OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 − Pure Maths

Friday 21 June 2019 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables (enclosed).
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
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7	
8	
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10	
11	
12	
TOTAL	

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Answer **all** questions in the spaces provided.

1 (a) Show that

$$\frac{1}{2r+1}-\frac{1}{2r+3}=\frac{k}{(2r+1)(2r+3)}$$

where k is a constant.

[2 marks]

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1 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)} = \frac{n}{p(2n+3)}$$

where p is an integer.

[4 marks]

6

Turn over for the next question

Do not write outside the box

2 (a) Use the definition of $\cosh x$ in terms of e^x and e^{-x} to show that

$$2\cosh^2 x - 1 = \cosh 2x$$

[3 marks]

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2 (b) A curve has equation

$$y = 3\sinh 2x - 5\sinh x + 4x$$

Using the result in part **(a)**, prove that this curve has no stationary points.

[6 marks]

9

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3 The roots of the cubic equation

$$3z^3 + 9z + r = 0$$

where r is real, are α, β and γ.

3 (a) (i) Write down the value of $\alpha\beta + \beta\gamma + \gamma\alpha$.

[1 mark]

$\alpha\beta + \beta\gamma + \gamma\alpha =$ ________________

3 (a) (ii) Hence show that $\alpha^2 + \beta^2 + \gamma^2 = -6$

[3 marks]

Do not write outside the box

3 (a) (iii) Hence explain why the cubic equation must have two non-real roots and one real root.

[2 marks]

3 (b) (i) Given that $\alpha = 1 + \sqrt{6}\,\mathrm{i}$, find the value of $\alpha\beta\gamma$.

[3 marks]

$\alpha\beta\gamma =$ ______

3 (b) (ii) Hence write down the value of r.

[1 mark]

$r =$ ______

10

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4 Three planes have equations

$$x + 3y + cz = c + 4$$
$$x + 2y + 3z = 6$$
$$x + y + z = d$$

where c and d are constants.

The three planes do not intersect at a unique single point.

4 (a) Show that $c = 5$

[2 marks]

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4 (b) In the case where the three planes also share a common line of intersection, determine the value of d.

[5 marks]

$d =$ ____________________

7

Turn over for the next question

Do not write outside the box

5 (a) Given that

$$f(k) = 2^{k+2} + 3^{2k+1}$$

show that

$$f(k+1) - 2\,f(k) = a \times 3^{2k+1}$$

where a is an integer.

[3 marks]

Do not write outside the box

5 (b) Hence prove by induction that $2^{n+2} + 3^{2n+1}$ is a multiple of 7 for all integers $n \geq 1$

[4 marks]

7

Turn over ►

Do not write outside the box

6 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 20e^{2x} + 18$$

6 (a) Find the values of the constants p and q for which $p + qxe^{2x}$ is a particular integral of this differential equation.

[5 marks]

$p =$ ____________ $q =$ ____________

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6 (b) Hence solve the differential equation, expressing y in terms of x, given that $y = 5$ when $x = 0$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} \to 0$ as $x \to -\infty$

[7 marks]

Answer

12

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7 The matrix $\mathbf{M} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 5 & 7 \\ k & 1 & 1 \end{bmatrix}$, where k is a positive integer.

An eigenvector of $\mathbf{M}$ is $\begin{bmatrix} -1 \\ 7 \\ 1 \end{bmatrix}$ and its corresponding eigenvalue is λ_1

7 (a) Find the value of λ_1 and the value of k.

[4 marks]

$\lambda_1 =$ ____________ $k =$ ____________

7 (b) Show that -2 is the least eigenvalue of $\mathbf{M}$.

[4 marks]

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7 (c) Find an eigenvector corresponding to the eigenvalue -2

[3 marks]

Answer

7 (d) The transformation T has matrix $\mathbf{M}$.

Write down the Cartesian equations for any one of the invariant lines of T.

[1 mark]

Answer

12

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8 (a) Use integration to show that $\frac{x}{x+1}$ is an integrating factor for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{x(x+1)}y = 2x + 3\,, \quad x > 0$$

[4 marks]

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8 (b) Hence find the general solution of this differential equation.

