

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

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Forename(s)

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Candidate signature

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I declare this is my own work.

# INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Wednesday 4 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

## Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
<b>TOTAL</b>	





- 1 (b) Show how the answer to **part (a)** can be used to find the gradient of the curve at the point where  $x = -\frac{5}{4}$

[2 marks]

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6

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- 3** For each of the improper integrals below, either find its exact value or explain why it has no finite value.

Show **all** necessary working.

**3 (a)**  $\int_3^{\infty} \frac{1}{\sqrt[5]{x}} dx$

**[3 marks]**

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Answer \_\_\_\_\_

**3 (b)**  $\int_3^{\infty} \frac{1}{x^2} dx$

**[3 marks]**

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Answer \_\_\_\_\_

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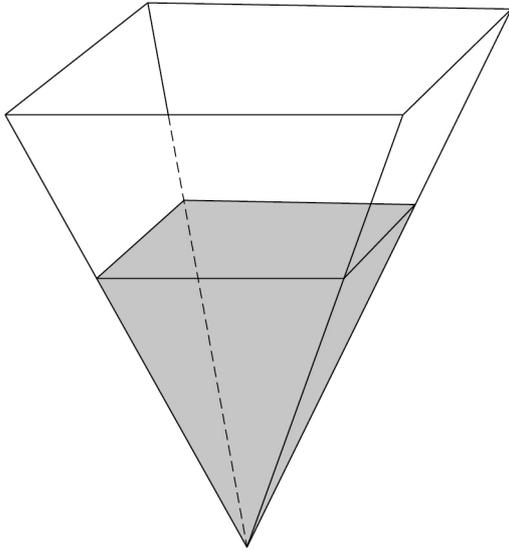




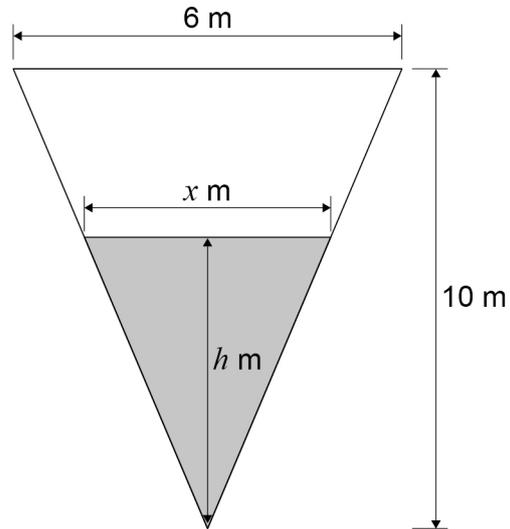
- 7 [The volume of a square-based pyramid is given by the formula  $V = \frac{1}{3}a^2h$  where  $a$  is the length of the side of the square base, and  $h$  is the height.]

A water tank in the shape of an inverted square-based pyramid has height 10 metres, and the length of the side of the square base is 6 metres.

Not drawn to scale



3D image



Cross-section

The tank fills with water at a rate of  $0.54 \text{ m}^3$  per minute.

At time  $t$  minutes the depth of water in the tank is  $h$  metres.

The horizontal surface of the water at time  $t$  minutes is in the shape of a square of side  $x$  metres.

- 7 (a) Explain why  $x = 0.6h$

[1 mark]

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8 The ellipse  $E_1$  has equation

$$\frac{x^2}{4} + y^2 = 1$$

The ellipse  $E_2$  has equation

$$\frac{x^2}{4} + \frac{y^2}{k} = 1$$

where  $k$  is a positive constant.

8 (a) Describe fully the transformation that maps  $E_1$  onto  $E_2$

[2 marks]

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8 (b) It is given that the line  $L$  has equation

$$y = 2x + c$$

where  $c$  is a constant.

8 (b) (i) Show that if  $L$  intersects  $E_2$  then

$$c^2 - k \leq A$$

where  $A$  is a constant to be found.

[5 marks]

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8 (b) (ii) Given that the line  $y = 2x + 7$  is a tangent to  $E_2$  find the value of  $k$

[2 marks]

$k =$  \_\_\_\_\_

9

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9 The function  $f$  is defined by

$$f(x) = \frac{4x+5}{x^2+4x+5}$$

9 (a) (i) Explain why the graph of  $y = f(x)$  has no vertical asymptotes.

[2 marks]

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9 (a) (ii) Write down the equation of the horizontal asymptote of the graph of  $y = f(x)$

[1 mark]

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Answer \_\_\_\_\_

9 (b) Prove that if the line  $y = k$  where  $k$  is a constant, intersects the graph of  $y = f(x)$  then

$$k^2 + 3k - 4 \leq 0$$

[4 marks]

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- 9 (c) Use the result in **part (b)** to find the coordinates of the stationary points of the graph of  $y = f(x)$

[5 marks]

