OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Please write clearly in block capitals.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 11 January 2022 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
TOTAL	

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) The matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Describe fully the **single** transformation represented by the matrix **A**

[2 marks]

1 (b) The matrix $\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

State the line of invariant points for the transformation represented by the matrix **B**

[1 mark]

Answer ______________________

3

Do not write outside the box

2 The vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are such that

$$\mathbf{v} \times \mathbf{w} = 5\mathbf{i} \quad \text{and} \quad \mathbf{u} \times \mathbf{v} = 2\mathbf{j}$$

Simplify

$$(4\mathbf{u} + 3\mathbf{v} + 6\mathbf{w}) \times (2\mathbf{u} - 4\mathbf{v} + 3\mathbf{w})$$

giving your answer in the form $a\mathbf{i} + b\mathbf{j}$ where a and b are integers.

[5 marks]

Answer ______________________

5

Do not write outside the box

3 The sequence $u_1,\ u_2,\ u_3,\ \ldots$ is defined by

$$u_1 = 3 \quad \text{and} \quad u_{n+1} = \frac{9u_n - 5}{5u_n - 1}$$

Prove by induction that for all integers $n \geq 1$

$$u_n = \frac{5n+1}{5n-3}$$

[6 marks]

6

Do not write outside the box

4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2\sin 2x + 14\cos 2x$$

[7 marks]

$y =$ ______________________________

7

Turn over ►

5 (a) Use the trigonometric identity

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

to show that

$$\frac{1}{2}\left[\sin(2r+1)x - \sin(2r-1)x\right] = \cos 2rx \sin x$$

[1 mark]

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \sin^2 rx = \frac{n}{2} - \frac{\sin nx \cos(n+1)x}{2\sin x}$$

[6 marks]

Do not write outside the box

7

Turn over for the next question

Do not write outside the box

6 Evaluate the improper integral

$$\int_0^{\infty} \left(x^2+1\right)\mathrm{e}^{-x}\ \mathrm{d}x$$

showing the limiting process used.

[8 marks]

Do not write outside the box

Answer ______________________

8

Turn over for the next question

Do not write outside the box

7 A curve C is defined for $x > 0$

All points (x, y) on the curve satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{1}{x} - \frac{1}{x+2}\right)y = x$$

7 (a) Use an integrating factor to find the general solution of this differential equation.

[7 marks]

Do not write outside the box

Answer ______________________

7 (b) The curve C has a stationary point when $x = 2$

Find the equation of the curve C giving your answer in the form $y = \mathrm{f}(x)$

[3 marks]

$y =$ ______________________

10

Do not write outside the box

8 The plane Π_1 has vector equation

$$\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 14$$

8 (a) Find the shortest distance from the origin to the plane Π_1 giving your answer in an exact form.

[2 marks]

Answer ______________________

8 (b) The line L has Cartesian equations

$$\frac{x-2}{3} = \frac{y+1}{2} = 2z-4$$

The line L intersects the plane Π_1 at the point P

8 (b) (i) Find the coordinates of P

[3 marks]

Answer ______________________

Do not write outside the box

8 (b) (ii) Calculate the acute angle between the line L and the plane Π_1 giving your answer to the nearest 0.1°

[4 marks]

Answer ______________________

8 (c) The plane Π_2 has vector equation $\mathbf{r} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 4$

Find direction cosines for the line of intersection of the planes Π_1 and Π_2

[3 marks]

Answer ______________________

12

Do not write outside the box

9 The matrix $\mathbf{M}$ is defined as

$$\mathbf{M} = \begin{bmatrix} 4 & 3 & k \\ 5 & 4 & k+1 \\ 1 & 1 & 3 \end{bmatrix}$$

where k is a constant.

9 (a) Show that $\mathbf{M}$ is a non-singular matrix.