[5 marks]

Answer

9

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9 Plane Π_1 has vector equation $\mathbf{r}.\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = 3$

Plane Π_2 has vector equation $\mathbf{r}.\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = 4$

9 (a) Calculate the acute angle between the planes Π_1 and Π_2 giving your answer to the nearest 0.1°

[4 marks]

Answer ________________________

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9 (b) Find a vector equation for the line of intersection of Π_1 and Π_2

[5 marks]

Answer ______________________________

9

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10 A curve C is given parametrically by the equations

$$x = t - \sin t \ , \qquad y = 2\sin^2\left(\frac{t}{2}\right)$$

10 (a) Show that $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 4\sin^2\left(\dfrac{t}{2}\right)$.

[4 marks]

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10 (b) The arc of C between the points where $t = \frac{\pi}{3}$ and $t = \frac{\pi}{2}$ is rotated through 2π radians about the x-axis.

Find the area of the surface generated, giving your answer in the form $\frac{2\pi}{3}\left(p\sqrt{3} + q\sqrt{2}\right)$, where p and q are integers.

[6 marks]

Answer ____________________

10

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11 Given that $z = \cos\theta + \mathrm{i}\sin\theta$:

11 (a) (i) use de Moivre's theorem to show that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$.

[3 marks]

11 (a) (ii) write down an expression for $z - \dfrac{1}{z}$ in terms of $\sin\theta$.

[1 mark]

Answer

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11 (b) Hence express $64\sin^6\theta$ in the form

$$20 + a\cos 2\theta + b\cos 4\theta + c\cos 6\theta$$

where a, b and c are integers.

[5 marks]

Answer

Question 11 continues on the next page

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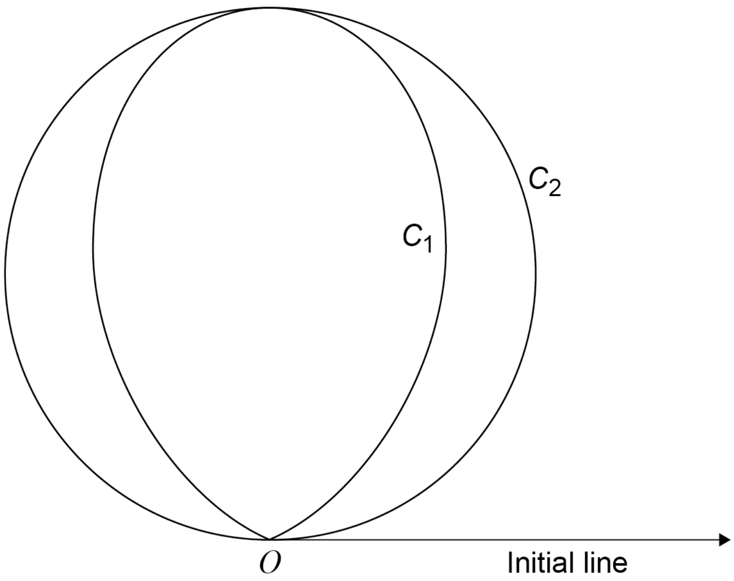
11 (c) A leaf lies flat on a thin circular disc.

The diagram shows a curve C_1 which models the leaf and a circle C_2 which models the disc.

The pole O and the initial line are also shown.



The polar equation of the curve C_1 is $r = 2\sin^3\theta$, $0 \leq \theta \leq \pi$.

The polar equation of the circle C_2 is $r = 2\sin\theta$, $0 \leq \theta \leq \pi$.

Using your answer to part **(b)**, find what percentage of the area of the circular disc is **not** covered by the leaf.

[5 marks]

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Answer

14

Turn over ►

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12 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^5

[1 mark]

Answer

12 (b) (i) Given that

$$\ln y = \tan^{-1} x$$

prove that

$$(2x-1)\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

[5 marks]

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12 (b) (ii) Hence, given that the first five terms in the Maclaurin series expansion in ascending powers of x of $e^{\tan^{-1} x}$ are

$$1 + x + \frac{x^2}{2} + px^3 + qx^4$$

show that $p = -\frac{1}{6}$ and find the value of q.

[6 marks]

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$q =$ ______

12 (c) Hence show that $\lim_{x \to 0} \left[\frac{e^{\tan^{-1} x} - e^{x}}{2x - \sin 2x} \right]$ exists and find its value.

[3 marks]

Answer ______

15

END OF QUESTIONS

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