \frac{n\pi}{2}+(-1)^n\frac{\pi}{8}-\frac{\pi}{6}$	A1	Obtains a correct general expression (or pair of expressions) for x eg $n\pi+\frac{\pi}{12}\pm\frac{\pi}{8}$ Condone a missing definition of n Accept two separate expressions eg $n\pi-\frac{\pi}{24}$, $n\pi+\frac{5\pi}{24}$ eg $\frac{n\pi}{2}-\frac{7\pi}{24}$ (n odd) , $\frac{n\pi}{2}-\frac{\pi}{24}$ (n even)
		4	

Q	Answer	Marks	Comments
3(b)	$x = \frac{5\pi}{24}, \frac{23\pi}{24}, \frac{29\pi}{24}, \frac{47\pi}{24}$	M1	Identifies at least two correct positive solutions. May be unsimplified.
	sum $= \frac{13\pi}{3}$	A1	**ACF** Obtains the correct sum
		2	

	Question 3 Total	6	

Q	Answer	Marks	Comments
4(a)	$\alpha + \beta = -4$ and $\alpha\beta = c$	B1	Writes/uses a correct value of $\alpha + \beta$
		B1	Writes/uses a correct value of $\alpha\beta$
	$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ $\alpha^3 + \beta^3 = (-4)^3 - 3c(-4)$	M1	Writes/uses a correct expression for $\alpha^3 + \beta^3$ in terms of $\alpha + \beta$ and $\alpha\beta$
	$= 12c - 64$	A1	Obtains the correct simplified expression for $\alpha^3 + \beta^3$
		4	

Q	Answer	Marks	Comments
4(b)(i)	sum = $\alpha^2-\beta+\beta^2-\alpha$	**M1**	Writes any expression for the sum (or $-$sum)
	$=\alpha^2+\beta^2-(\alpha+\beta)$		
	$=(\alpha+\beta)^2-2\alpha\beta-(\alpha+\beta)$	**m1**	Writes/uses a correct expression for the sum (or $-$sum) in terms of $\alpha+\beta$ and $\alpha\beta$
	$=(-4)^2-2c-(-4)$ $=20-2c$		
	$p=2c-20$	**A1**	Obtains the correct simplified expression for p
		3	

Q	Answer	Marks	Comments
4(b)(ii)	product = $(\alpha^2-\beta)(\beta^2-\alpha)$	**M1**	Writes any correct expression for the product
	$=(\alpha\beta)^2-(\alpha^3+\beta^3)+\alpha\beta$	**m1**	Writes/uses a correct expression for the product in terms of $\alpha+\beta$ and $\alpha\beta$ and $\alpha^3+\beta^3$ **ft** Their $\alpha^3+\beta^3$ from **part (a)** Accept $\alpha^2\beta^2$ for $(\alpha\beta)^2$
	$=c^2-(12c-64)+c$		
	$q=c^2-11c+64$	**A1**	Obtains the correct simplified expression for q
		3	

