OXFORD AQA
INTERNATIONAL QUALIFICATIONS

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 14 January 2025 07:00 GMT Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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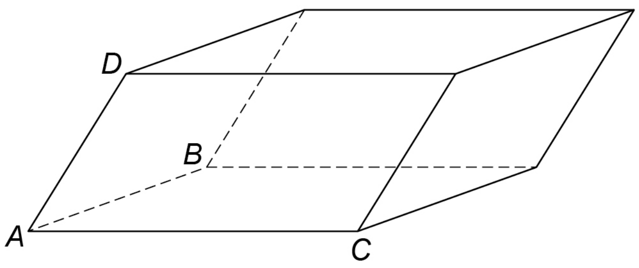
Answer **all** questions in the spaces provided.

1 The diagram shows a parallelepiped and the four points *A, B, C* and *D*



The points *A, B, C* and *D* have position vectors

$$\mathbf{a} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} k+4 \\ 6 \\ 3 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 6 \\ 7 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} k+5 \\ 3 \\ 5 \end{bmatrix}$$

where k is a constant.

It is given that the volume of the parallelepiped has a magnitude of 4 cubic units.

Find the possible values of k

[6 marks]

Do not write outside the box

Answer

6

2 (a) Use the definitions of $\cosh x$ and $\sinh x$ in terms of e^{x} and e^{-x} to show that

$$\cosh^2 x - \sinh^2 x = 1$$

[2 marks]

2 (b) Use the result given in **part (a)** to solve the equation

$$\sinh^2 x - \cosh x - 5 = 0$$

Give any solutions as an exact natural logarithm.

[5 marks]

Answer

7

Turn over for the next question

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3 Three non-singular square matrices **A**, **B** and **R** are such that

$$\mathbf{AR} = \mathbf{B}$$

The matrix **R** represents a rotation about the x-axis through an angle θ

The matrix **B** is defined as

$$\mathbf{B} = \begin{bmatrix} 0 & \cos\theta & -\sin\theta \\ 1 & 0 & 0 \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

3 (a) Show that **A** is independent of θ

[3 marks]

Do not write outside the box

3 (b) Describe the single transformation represented by **A**

[2 marks]

5

Turn over for the next question

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4 (a) Explain why $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$ is an improper integral.

[1 mark]

4 (b) Evaluate $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$ showing the limiting process used.

[3 marks]

Answer ______________________

4

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5 It is given that

$$f(n) = 7^{n+1} + 11^n$$

Prove by induction that $f(n)$ is a multiple of 4 for all integers $n \geq 1$

[5 marks]

5

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6 (a) Show that

$$(r+1)^5-(r-1)^5=10r^4+20r^2+2$$

[1 mark]

6 (b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} r^4=\frac{1}{30}n(n+1)(2n+1)\left(3n^2+pn+q\right)$$

where p and q are integers

[7 marks]

Do not write outside the box

8

Do not write outside the box

7 A curve C is defined by the polar equation

$$r = \sec^2\theta\sqrt{2(1+\tan\theta)} \qquad \text{where} \qquad -\frac{\pi}{4} \le \theta < \frac{\pi}{2}$$

The point A on C is where $\theta = 0$

The point B on C is where $\theta = \frac{\pi}{6}$

The point O is the pole.

Show that the area of the region bounded by the curve C and the lines OA and OB is

$$\frac{m+n\sqrt{3}}{108}$$

where m and n are integers.

[5 marks]

Do not write outside the box

5

Do not write outside the box

8 Three planes have equations

$$\begin{aligned} 2x \quad - \quad y \quad + \quad 3z &= 1 \\ x \quad + (k-1)y - \quad z &= 3 \\ (k-3)x - \quad y \quad + \quad z &= 1 \end{aligned}$$

where k is a constant.

The planes **do not** meet at a unique point.

8 (a) Find the possible values of k

[3 marks]

Answer

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8 (b) (i) Determine the number of solutions of the three equations when $k = 3$

Fully justify your answer.

