OXFORD AQA
INTERNATIONAL QUALIFICATIONS

INTERNATIONAL A-LEVEL
FURTHER MATHEMATICS
(9665/FM03) Unit FP2 Pure Mathematics

Thursday 29 May 2025   07:00 GMT   Time allowed: 2 hours 30 minutes

Materials
- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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11	
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15	
TOTAL	

Answer **all** questions in the spaces provided.

1 The position vectors of the points A, B and C are

$$\mathbf{a} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \qquad \mathbf{b} = 11\mathbf{i} - \mathbf{j} + 5\mathbf{k} \qquad \mathbf{c} = 9\mathbf{i} + 4\mathbf{k}$$

Calculate the area of the triangle ABC

Give your answer in an exact form.

[4 marks]

Answer _____

4

2 The transformation S is represented by the matrix **M**

The transformation T is represented by the matrix **N**

A three-dimensional shape P is mapped onto the three-dimensional shape Q by a sequence of two transformations:

- a transformation S

followed by

- a transformation T

2 (a) Write down, in terms of **M** and **N** the matrix product that maps P onto Q

[1 mark]

Answer _____

2 (b) It is given that:

$\det \mathbf{N} = 2$

the volume of the three-dimensional shape P is 9 cubic units

the volume of the three-dimensional shape Q is 7 cubic units.

Find the possible values of $\det \mathbf{M}$

[3 marks]

Answer _____

4

Turn over ▶

3 The roots of the equation

$$z^3 + pz^2 + 65z + q = 0$$

are α, β and γ where p and q are real.

It is given that $\alpha = 4 - 3i$

3 (a) Find the values of β and γ

[3 marks]

$\beta = \underline{\hspace{4cm}}$ $\gamma = \underline{\hspace{4cm}}$

3 (b) Hence find the value of p and the value of q

[2 marks]

$p = $ _____ $q = $ _____

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4 Solve the equation $z^4 = 16i$

Give your solutions in the form $r\,e^{i\theta}$ where $r > 0$ and $0 \le \theta < 2\pi$

[5 marks]

Answer _____

5

Turn over ►

5 (a) Express $\dfrac{1}{2}(k+1)\left(4(k+1)^2+7(k+1)+1\right)$ in the form

$$\frac{1}{2}\left(4k^3+bk^2+ck+d\right)$$

where b, c and d are integers.

[2 marks]

Answer _____

5 (b) Prove by induction that for all integers $n \geq 1$

$$2\times3 + 5\times5 + 8\times7 + 11\times9 + \cdots + (3n-1)\times(2n+1) = \frac{1}{2}n\left(4n^2+7n+1\right)$$

[6 marks]

8

6   A curve C is given parametrically by the equations

$$x = \frac{t^3}{3} - \frac{t^2}{2} \quad \text{and} \quad y = \frac{4t^{\frac{5}{2}}}{5} \quad \text{for } 0 \le t \le 3$$

The point P on C is where $t = 0$

The point Q on C is where $t = \sqrt{2}$

Find the length of the arc PQ of the curve C

Give your answer in the form $\dfrac{a\sqrt{2}}{b} + c$ where a, b and c are integers.

[6 marks]

Answer _____

6

Turn over for the next question

Turn over ▶

7 Three planes have equations

$$(2-k)x + y + 2z = 5$$
$$x + 3y - 2z = -1$$
$$8x + 4y - (5k+1)z = 12$$

where k is a constant.

The planes **do not** meet at a unique point.

7 (a) Find the possible values of k

[3 marks]

Answer _____

7 (b) (i) Determine the number of solutions of the three equations for the smallest possible value of k

[3 marks]

7 (b) (ii) Hence state whether the three equations for the smallest possible value of k have consistent or inconsistent solutions.

[1 mark]

7 (b) (iii) Hence give a geometric interpretation for the three planes for the smallest possible value of k

[1 mark]

8

Turn over ▶

8 (a) Express $\dfrac{4r^2+8r+2}{r(r+1)(r+2)}$ in partial fractions.

[2 marks]

Answer _____

8 (b) Use your answer to **part (a)** with the method of differences to show that

$$\sum_{r=1}^{2n} \frac{(-1)^r\left(4r^2+8r+2\right)}{r(r+1)(r+2)} = \frac{4n+3}{(2n+1)(2n+2)} - \frac{3}{2}$$

[4 marks]

8 (c) Hence write down the value of

$$\sum_{r=1}^{\infty} \frac{(-1)^r \left(4r^2 + 8r + 2\right)}{r(r+1)(r+2)}$$

[1 mark]

Answer _____

7

Turn over ▶

9 It is given that $y = \cosh 2x$

9 (a) (i) Find $\dfrac{d^6 y}{dx^6}$ in terms of x

[1 mark]

Answer _____

9 (a) (ii) Hence show that the first four non-zero terms in ascending powers of x in the Maclaurin series expansion of $\cosh 2x$ are

$$1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6$$

[3 marks]

9 (b) Show that

$$\lim_{x \to 0} \left[\frac{\cosh 2x - \cos 2x}{\left(\ln(1+x) \right)^2} \right]$$

exists and calculate its value.