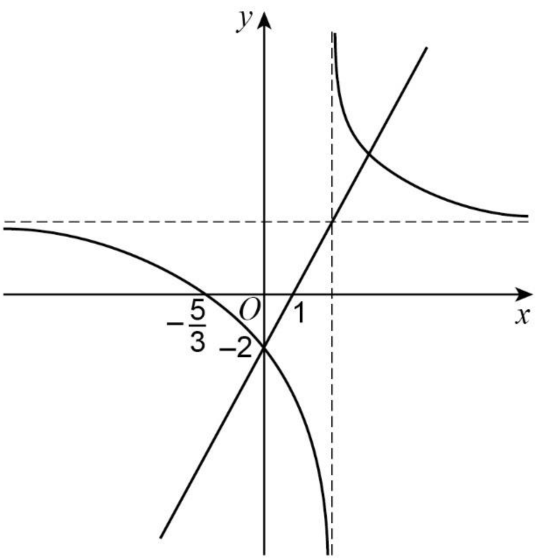
	Question 4 Total	**10**	

Q	Answer	Marks	Comments
5(a)	$\sum_{r=1}^{n}(3r^2-3r+1) = 3\sum_{r=1}^{n}r^2 - 3\sum_{r=1}^{n}r + \sum_{r=1}^{n}1$	**M1**	Writes the expression in terms of $\sum r^2$, $\sum r$ and $\sum 1$
	$= 3\times\frac{1}{6}n(n+1)(2n+1) - 3\times\frac{1}{2}n(n+1) + n$	**M1**	Replaces $\sum r^2$ with $\frac{1}{6}n(n+1)(2n+1)$ and $\sum r$ with $\frac{1}{2}n(n+1)$
	$= \frac{1}{2}n(n+1)(2n+1-3)+n$		
	$= \frac{1}{2}n(2n^2-2)+n$	**M1**	Replaces $\sum 1$ with n
	$= n^3-n+n$	**A1**	Obtains a correct expression in terms of n May be unsimplified
	$= n^3$, [which is a cube number for all n]	**A1**	Correctly demonstrates that the expression is a cube
		5	

Q	Answer	Marks	Comments
5(b)		**M1**	Writes $\sum_{r=p+1}^{4p}$ as the difference of two sums
	$\sum_{r=p+1}^{4p}(3r^2-3r+1) = (4p)^3 - p^3$	**A1**	Obtains a correct expression for $\sum_{r=p+1}^{4p}$ in terms of p only May be unsimplified
	$= 63p^3$		
	$= 3^2\times 7\times p^3$	**A1**	Obtains the correct prime factor decomposition.
		3	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6(a)	$x = \frac{5}{2}$ and $y = 3$	M1 A1	Identifies at least one correct asymptote Condone any incorrect asymptotes for this mark only Identifies both correct asymptotes and no incorrect equations
		2	

Q	Answer	Marks	Comments
6(b)		B1 B1 B1 B1	Correct RHS with correct asymptotic behaviour Correct LHS with correct asymptotic behaviour Must intersect the x-axis to the left of the origin and the y-axis below the origin Includes the correct axis intercepts for the curve C Draws a line through the asymptote intersection and the curve y-intercept (or a point labelled –2 if their curve does not intersect the y-axis)
		4	

Q	Answer	Marks	Comments
6(c)	*C* and *L* intersect when $6x+10 = (2x-5)(2x-2)$ $4x^2 - 20x = 0$	M1	Rearranges the equation/inequality into a 2 or 3-term quadratic equation/inequality
	$x = 0$, $x = 5$	A1	Obtains the correct x-coordinates of the intersections of *C* and *L* **PI**
	The inequality solution is $0 \le x < \frac{5}{2}$, $x \ge 5$	A1ft	Identifies at least one correct region. **ft** Their vertical asymptote or **ft** Their greatest x-intercept of *C* and *L* Condone strict inequalities for non-strict and vice versa. Ignore any incorrect regions.
		A1ft	Identifies two correct regions. **ft** Their vertical asymptote and their greatest x-intercept of *C* and *L* Condone strict inequalities for non-strict and vice versa. Ignore any incorrect regions.
		A1	Obtains the correct regions with no incorrect regions.
		5	

	Question 6 Total	**11**	

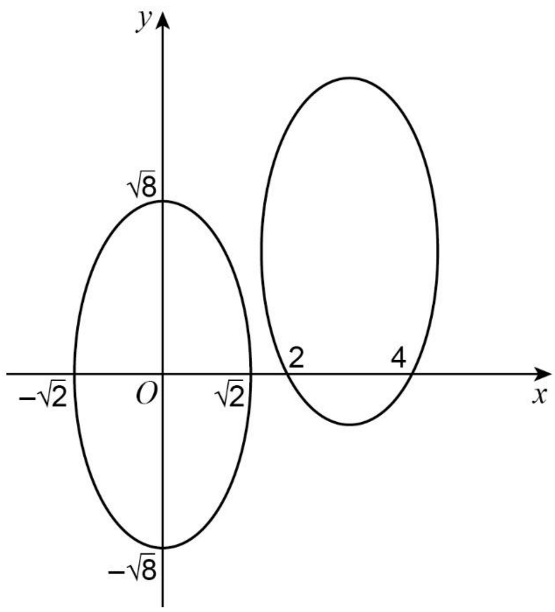
Q	Answer	Marks	Comments
7(a)	The integrand is not defined for $x = 0$	**E1**	Accept any equivalent explanation
		1	

Q	Answer	Marks	Comments
7(b)(i)	$a = 0$ and $b = 2$	**B1**	Writes the correct values for a and b
		1	

