OXFORD

INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

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Answer **all** questions in the spaces provided.

1 A curve has equation $y = x^3 - 4x^2$

1 (a) A line passes through two points on the curve, one where $x = 6$ and the other where $x = 6 + h$

Find the gradient of this line in the form $a + bh + h^2$, where a and b are constants.

[4 marks]

Answer _____

1 (b) Show how the answer to **part (a)** can be used to find the gradient of the curve at the point where $x = 6$

State the value of this gradient.

[2 marks]

Answer _____

6

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2 It is given that $z = \dfrac{a + 4\mathrm{i}}{7 + b\mathrm{i}}$, where a and b are real numbers.

Find the real part of z and the imaginary part of z

Give your answers in terms of a and b

[4 marks]

$\mathrm{Re}(z) =$ $\mathrm{Im}(z) =$

4

Turn over ►

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3 (a) Find the general solution of the equation

$$\tan\left(3x - \frac{\pi}{6}\right) = 1$$

giving your answer in terms of π

[3 marks]

Answer _____

Do not write outside the box

3 (b) Find the general solution of the equation

$$\tan^2\left(3x-\frac{\pi}{6}\right)=1$$

giving your answer in terms of π

[3 marks]

Answer _____

6

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4 (a) Explain why $\int_0^{16} x^{-\frac{1}{4}}\,\mathrm{d}x$ is an improper integral.

[1 mark]

4 (b) Find the exact value of the improper integral $\int_0^{16} x^{-\frac{1}{4}}\,\mathrm{d}x$

[3 marks]

Answer _____

4

Turn over ►

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5 It is given that

$$S_n = \sum_{r=2}^{n} \left(\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2} \right)$$

5 (a) Use the method of differences to show that

$$S_n = \frac{a}{b} - \frac{\mathrm{f}(n)}{n^2 (n+1)^2}$$

where a and b are integers and f is a function of n

[4 marks]

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5 (b) Hence find the exact value of

$$\sum_{r=11}^{\infty}\left(\frac{1}{(r-1)^2}-\frac{1}{(r+1)^2}\right)$$

You must show all your working.

[4 marks]

Answer _____

8

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6 The equation $x - 2 = \dfrac{2x}{x-5}$ has roots α and β

6 (a) Find the value of $\alpha + \beta$ and the value of $\alpha\beta$

[3 marks]

$\alpha + \beta =$ _____ $\alpha\beta =$ _____

6 (b) Show that $\alpha^2 + \beta^2 = 61$

[2 marks]

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6 (c) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha - \frac{1}{\beta^2} \text{ and } \beta - \frac{1}{\alpha^2}$$

[6 marks]

Answer

11

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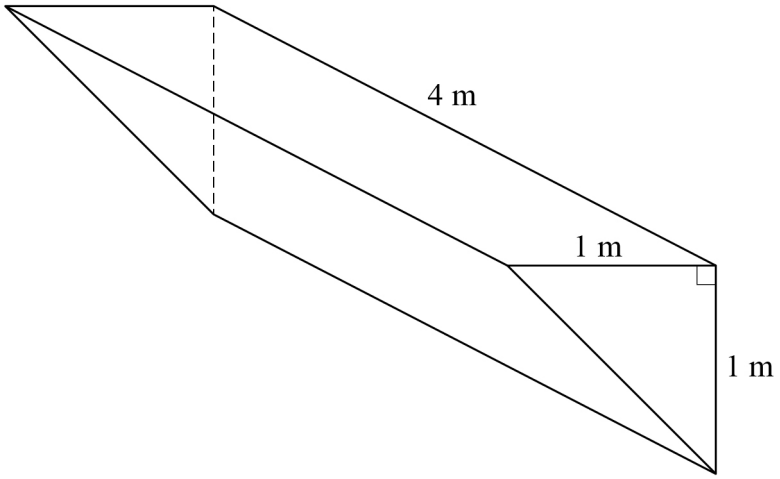
7 A water tank is in the shape of a triangular prism.



The length of the tank is 4 metres.

The cross-section of the tank is a right-angled triangle.

One side of the triangle is vertical and has length 1 metre.

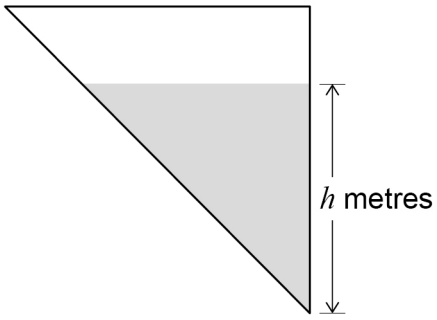
Another side of the triangle is horizontal and has length 1 metre.