[2 marks]

9 (b) Find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

Answer

Do not write outside the box

9 (c) The transformation represented by the matrix $\begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 6 \\ 1 & 1 & 3 \end{bmatrix}$ maps the straight line L

onto the straight line whose vector equation is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

Find the vector equation of the line L giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

[5 marks]

Answer ______________________

12

Turn over ►

Do not write outside the box

10 (a) Use the Maclaurin series for $\ln(1+x)$ to show that the first three non-zero terms in the Maclaurin series expansion in ascending powers of x of

$$\tanh^{-1}x \quad \text{are} \quad x+\frac{x^3}{3}+\frac{x^5}{5}$$

[3 marks]

10 (b) It is given that $y=\tan x$

10 (b) (i) Show that when $x=0$

$$\frac{d^5y}{dx^5}=16$$

[4 marks]

Do not write outside the box

10 (b) (ii) Show that the first non-zero term in the Maclaurin series expansion in ascending powers of x of

$$\tanh^{-1}x - \tan x \quad \text{is} \quad \frac{x^5}{15}$$

[3 marks]

10 (c) Hence show that

$$\lim_{x \to 0}\left[\frac{\tan x + \tanh^{-1}x - 2x}{x(1 - \cos 2x)}\right]$$

exists and find its value.

[4 marks]

Answer ______________________

14

Do not write outside the box

11 A curve C is given parametrically by the equations

$$x = t^2 \qquad y = 2t \quad \text{where} \quad t \geq 0$$

The origin O and the point P lie on the curve C

The x-coordinate of P is 3

11 (a) The arc OP of the curve C is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is $\frac{56}{3}\pi$

[5 marks]

Do not write outside the box

11 (b) Show that the length of the arc *OP* of the curve *C* is $2\sqrt{3}+\sinh^{-1}\left(\sqrt{3}\right)$

[7 marks]

12

Turn over ►

Do not write outside the box

12 (a) (i) Use de Moivre's theorem to show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

[3 marks]

12 (a) (ii) Given that

$$(2\,\mathrm{i}\sin\theta)^6(2\cos\theta)^2 = \left(z - \frac{1}{z}\right)^4\left(z^2 - \frac{1}{z^2}\right)^2$$

use the result in **part (a)(i)** to show that

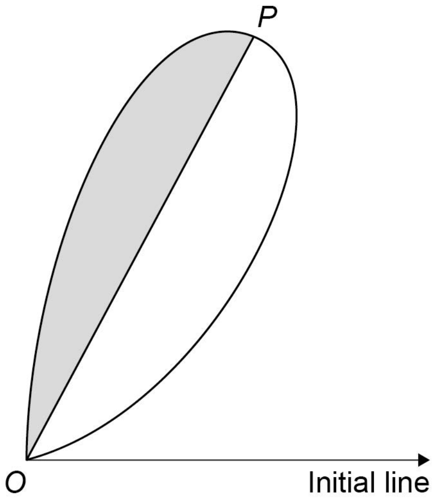
$$128\sin^6\theta\cos^2\theta = 5 - 4\cos 2\theta - 4\cos 4\theta + 4\cos 6\theta - \cos 8\theta$$

[4 marks]

Do not write outside the box

Question 12 continues on the next page

12 (b) The diagram shows a curve, the pole O, the initial line and a point P which lies on the curve.



P is the point on the curve that is furthest from the pole O

The curve has polar equation $r = 32 \sin^3\theta \cos\theta$ where $0 \le \theta \le \frac{\pi}{2}$

12 (b) (i) By differentiating r with respect to θ find the polar coordinates of the point P

[4 marks]

Answer ____________________

Do not write outside the box

12 (b) (ii) Find the area of the shaded region bounded by the line *OP* and the upper part of the curve.

Give your answer in the form $a\pi + b\sqrt{n}$ where a and b are rational and n is a prime number.

[4 marks]

Answer ____________________

15

13 The cubic equation

$$tx^3 + ux^2 + vx + w = 0$$

has coefficients t, u, v and w which are all real constants.

The three roots of this cubic equation can be arranged as successive terms of an arithmetic sequence.

13 (a) Show that

$$2u^3 - 9tuv + 27t^2w = 0$$

[3 marks]

Do not write outside the box

13 (b) It is given that the roots of the cubic equation

$$kx^3 - 36x^2 + mx - 3 = 0$$

where k and m are real constants, can be arranged as three successive terms of an arithmetic sequence with common difference d

13 (b) (i) Find an expression for d^2 in terms of k

[2 marks]

Answer ______________________

13 (b) (ii) Given that $m = 38$ find the possible values for d giving your values in an exact form.

[4 marks]

Do not write outside the box

Answer

9

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

Question number	**Additional page, if required. Write the question numbers in the left-hand margin.**

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED