Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM01) Unit FP1 Pure Mathematics

Monday 8 May 2023   07:00 GMT   Time allowed: 1 hour 30 minutes

Materials
* For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
* You may use a graphical calculator.

Instructions
* Use black ink or black ball-point pen. Pencil should only be used for drawing.
* Fill in the boxes at the top of this page.
* Answer **all** questions.
* You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
* If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
* Do all rough work in this book. Cross through any work you do not want to be marked.

Information
* The marks for questions are shown in brackets.
* The maximum mark for this paper is 80.

Advice
* Unless stated otherwise, you may quote formulae, without proof, from the booklet.
* Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
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8	
9	
10	
11	
TOTAL	

Answer **all** questions in the spaces provided.

1 By considering the derivative of $y = x^{\frac{1}{2}}$ when $x = 25$, find an estimate for $\sqrt{25.4}$

[6 marks]

Answer _____

6

2 The integral I is defined by

$$I = \int_4^\infty x^{-3} \, dx$$

2 (a) Explain why I is an improper integral.

[1 mark]

2 (b) Evaluate I showing the limiting process.

[3 marks]

Answer _____

4

Turn over ▶

3 (a) Show that

$$(x+1)^3 - (x-1)^3 = 6x^2 + 2$$

[1 mark]

3 (b) Use the method of differences to show that

$$\sum_{r=15}^{n} \left(6r^2 + 2\right) = n^3 + (n+1)^3 - k$$

where k is a constant.

[4 marks]

5

4 (a) Find the general solution of the equation

$$\sin\left(\frac{x}{3}+\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}$$

Give your answer in terms of π

[4 marks]

Answer _____

4 (b) Find the sum of the four smallest positive solutions of the equation

$$\sin\left(\frac{x}{3}+\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}$$

Give your answer in terms of π

[3 marks]

Answer _____

7

Turn over ▶

5 The equation

$$z^2 - az + (b + i) = 0$$

where a and b are real constants, has two complex roots.

One of the roots of the equation is $2 + i$

Find the other root of the equation.

[5 marks]

Answer _____

6 (a) A curve has equation $y = px^2 - 3x$ where p is a constant.

A line passes through two points on the curve, one where $x = 7$
and the other where $x = 7 + h$

Find the gradient of this line in terms of p and h

Give your answer in its simplest form.

[3 marks]

Answer _____

6 (b) The curve has a stationary point at the point where $x = 7$

Use your answer to **part (a)** to find the value of p

[2 marks]

Answer _____

5

7 The quadratic equation

$$3x^2 - 2x + 9 = 0$$

has roots α and β

7 (a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

[2 marks]

$\alpha + \beta = $ _____ $\alpha\beta = $ _____

7 (b) Hence show that $\alpha^2 + \beta^2 = -\dfrac{50}{9}$

[2 marks]

7 (c) Find a quadratic equation, with integer coefficients, which has roots α^4 and β^4

[4 marks]

Answer _____

8

8 The function f is defined by

$$f(x) = \frac{x^2}{(x-1)(x+2)}$$

8 (a) Write down the equations of the asymptotes of the graph of $y = f(x)$

[2 marks]

Answer

8 (b) It is given that the line $y = k$, where k is a constant, intersects the graph of $y = f(x)$

Find the set of possible values of k

[3 marks]

Answer

8 (c) Hence find the coordinates of the stationary points of the graph of $y = f(x)$

[3 marks]

Answer

8 (d) Show that the graph of $y = f(x)$ intersects its horizontal asymptote at one point.

Find the coordinates of this point.

[2 marks]

Answer

8 (e) Sketch the graph of $y = f(x)$ on the axes below.

Show the coordinates of the stationary points

Show the coordinates of the point of intersection of the graph with its horizontal asymptote.

[3 marks]

13

Turn over ▶

9 (a) Show that

$$\sum_{r=1}^{n}\left(r^3 + r^2\right) = \frac{1}{12}n(n+a)(n+b)(cn+a)$$

where a, b and c are integers.

[4 marks]

9 (b) Find all the possible values of n in the range $1 \leq n \leq 50$ such that $\sum_{r=1}^{n} \left(r^3 + r^2 \right)$

is divisible by 37

[3 marks]

