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(9665/FM03) Unit FP2 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$\overrightarrow{AB}=\begin{bmatrix}5\\1\\2\end{bmatrix}$, $\overrightarrow{AC}=\begin{bmatrix}3\\2\\1\end{bmatrix}$	B1	Two correct directional vectors $\overrightarrow{BC}=\begin{bmatrix}-2\\1\\-1\end{bmatrix}$
	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{bmatrix}5\\1\\2\end{bmatrix}\times\begin{bmatrix}3\\2\\1\end{bmatrix}=\begin{bmatrix}-3\\1\\7\end{bmatrix}$	M1	Cross product of their directional vectors – not position vectors $\overrightarrow{BA}\times\overrightarrow{BC}=\begin{bmatrix}3\\-1\\-7\end{bmatrix}$ $\overrightarrow{CA}\times\overrightarrow{CB}=\begin{bmatrix}-3\\1\\7\end{bmatrix}$
	$\text{Area}=\frac{1}{2}\left\lVert\begin{bmatrix}-3\\1\\7\end{bmatrix}\right\rVert$	M1	Uses correct formula for area of a triangle with their cross product
	$=\frac{\sqrt{59}}{2}$	A1	Correct exact answer

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	**NM**	B1	
		1	

Q	Answer	Marks	Comments
2(b)	$\det(\mathbf{NM}) \times 9 = \pm 7$	M1	Use of $\det(\mathbf{NM}) \times \text{Volume}\,P = \text{Volume}\,Q$ Condone $\det(\mathbf{NM})$ and lack of $\pm$ here
	$2 \times \det(\mathbf{M}) \times 9 = \pm 7$	M1	Use of $\det(\mathbf{NM}) = \det(\mathbf{N}) \times \det(\mathbf{M})$ Condone use of $\det(\mathbf{MN}) = \det(\mathbf{M}) \times \det(\mathbf{N})$ and lack of $\pm$ here
	$\det(\mathbf{M}) = \pm \frac{7}{18}$	A1	Must have **NM** in **part (a)** or clearly shown use of **NM** in **part (b)** condone $\pm$ appearing at the end
		3	

	Question 2 Total	4	

Q	Answer	Marks	Comments
3(a)	$\beta = 4+3\text{i}$	B1	Correctly states the value of β
	$65 = (4-3\text{i})\times(4+3\text{i})$ $+(4-3\text{i})\gamma+\gamma(4+3\text{i})$	M1	Correct use of $\frac{c}{a} = \alpha\beta+\beta\gamma+\gamma\alpha$
	$65 = 25+8\gamma$		
	$\gamma = 5$	A1	Correct value of γ
		3	

Q	Answer	Marks	Comments
3(b)	$\frac{-p}{1} = 4-3\text{i}+4+3\text{i}+5$ $p=-13$ $\frac{-q}{1} = (4-3\text{i})\times(4+3\text{i})\times 5$ $q=-125$	M1	Correct use of $\alpha+\beta+\gamma = \frac{-b}{a}$ and $\alpha\beta\gamma = \frac{-d}{a}$ with their roots to get values for p and q
	$p=-13$ $q=-125$	A1	Obtains the correct values of p and q
			If 0/2 scored **SC1** for writing $p = -(\alpha+\beta+\gamma)$ **and** $q = -\alpha\beta\gamma$
		2	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4	$z^4 = 16\mathrm{i} = 16\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}$	B1	Correct Eulerian form of $16\mathrm{i}$
	$z^4 = 16\mathrm{e}^{\left(\frac{\pi}{2}+2\pi k\right)\mathrm{i}}$		
	$z = 2\mathrm{e}^{\frac{\pi(1+4k)}{8}\mathrm{i}}$	M1	Divides their argument by 4
	$r = 2$	B1	Correct value of r
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}\left(\frac{-7\pi}{8}\right), \frac{13\pi}{8}\left(\frac{-3\pi}{8}\right)$	A1	Correct angles (mod 2π)
	$z = 2\mathrm{e}^{\frac{\pi}{8}\mathrm{i}}, 2\mathrm{e}^{\frac{5\pi}{8}\mathrm{i}}, 2\mathrm{e}^{\frac{9\pi}{8}\mathrm{i}}, 2\mathrm{e}^{\frac{13\pi}{8}\mathrm{i}}$	A1	Correct with $0 \le \theta < 2\pi$

	Question 4 Total	**5**	

Q	Answer	Marks	Comments
5(a)	$\frac{1}{2}(k+1)\left[4(k+1)^2+7(k+1)+1\right]$		
	$=\frac{1}{2}(k+1)\left(4k^2+8k+4+7k+7+1\right)$	M1	Correctly expands and collects to get at least one of b, c or d correct
	$=\frac{1}{2}(k+1)\left(4k^2+15k+12\right)$	A1	Correctly obtains the required result
	$=\frac{1}{2}\left(4k^3+19k^2+27k+12\right)$		
		2	

Q	Answer	Marks	Comments
5(b)	LHS $= 2\times 3 = 6$ RHS $= \frac{1}{2}\times 1\times\left(4\times 1^2+7\times 1+1\right)=6$ [True for $n=1$]	B1	Demonstrates result is true for $n=1$
	$n=k+1$ Assume true $n=k$ (*) $2\times 3+5\times 5+8\times 7+11\times 9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\sum_{r=1}^{k}(3r-1)\times(2r+1)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\frac{1}{2}k\left(4k^2+7k+1\right)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$	M1	Assumes true for $n=k$ and adds $\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ to $=\frac{1}{2}k\left(4k^2+7k+1\right)$ Condone lack of $2\times 3+5\times 5+8\times 7+11\times 9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$
	$=\frac{1}{2}k\left(4k^2+7k+1\right)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\frac{1}{2}\left(4k^3+7k^2+k\right)+\left(6k^2+13k+6\right)$ $=\frac{1}{2}\left(4k^3+19k^2+27k+12\right)$	M1	Expands and collects terms
		A1	Expands and collects terms correctly. Condone lack of $2\times 3+5\times 5+8\times 7+11\times 9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$
	$=\frac{1}{2}(k+1)\left[4(k+1)^2+7(k+1)+1\right]$ from **(a)**	A1	Shows result is true for $n=k+1$ Must include $2\times 3+5\times 5+8\times 7+11\times 9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$ or $\sum_{r=1}^{k+1}(3r-1)\times(2r+1)=$
	Hence true for $n=k+1$ (**) and since true for $n=1$ (***) By induction true for all integers $n\geq 1$ (****)	E1	Must have (*), (**), (***) present, previous 5 marks scored and a final statement (****) clearly indicating that it relates to all integers $n\geq 1$ and induction
		6	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6	$\frac{dx}{dt} = t^2 - t$		
	$\frac{dy}{dt} = 2t^{\frac{3}{2}}$	B1	Correct expression for $\frac{dx}{dt}$ and $\frac{dy}{dt}$
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^4 - 2t^3 + t^2 + 4t^3$	M1	Correctly expands their $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^4 + 2t^3 + t^2$ $= \left(t^2 + t\right)^2$	A1	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(t^2 + t\right)^2$
	$PQ = \int_0^{\sqrt{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$		
	$PQ = \int_0^{\sqrt{2}} \left(t^2 + t\right)\ dt$	B1	Their correct integrand, with dt and correct limits. All seen here
	$PQ = \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_0^{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^3}{3} + \frac{\left(\sqrt{2}\right)^2}{2}$	M1	Correct use of limits in an expression of the form $mt^3 + nt^2$
	$PQ = \frac{2\sqrt{2}}{3} + 1$	A1	**CSO** – form of answer given

	Question 6 Total	**6**	

Q	Answer	Marks	Comments
7(a)	$0 = \begin{vmatrix} 2-k & 1 & 2 \\ 1 & 3 & -2 \\ 8 & 4 & -5k-1 \end{vmatrix}$ $= (2-k)\{3(-5k-1)+8\}$ $-(-5k-1+16)+2(4-24)$	**M1**	Sets determinant = 0 and attempts to expand
	$0 = 15k^2 - 35k + 10 + 5k - 55$ $= 15k^2 - 30k - 45$ $0 = k^2 - 2k - 3$	**A1**	Obtains a correct quadratic equation
	$k = -1$ or $k = 3$	**A1**	Obtains correct values of k
		3	

Q	Answer	Marks	Comments
7(b)(i)	$3x + y + 2z = 5 \quad (\text{I})$ $x + 3y - 2z = -1 \quad (\text{II})$ $8x + 4y + 2z = 5 \quad (\text{III})$	B1ft	Correct system of equations for their smallest k
	$(\text{I})+(\text{II}) \Rightarrow 4x + 4y = 4$ $(\text{III})+2(\text{II}) \Rightarrow 10x + 10y = 10$	M1	Eliminating one variable in order to compare two simultaneous equations
	Infinite solutions	A1	From comparing correct equations
		3	

Q	Answer	Marks	Comments
7(b)(ii)	Consistent	E1ft	Must follow from their answer to **part (b)(i)**
		1	

Q	Answer	Marks	Comments
7(b)(iii)	Line of intersection	E1	**CAO**, **oe** Must come from correct working with correct equations in **part (b)(i)**
		1	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8(a)	$\frac{A}{r}+\frac{B}{r+1}+\frac{C}{r+2}$ $4r^2+8r+2=A(r+1)(r+2)+Br(r+2)+Cr(r+1)$ $r=0:2=2A$; $r=-1:\ -2=-B$; $r=\ -2:2=2C$	M1	$4r^2+8r+2=A(r+1)(r+2)$ $+Br(r+2)+Cr(r+1)$ used to form either at least two correct equations in A, B or C or two correct simultaneous equations in A, B and C
	$\frac{1}{r}+\frac{2}{r+1}+\frac{1}{r+2}$	A1	Correct partial fractions
		2	

Q	Answer	Marks	Comments
8(b)	$\sum_{r=1}^{2n} \frac{(-1)^r (4r^2+8r+2)}{r(r+1)(r+2)}$ $= -\frac{1}{1} - \frac{2}{2} - \frac{1}{3}$ $+\frac{1}{2} + \frac{2}{3} + \frac{1}{4}$ $-\frac{1}{3} - \frac{2}{4} - \frac{1}{5}$ $\vdots$ $+\frac{1}{2n-2} + \frac{2}{2n-1} + \frac{1}{2n}$ $-\frac{1}{2n-1} - \frac{2}{2n} - \frac{1}{2n+1}$ $+\frac{1}{2n} + \frac{2}{2n+1} + \frac{1}{2n+2}$	M1 M1	Writes at least three lines of subtracting terms, using their **part (a)** Writes at least three consecutive lines, showing cancellation
	$= -\frac{1}{1} - \frac{1}{2} + \frac{1}{2n+1} + \frac{1}{2n+2}$	M1	Correctly reduces the expression to four terms
	$= -\frac{3}{2} + \frac{4n+3}{(2n+1)(2n+2)}$	A1	Obtains the required result This mark is only available if at least the first three lines and the last three lines are present. **AG**
		4	

Q	Answer	Marks	Comments
8(c)	$-\frac{3}{2}$	B1	**CAO**
		1	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)(i)	$\frac{d^6y}{dx^6} = 64\cosh 2x$	B1	ACF
		1	

Q	Answer	Marks	Comments
9(a)(ii)	When $x = 0$: $y = 1,\ \frac{dy}{dx} = 0,\ \frac{d^2y}{dx^2} = 4,\ \frac{d^3y}{dx^3} = 0,$ $\frac{d^4y}{dx^4} = 16,\ \frac{d^5y}{dx^5} = 0,\ \frac{d^6y}{dx^6} = 64$	B1	
	Maclaurin series: $= 1 + x \times 0 + \frac{x^2}{2!} \times 4 + \frac{x^3}{3!} \times 0$ $+ \frac{x^4}{4!} \times 16 + \frac{x^5}{5!} \times 0 + \frac{x^6}{6!} \times 64$	M1	**ft** Their derivatives
	$\cosh 2x = 1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6$	A1	**AG** Must be convincingly shown
		3	

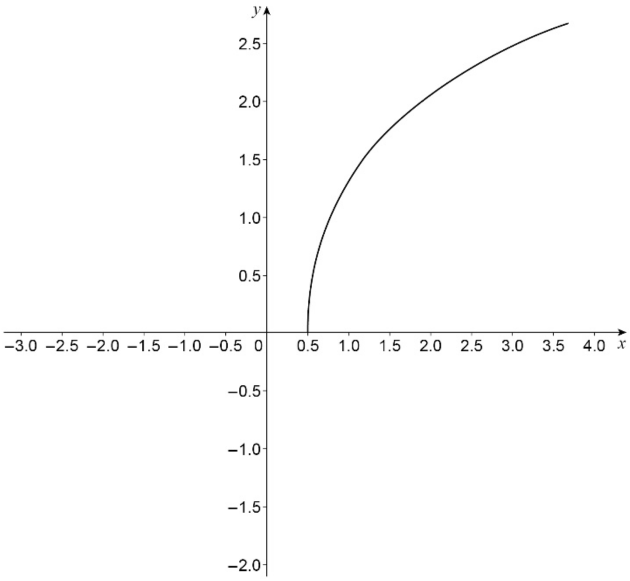
Q	Answer	Marks	Comments
9(b)	$\cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \ldots$	**B1**	Condone $\cos 2x = 1 - \frac{4}{2!}x^2 + \frac{16}{4!}x^4 - \frac{64}{6!}x^6$
	$\left[\ln\left(1+x^2\right)\right]^2 = \left[x - \frac{x^2}{2}\right]^2 = x^2 - x^3 + \ldots$	**B1**	
	$\frac{\cosh 2x - \cos 2x}{\left[\ln(1+x)\right]^2} = \frac{4x^2 + \frac{8}{45}x^6 + \ldots}{x^2 - x^3 + \ldots}$	**B1**	
	$\lim_{x \to 0}\left[\frac{\cosh 2x - \cos 2x}{\left[\ln(1+x)\right]^2}\right]$ $= \lim_{x \to 0}\left[\frac{4x^2 + \frac{8}{45}x^6 + \ldots}{x^2 - x^3 + \ldots}\right]$		
	$= \lim_{x \to 0}\left[\frac{4 + \frac{8}{45}x^4 + \ldots}{1 - x + \ldots}\right]$	**M1**	May be on last line May see $O\left(x^4\right)$ and/or $O(x)$
	So limit exists	**E1**	With comment
	$= 4$	**A1**	Can score **B1 B1 B1 M1 E0 A1**
		6	