Answer \_\_\_\_\_

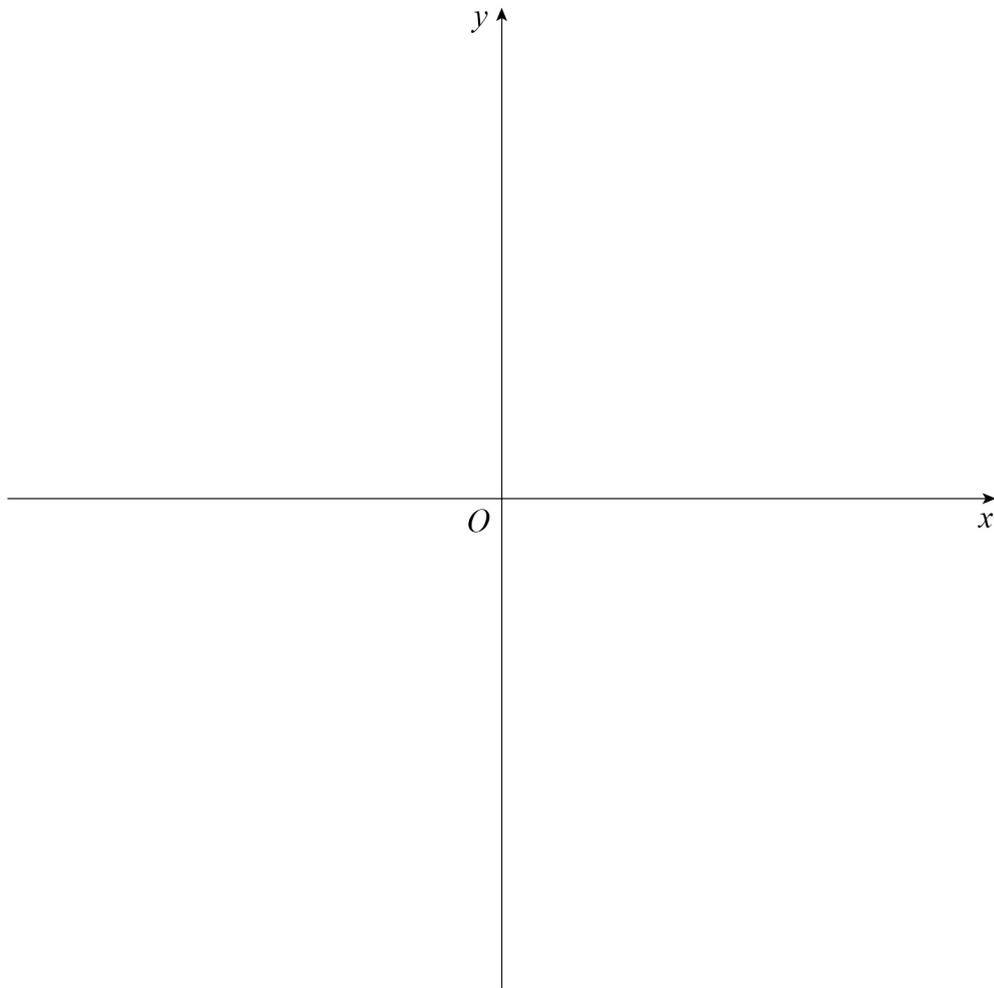
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- 9 (d) Sketch the graph of  $y = f(x)$  on the axes below showing the coordinates of the stationary points.

[3 marks]



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**10** The circle  $C$  is the locus of points on an Argand diagram such that

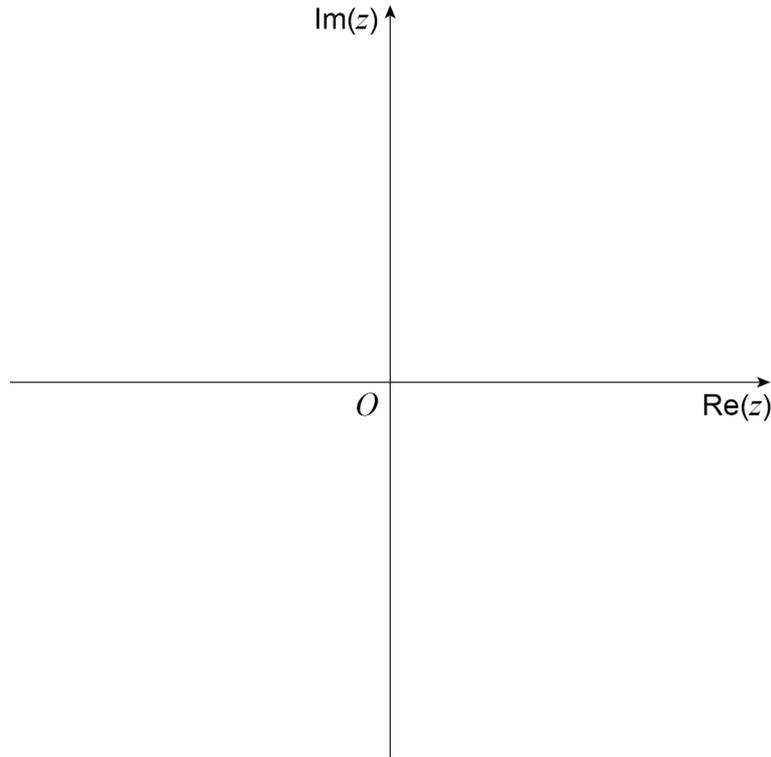
$$|z| = 5$$

The line  $L$  is the locus of points on an Argand diagram such that

$$\operatorname{Re}(z) = 3$$

**10 (a)** Draw  $C$  and  $L$  on the Argand diagram.

**[2 marks]**



**10 (b)** Find all the points which are **both** a distance of exactly one unit from  $C$  **and** a distance of exactly one unit from  $L$

**[6 marks]**

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