Answer _____

7

Turn over for the next question

Turn over ▶
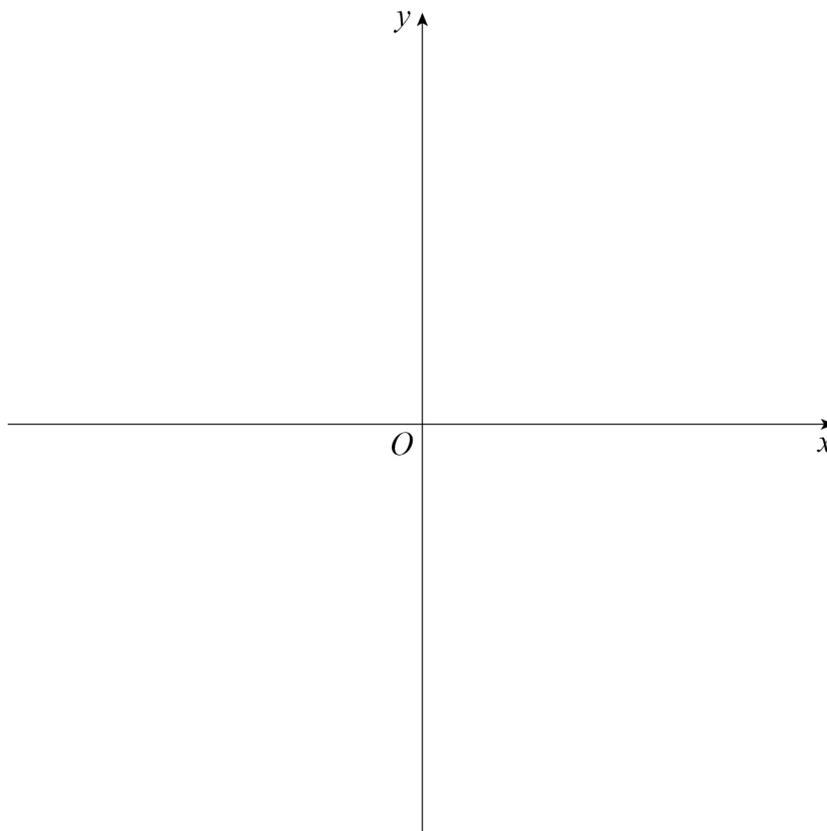
10   The locus of a point P is such that the distance from P to the point $(1, 0)$ is equal to **half** the distance from P to the line $x = 4$

The locus of P is the curve E

10 (a)   Show that the equation of E is

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

[3 marks]

10 (b)   The rectangular hyperbola H has equation

$$xy = \sqrt{3}$$

Find the coordinates of the two points of intersection of H and E

[4 marks]

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Answer _____

10 (c) Sketch H and E on the axes below, showing all significant features.

[4 marks]
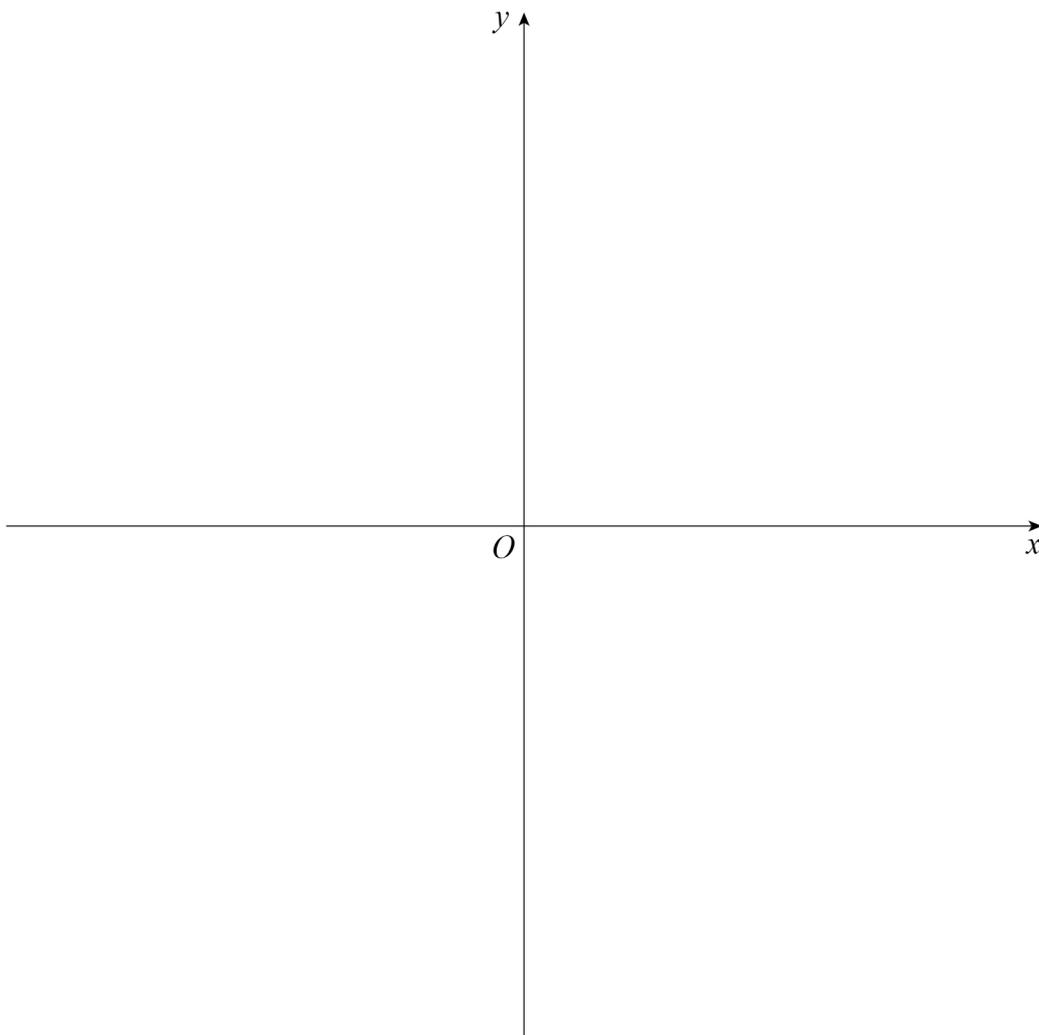
11 The circle C is the locus of points on an Argand diagram such that

