OXFORD

INTERNATIONAL AQA EXAMINATIONS

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Pure Maths Unit FP1

Tuesday 22 January 2019   07:00 GMT   Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

1 A curve has equation $y = x^3 - 12x$

1 (a) A line passes through two points on the curve.
At one point $x = -2$ and at the other point $x = -2 + h$

Find the gradient of the line in the form $ph + qh^2$, where p and q are integers.
[4 marks]

Answer

1 (b) Use your answer to part **(a)** to explain why the point on the curve where $x = -2$ is a stationary point.
[2 marks]

6

2 The series S_n is defined for $n \geq 2$ by

$$S_n = \sum_{r=1}^{n}\left(2r^3 + 3r^2 - 5r\right)$$

2 (a) Show that

$$S_n = kn(n + a)(n + b)(n + c)$$

where k is a fraction and a, b and c are integers.

[5 marks]

_____

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2 (b) Explain why S_n is always a multiple of 3

[2 marks]

_____

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7

Turn over ▶

3 One of the roots of the quadratic equation

$$x^2 + 3x + c = 0$$

is $p + 2i$, where p and c are real numbers.

Find the value of p and the value of c.

[5 marks]

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$p =$ _____

5

$c =$ _____

4 For each of the improper integrals below, either find its exact value or explain why it has no finite value.

Show all necessary working.

4 (a) $\displaystyle\int_0^{12} \frac{1}{\sqrt{x}}\,dx$

[3 marks]

Answer _____

4 (b) $\displaystyle\int_0^{12} \frac{1}{x^4}\,dx$

[3 marks]

Answer _____

6

Turn over ▶

5 (a) Find the general solution of the equation

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

[4 marks]

Answer _____

5 (b) Find the **sum** of all the solutions of the equation

$$\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

between -18π and 18π.

Give your answer in terms of π.

[5 marks]

Answer _____

9

6 By considering the derivative of $y = x^{\frac{1}{3}}$ when $x = 8$, find an estimate for $\sqrt[3]{8.06}$

[6 marks]

Answer _____

6

Turn over for the next question

Turn over ▶

7 For the complex number $z_1 = x + iy$, where x and y are real numbers,

$$2z_1^* + 3i = iz_1$$

7 (a) Find the value of x and the value of y.

[6 marks]

$x =$ _____

$y =$ _____

7 (b) Find the modulus and argument of z_1

[3 marks]

$$|z_1| = \rule{5cm}{0.4pt}$$

$$\arg(z_1) = \rule{5cm}{0.4pt}$$

7 (c) Find the shortest distance on an Argand diagram from the point representing z_1 to the half-line $\arg z = \dfrac{\pi}{4}$

[4 marks]

Answer \rule{5cm}{0.4pt}

13

Turn over ▶

8 The curve C has equation

$$y = \frac{x-2}{x-3}$$

8 (a) State the equations of the asymptotes of C.

[2 marks]

Answer _____

8 (b) The line L has equation

$$y = \frac{1}{2}x$$

Find the coordinates of the points of intersection of L and C.
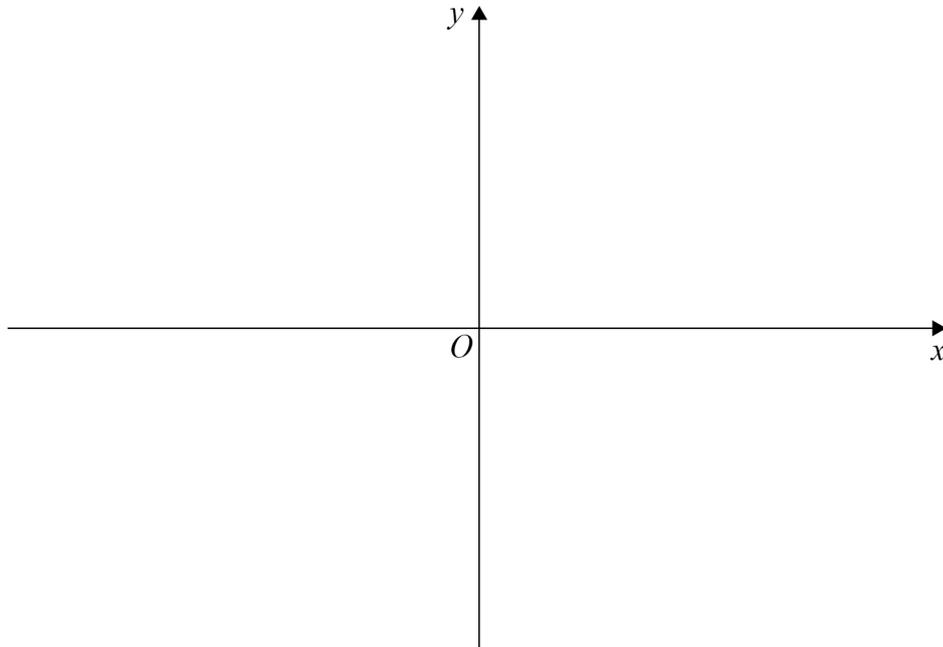
[4 marks]

Answer _____

8 (c)   Sketch C and L on the same axes.

You are given that C has no stationary points.

[4 marks]

8 (d)   Solve the inequality

$$\frac{x-2}{x-3} \le \frac{1}{2}x$$

[2 marks]

Answer

12

9 The ellipse E_1 has equation

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

The locus of a point P is such that the distance from P to the point (6, 0) is half the distance from P to the y-axis.

9 (a) The locus of P is the curve E_2

Show that E_2 is an ellipse which is a translation of E_1

Write down the vector for this translation.

[7 marks]

Answer _____

9 (b) Find the coordinates of the points where E_2 meets the x-axis.

[1 mark]

Answer _____

9 (c) Show that, if the line $y = mx + c$ meets the ellipse E_2, then

$$(3 + 4m^2)x^2 + (8mc - 48)x + (4c^2 + 144) = 0$$

[3 marks]

Question 9 continues on the next page

Turn over ▶

9 (d) Hence find the equations of the tangents to the ellipse E_2 which pass through the point (0, 8).

No credit will be given for solutions based on differentiation.

[5 marks]

Answer

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END OF QUESTIONS

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ANSWER IN THE SPACES PROVIDED

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