Q	Answer	Marks	Comments
7(b)(ii)	$I = \lim\limits_{p \to 0^-} \int_{-1}^{p} \left(\frac{2}{\sqrt[3]{x}}\right) \mathrm{d}x + \lim\limits_{q \to 0^+} \int_{q}^{2} \left(\frac{2}{\sqrt[3]{x}}\right) \mathrm{d}x$	**M1**	Replaces the zero limit of at least one of their integrals with a letter, condoning x
	$= \lim\limits_{p \to 0^-} \left[3x^{\frac{2}{3}}\right]_{-1}^{p} + \lim\limits_{q \to 0^+} \left[3x^{\frac{2}{3}}\right]_{q}^{2}$	**M1**	Substitutes a limit of –1 or 2 into an expression of the form $kx^{\frac{2}{3}}$ where k is a non-zero constant.
	$= \lim\limits_{p \to 0^-} \left(3p^{\frac{2}{3}} - 3\right) + \lim\limits_{q \to 0^+} \left(3 \times 2^{\frac{2}{3}} - 3q^{\frac{2}{3}}\right)$	**M1**	Correct use of the limiting notation at any stage for at least one term. Condone 0 replaced with x
	$= 0 - 3 + 3 \times 2^{\frac{2}{3}} - 0$ $= 3\left(\sqrt[3]{4} - 1\right)$	**A1**	Obtains the required result following the correct limiting process seen on both terms. Accept $\to 0$ instead of $\to 0^+$ (or $\to 0^-$) throughout. Condone the same letter used in both integrals. Do not condone 0 replaced with x
		4	

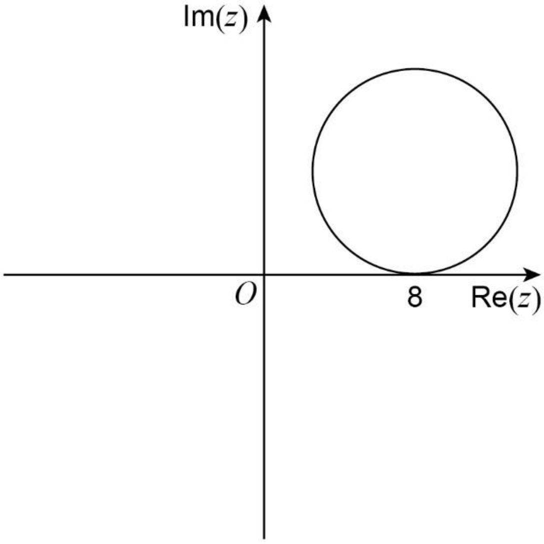
	Question 7 Total	**6**	

Q	Answer	Marks	Comments
8(a)	$4\left(x^2-6x\right)+y^2-4y=-p$		
	$4\left(\left(x-3\right)^2-9\right)+\left(y-2\right)^2-4=-p$	M1	Attempts to complete the square on either the x terms or the y terms. Condone $\left(x+3\right)^2$ or $\left(y+2\right)^2$ **or** Replaces x with $x-a$ and y with $y-b$ in C_1 (or $x+a$ and $y+b$ in C_2)
	$a=3$ and $b=2$	A1	Obtains the correct value for one of a or b **or** Compares coefficients to form two correct equations in a and b
		A1	Obtains the correct values for a and b
	$4\left(x-3\right)^2+\left(y-2\right)^2=40-p$		
	$40-p=8$	M1	Forms a correct equation in p
	$p=32$	A1	Obtains the correct value of p
		5	

Q	Answer	Marks	Comments
8(b)	$C_1: \quad \dfrac{x^2}{2}+\dfrac{y^2}{8}=1$	**B1**	Sketches an ellipse centred on the origin
	C_1 axis intercepts: $x=\pm\sqrt{2}$, $y=\pm\sqrt{8}$	**B1**	States the correct C_1 axis intercepts
	$y=0 \Rightarrow 4x^2-24x+32=0$	**M1**	Substitutes $y=0$ into the C_2 equation
	x-intercepts are $(2,0)$ and $(4,0)$	**A1**	Obtains the correct C_2 axis intercepts
		**M1**	Sketches a translation of C_1 by $\begin{bmatrix} a \\ b \end{bmatrix}$ **ft** Their a and b
		A1	Sketches an ellipse with its centre in the 1st quadrant, intersecting the x-axis twice, no intersections with the y-axis, and no intersections with C_1
		6	