[3 marks]

8 (b) (ii) Hence give a geometric interpretation for the three planes when $k = 3$

[1 mark]

7

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9 A curve is given parametrically by the equations

$$x = \sin\theta\cos\theta \quad \text{and} \quad y = \sin^2\theta \quad \text{for} \quad 0 \le \theta < \pi$$

The arc of the curve from $\theta = 0$ to $\theta = \frac{\pi}{6}$ is rotated through 2π radians about the x-axis to generate a surface with area S

Find the value of S

Give your answer in the form $\frac{\pi}{12}\left(p\pi + q\sqrt{3}\right)$ where p and q are integers.

[8 marks]

Do not write outside the box

Answer

8

Turn over ►

Do not write outside the box

10 The roots of the quartic equation

$$z^4 - 2z^3 - 21z^2 + pz + q = 0$$

are α, β, γ and δ, where p and q are real.

It is given that α, β, γ and δ form an arithmetic sequence.

10 (a) Find the possible values of α, β, γ and δ

[6 marks]

Do not write outside the box

Answer ______

10 (b) Hence find the value of p and the value of q

[3 marks]

$p =$ ______ $q =$ ______

9

Turn over ►

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11 The line L_1 has Cartesian equation

$$2x - 12 = \frac{y-7}{2} = 5 - z$$

The line L_2 has equation

$$\left(\mathbf{r} - \begin{bmatrix} 8 \\ w \\ 6 \end{bmatrix}\right) \times \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} = \mathbf{0} \quad \text{where } w \text{ is a constant.}$$

11 (a) It is given that L_1 and L_2 intersect at a point.

Find the value of w

[5 marks]

Answer ______________________

11 (b) It is given that L_1 and L_2 lie in the plane Π

Find an equation of the plane Π

Give your answer in the form $\mathbf{r} \bullet \mathbf{n} = d$

[3 marks]

Answer

Question 11 continues on the next page

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11 (c) The point P has position vector $\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$

11 (c) (i) The point Q is the image of P in a reflection in the plane Π

Find the position vector of Q

[4 marks]

Answer

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11 (c) (ii) Find the exact distance of P from Π

[2 marks]

Answer

14

Turn over for the next question

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12 It is given that $y = \mathrm{f}(x)$ satisfies the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 6\mathrm{e}^{-x} - 10\cos x$$

and when $x = 0$ it is given that both $y = 10$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$

Find $\mathrm{f}(x)$

[12 marks]

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$f(x) =$

12

Turn over ►

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13 The matrix **M** is defined as

$$\mathbf{M} = \begin{bmatrix} 4 & 1 & -(k+1) \\ 1 & 3 & k+2 \\ 2 & 1 & 1 \end{bmatrix}$$

where k is a constant.

It is given that **M** is a non-singular matrix.

13 (a) (i) Find any restrictions on the value of k

[2 marks]

Answer

13 (a) (ii) Find $\mathbf{M}^{-1}$ in terms of k

[5 marks]

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Answer

13 (b) Use your answer to **part (a)(ii)** to solve

$$\begin{aligned} 4x + y - (k+1)z &= 4 \\ x + 3y + (k+2)z &= 3 \\ 2x + y + z &= 1 \end{aligned}$$

Give your answer in terms of k

[3 marks]

$x =$ ______ $y =$ ______ $z =$ ______

10

14 (a) Use de Moivre's theorem to express $\sin^6\theta$ in terms of $\cos 6\theta$, $\cos 4\theta$ and $\cos 2\theta$

[6 marks]

Answer

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14 (b) Use your answer to **part (a)** to show that

$$\int_0^{\frac{\pi}{6}} \sin^6\theta \, \mathrm{d}\theta = \frac{1}{96}\left(a\pi + b\sqrt{3}\right)$$

where a and b are integers.

[3 marks]

9

Turn over for the next question

15 The general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}\cos x + y\sin x = \frac{2x+5}{x^2+4x+5}\cos^2 x$$

can be written in the form $y = \mathrm{f}(x)$

15 (a) Find $\mathrm{f}(x)$

[9 marks]

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$\mathrm{f}(x) =$

15 (b) Find the particular solution of the differential equation where $\mathrm{f}(0) = \tan^{-1}(2)$

[2 marks]

$y =$

11

END OF QUESTIONS

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