7 (a) The tank contains water.

The height of the water is h metres.

The volume of water in the tank is V m^3

cross-section



Find an expression for V in terms of h

[1 mark]

$V =$ _____

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7 (b) Water flows out of the tank at a constant rate of 0.006 m^3 per minute.

Find the rate at which the **height** of the water is decreasing when the height of the water is 0.75 metres.

Give your answer in metres per minute.

[5 marks]

Answer _____

6

Turn over ►

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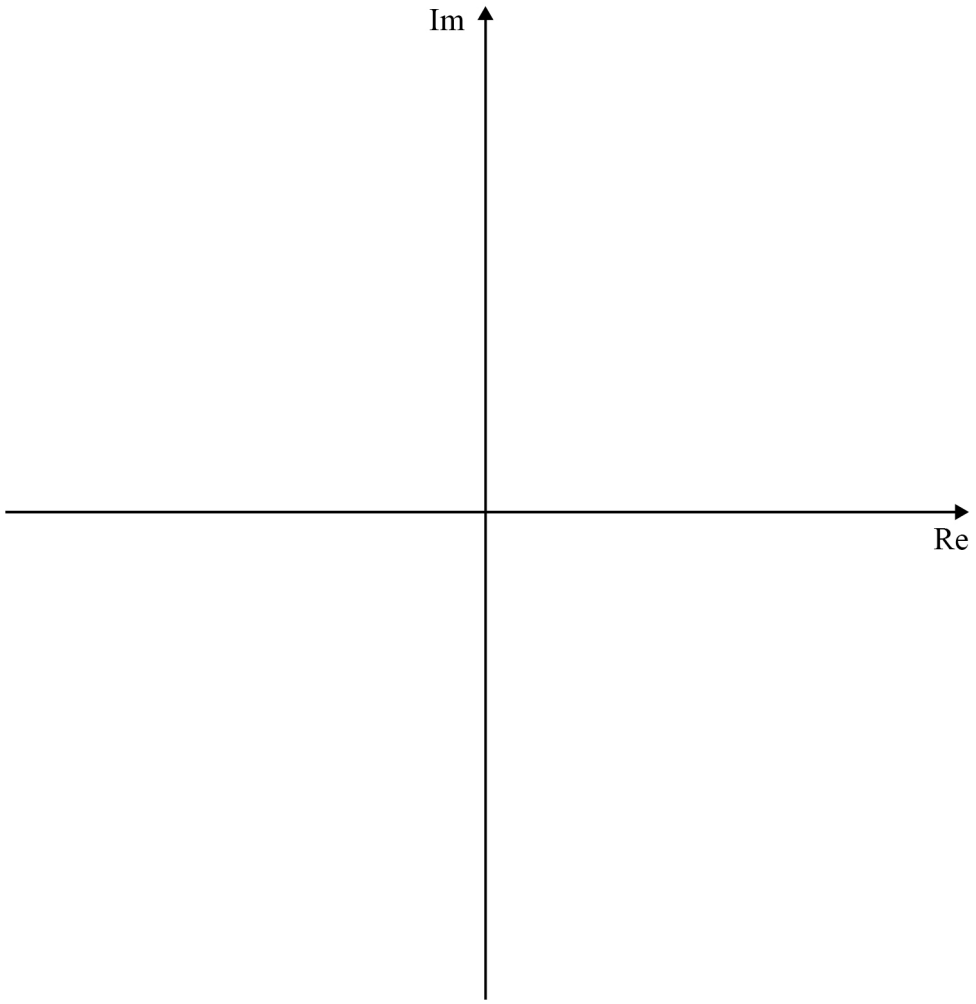
8 The circle C is the locus of points on an Argand diagram such that

$$\left| z + 5 - 6\mathrm{i} \right| = 2$$

8 (a) Draw the circle C on the Argand diagram showing clearly both its centre and radius.

[2 marks]



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8 (b) The complex number z_1 lies on C and is such that $\left| z_1 - 3 \right|$ has its greatest possible value.

Find z_1

[5 marks]

Answer _____

7

Turn over ►

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9 The function f is defined by

$$\mathrm{f}(x) = \frac{4x+7}{x+2}$$

9 (a) Write down the equations of the asymptotes of the graph of $y = \mathrm{f}(x)$

[2 marks]

Answer _____

9 (b) Find the exact values of the coordinates of the points of intersection of the graph of $y = \mathrm{f}(x)$ with the line $y = x - 2$

[4 marks]

Answer _____

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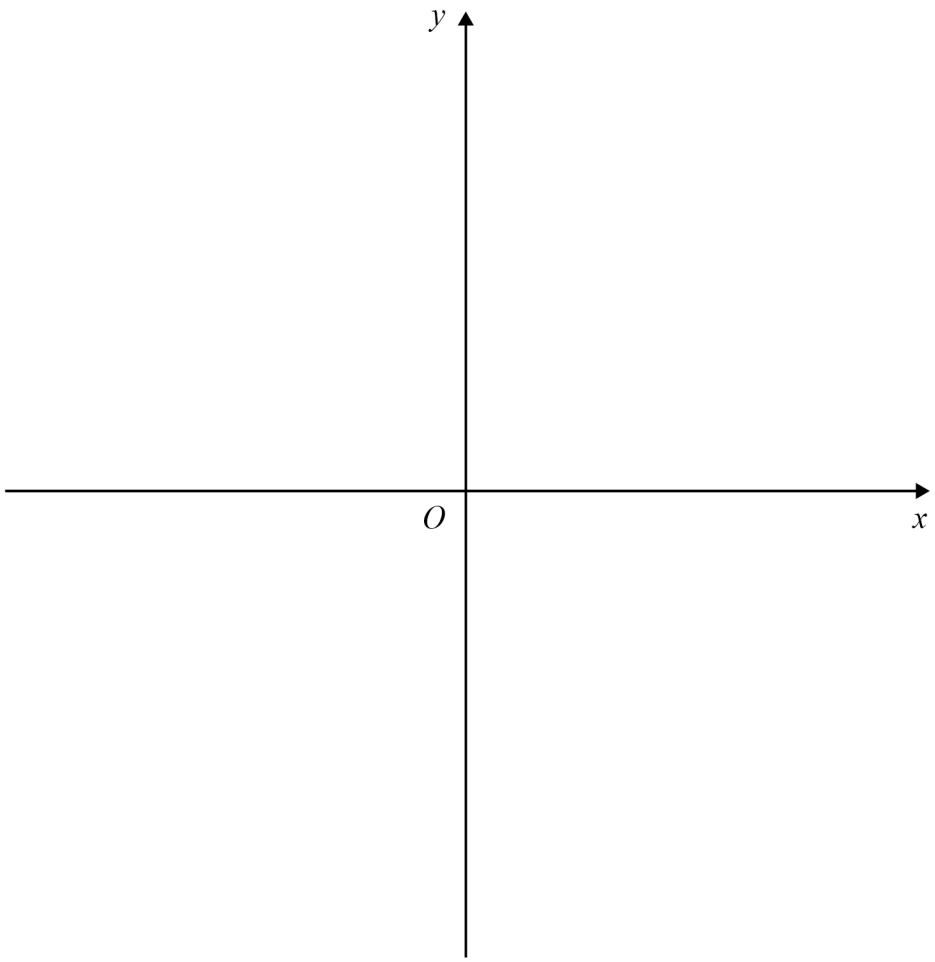
9 (c) Sketch the graph of $y = \mathrm{f}(x)$ and the line $y = x - 2$ on the axes below.

[4 marks]



9 (d) Solve the inequality

$$\frac{4x+7}{x+2} \leq x - 2$$

[2 marks]

Answer _____

12

Turn over ►

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10 The circle C_1 has equation $x^2 + y^2 = 1$

The circle C_2 has equation $(x-7)^2 + y^2 = 1$

10 (a) Describe fully the transformation that maps C_1 onto C_2

[2 marks]

10 (b) The line $y = mx - 4$ meets C_2

Find the set of possible values of m

No credit will be given for solutions using differentiation.

[6 marks]

Answer _____

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10 (c) C_1 is transformed to a curve C_3 by a stretch, parallel to the x-axis, scale factor a, where $a > 1$

10 (c) (i) State what type of curve C_3 is.

[1 mark]

Answer _____

10 (c) (ii) Write down the equation of C_3

[1 mark]

Answer _____

10 (d) Find the set of possible values of a if the line $y = mx - 4$ meets **both** C_2 **and** C_3

[6 marks]

Answer _____

16

END OF QUESTIONS

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