Q	Answer	Marks	Comments
8(c)	Area $= \left(\sqrt{2}+3+\sqrt{2}\right)\times\left(\sqrt{8}+2+\sqrt{8}\right)$	**M1**	Calculates the rectangle width. May be unsimplified. **ft** Their a and b and their C_1 x-intercepts
		M1	Calculates the rectangle height. May be unsimplified. **ft** Their a and b and their C_1 y-intercepts
	$= 22+16\sqrt{2}$	**A1**	Obtains the correct area in the required form
		3	

	Question 8 Total	**14**	

Q	Answer	Marks	Comments
9(a)		M1 M1 A1	Draws a circle Draws a circle in the 1st quadrant which either touches the real axis or does not intercept the imaginary axis Draws a circle in the 1st quadrant touching the real axis but not the imaginary axis and includes the correct real intercept Accept any indication of the real axis intercept as 8
		3	

Q	Answer	Marks	Comments
9(b)	least $\arg(z) = 0$	B1	States the correct argument.
		1	

Q	Answer	Marks	Comments
9(c)	$\arg(8+6\text{i}) = \tan^{-1}\left(\frac{6}{8}\right)$	M1	Correct method for one of the acute angles in a 3,4,5 triangle. Condone degrees **PI** by **AWRT** 0.64 or 0.93 or 36.9 or 53.1 or $\frac{24}{7}$
	maximum $\arg(z) = 2\times\tan^{-1}\left(\frac{6}{8}\right)$	m1	Full method for required angle. Condone degrees. **PI** by **AWRT** 73.7 or 74 or $\tan^{-1}\left(\frac{24}{7}\right)$
	= 1.29 (2dp)	A1	Obtains required angle. Accept **AWRT** 1.29
		3	

Q	Answer	Marks	Comments
9(d)(i)	$\cos\alpha = \dfrac{10^2 + 12^2 - 6^2}{2\times 10\times 12}$	**M1**	Uses cosine rule on a 6,10,12 triangle to calculate the angle between z_2 and *OP* where *P* is $8+6\mathrm{i}$ or Forms two real equations in a and b where $z_2 = a + \mathrm{i}b$
	$\alpha = \cos^{-1}\left(\dfrac{13}{15}\right)$	**A1**	Calculates the angle at the origin. Condone degrees. **PI** by **AWRT** 0.52 or 29.9 or Obtains **AWRT** $11.9 + 1.45\mathrm{i}$ or $4.73 + 11\mathrm{i}$
	$\arg(z_2) = \tan^{-1}\left(\dfrac{6}{8}\right) \pm \cos^{-1}\left(\dfrac{13}{15}\right)$	**m1**	Full method for at least one of the required arguments. Condone degrees. **PI** by **AWRT** 66.8 or 6.9
	$= 0.12$ or 1.17 (2 dp)	**A1**	Obtains both arguments. Accept **AWRT** 0.12 and **AWRT** 1.17
		4	

Q	Answer	Marks	Comments
9(d)(ii)	$\cos\alpha = \dfrac{10^2 + x^2 - 6^2}{2\times 10\times x}$	**M1**	Forms a correct equation in $\lvert z_1\rvert$ or the chord length or half the chord length or OM where M is the centre of the chord. Accept $(r\cos\theta - 8)^2 + (r\sin\theta - 6)^2 = 6^2$ Note: must be a method which leads to an exact answer
	$\dfrac{13}{15} = \dfrac{x^2+64}{20x}$		
	$3x^2 - 52x + 192 = 0$	**A1**	Obtains a correct 3-term quadratic equation in $\lvert z_1\rvert$ only or Obtains $\dfrac{20}{3}$ or $\dfrac{10}{3}$ or $\dfrac{26}{3}$ or Identifies 8^2 as the product of $\lvert z_1\rvert$ and $\lvert z_2\rvert$ **PI**
	$x = \dfrac{16}{3}$ or 12	**m1**	Uses a full method for $\lvert z_1\rvert$
	$\lvert z_1\rvert = \dfrac{16}{3}$	**A1**	Obtains the required modulus in exact form. Must reject 12 if seen.
		4	

	Question 9 Total	**15**	