[6 marks]

Answer _____

10

Turn over ▶

10 (a) Use an integrating factor to find the general solution of the differential equation

$$\frac{dy}{dx} - \frac{y}{x} = \frac{x}{\sqrt{1-x^2}}\sin^{-1}x$$

Give your answer in the form $y = f(x)$

[7 marks]

$y = $ _____

10 (b) Find the particular solution of the differential equation for which $y = \dfrac{\pi^2}{4}$ when $x = 1$

Give your answer in the form $y = \mathrm{f}(x)$

[2 marks]

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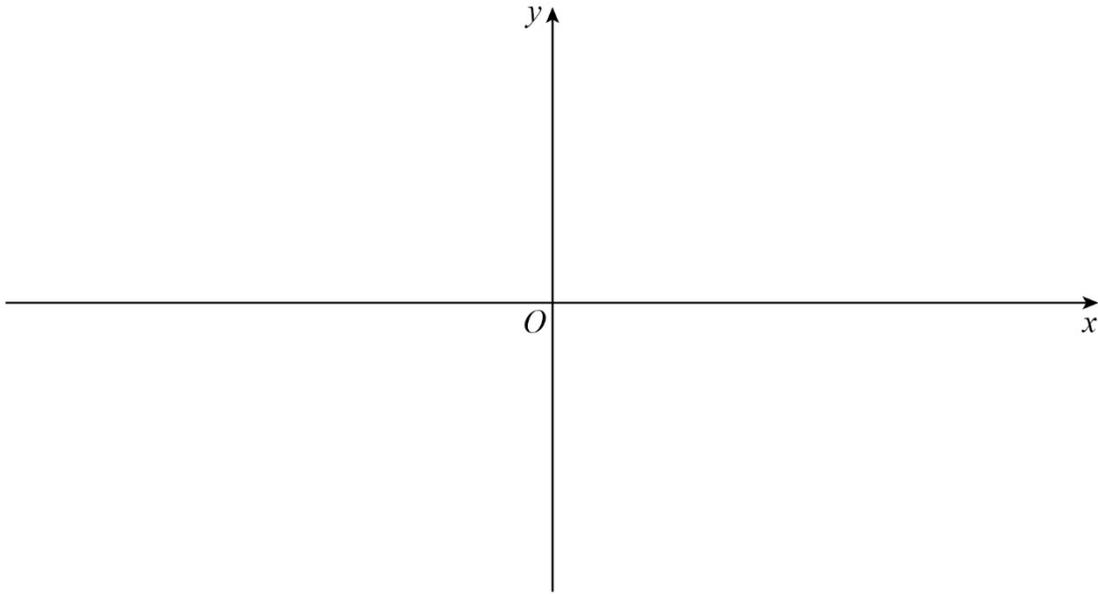
$y = $ _____

9

Turn over for the next question

Turn over ▶

11 (a)(i) On the axes below sketch the graph of $y = \cosh^{-1}(2x)$

[2 marks]

11 (a)(ii) By writing $y = \cosh^{-1}(2x)$ as $\cosh y = 2x$, prove that

$$\frac{dy}{dx} = \frac{2}{\sqrt{4x^2 - 1}}$$

[4 marks]

11 (b) A curve has equation

$$f(x) = 4 - x^2 + \cosh^{-1}(2x)$$

Prove that the curve has a single stationary point P

Express the x-coordinate of P in exact form.

[6 marks]

$x =$ _____

Turn over ▶

12 (a) The points $A(1, 1, 2)$, $B(2, 3, 6)$ and $C(-1, -2, -1)$ lie in the plane Π_1

Find the equation of the plane Π_1

Give your answer in the form $\mathbf{r} = \mathbf{w} + \lambda\mathbf{p} + \mu\mathbf{q}$ where $\mathbf{w}$, $\mathbf{p}$ and $\mathbf{q}$ are column vectors.

[3 marks]

Answer

12 (b) The plane Π_2 has equation $2x - y - 3z = 7$

The line L is the line of intersection of planes Π_1 and Π_2

Find the equation of L

Give your answer in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$ where $\mathbf{u}$ and $\mathbf{v}$ are column vectors.

[8 marks]

Answer _____

11

Turn over ▶

13 Find the solution of the differential equation

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} - 8y = 12e^{-2x} + 8$$

given that $y \to -1$ as $x \to \infty$ and that $y = 3$ when $x = 0$

Give your answer in the form $y = f(x)$

Clearly show any limiting processes used.

[11 marks]

$y =$ _____

11

Turn over ▶

14 The matrix **M** is given by

$$\mathbf{M} = \frac{1}{5}\begin{bmatrix} a & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}$$

where a is a real number.

The eigenvectors of **M** are $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$

The corresponding eigenvalues of **M** are λ_1, λ_2 and λ_3

14 (a) (i) It is given that $\lambda_1 = 2$ and $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Find the value of a

[2 marks]

$a =$

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14 (a) (ii) It is given that $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$ where $b > 0$

Find the value of b and the value of λ_2

[4 marks]

$b =$ _____ $\quad \lambda_2 =$ _____

Question 14 continues on the next page

Turn over ▶

14 (a) (iii) It is given that $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Find the value of λ_3

[1 mark]

$\lambda_3 =$ _____

14 (b) (i) Show that the eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are all perpendicular to each other.

[4 marks]

14 (b) (ii) Write down the Cartesian equation of the line of invariant points of the transformation represented by **M**

[1 mark]

Answer _____

Turn over for the next question

Turn over ▶

15 The curve C has polar equation

$$r^2 = \frac{64}{7 + 6\sin^2\theta + 6\sqrt{3}\sin\theta\cos\theta}$$

15 (a) Show that the Cartesian equation of C can be written in the form $\mathrm{f}(x, y) = 64$ where f is a function of x and y

[3 marks]

Answer _____

15 (b) The curve C is mapped onto the curve D by a reflection in the line $y = \dfrac{1}{\sqrt{3}}x$

Find the Cartesian equation of the curve D in an exact form.

[5 marks]

Answer _____

8

END OF QUESTIONS

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