$$|z - 3| = 2$$

The point Q is the centre of C

The line L is the locus of points on an Argand diagram such that

$$|z - 3| = |z + 3 - 3i|$$

The point P is the point on L which is closest to C

11 (a) On the Argand diagram, draw C and L, and mark the points P and Q

[4 marks]

11 (b) *PT* and *PS* are tangents to *C*, touching the circle at *T* and *S*

Find the exact area of the quadrilateral *PTQS*

[5 marks]

Answer _____

END OF QUESTIONS

9
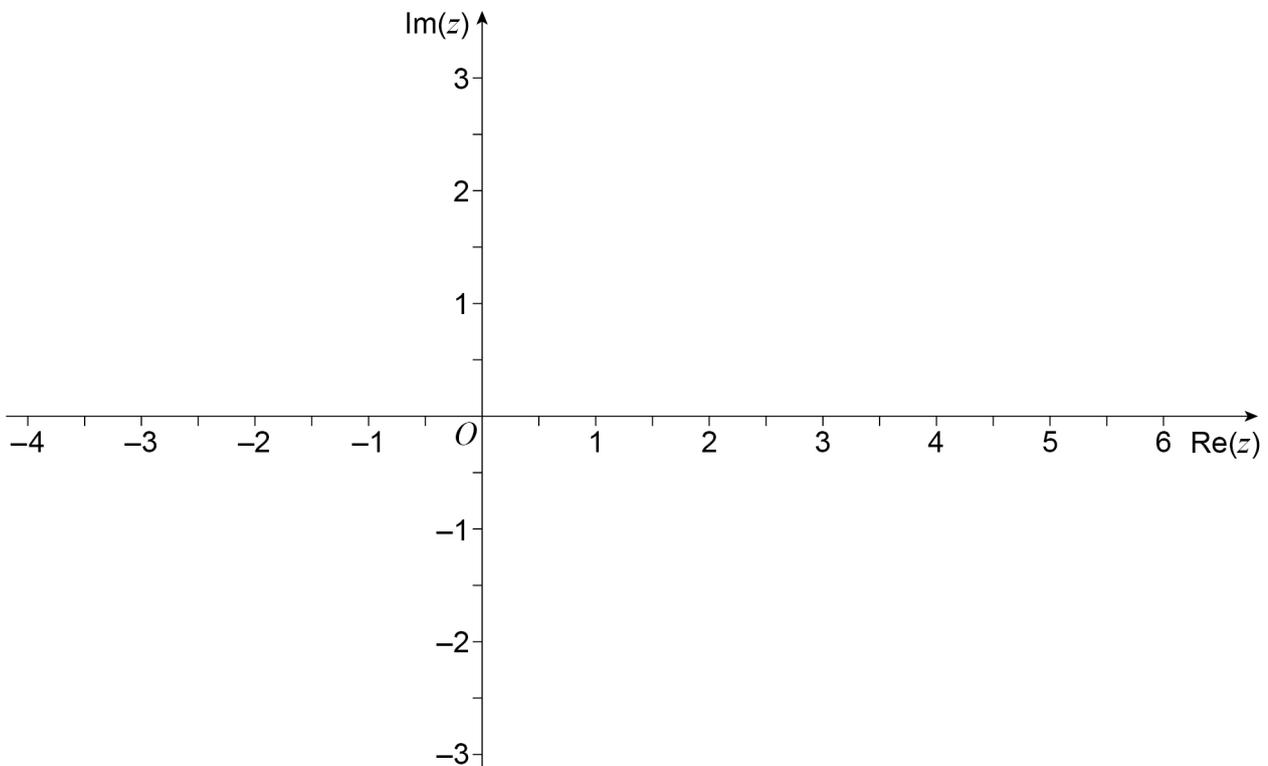
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ANSWER IN THE SPACES PROVIDED**

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