	Question 9 Total	**10**	

Q	Answer	Marks	Comments
10(a)	I.F. is $e^{-\int \frac{1}{x} dx} = e^{-\ln(x)}$	M1	I.F. identified and integration attempted
	$= \frac{1}{x}$	A1	Correct integrating factor
	$\frac{y}{x} = \int \left(\frac{1}{\sqrt{1-x^2}} \sin^{-1}x \right) dx$	m1	Multiplying both sides of the Standard Form by their I.F. and integrating LHS to get $y \times$I.F.
	$u = \sin^{-1}x \qquad \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad v = \sin^{-1}x$	M1	Uses Integration by Parts
	$\int \left(\frac{1}{\sqrt{1-x^2}} \sin^{-1}x \right) dx$ $= \left[\left(\sin^{-1}x \right)^2 \right] - \int \left(\frac{1}{\sqrt{1-x^2}} \sin^{-1}x \right) dx$	A1	Correct use of Integration by Parts
	$\int \left(\frac{1}{\sqrt{1-x^2}} \sin^{-1}x \right) dx = \frac{1}{2} \left[\left(\sin^{-1}x \right)^2 \right]$	M1	Rearranges to get equation only in $\int \left(\frac{1}{\sqrt{1-x^2}} \sin^{-1}x \right) dx$
	$y = x \left[\frac{1}{2} \left(\sin^{-1}x \right)^2 + c \right]$	A1	Correct result ACF, must be $y = \cdots$ **ISW**
		7	

Q	Answer	Marks	Comments
10(b)	$\frac{\pi^2}{4} = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + c$ $c = \frac{\pi^2}{8}$	M1	Substituting $y = \frac{\pi^2}{4}$ and $x = 1$ into their GS (with a constant of integration) to find a value of c
	$y = x\left[\frac{1}{2}\left(\sin^{-1}x\right)^2 + \frac{\pi^2}{8}\right]$	A1	Correct PS ACF, must be $y = \cdots$ **ISW**
		2	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11(a)(i)		M1 A1	Correct shape with x intercept clearly >0 (does not need to be labelled) Fully correct with correct x intercept labelled (0.5)
		2	

Q	Answer	Marks	Comments
11(a)(ii)	$\cosh y = 2x$		
	$\frac{dy}{dx}\sinh y = 2$	B1	Correctly differentiates equation
	$\frac{dy}{dx} = \frac{2}{\sinh y} = \frac{\pm 2}{\sqrt{\cosh^2 y - 1}}$	M1	Use of $\sinh^2 y = \cosh^2 y - 1$
	$\frac{dy}{dx} = \frac{\pm 2}{\sqrt{4x^2 - 1}}$	A1	Correct equation including $\pm$
	From graph $\frac{dy}{dx} > 0$ $\therefore \frac{dy}{dx} = \frac{2}{\sqrt{4x^2 - 1}}$	E1	Correct explanation leading to required result. **AG**
		4	

Q	Answer	Marks	Comments
11(b)	$f'(x) = -2x + \dfrac{2}{\sqrt{4x^2 - 1}} = 0$	B1	Correctly differentiates $f(x)$
	$x = \dfrac{1}{\sqrt{4x^2 - 1}}$ $x\sqrt{4x^2 - 1} = 1$ $x^2(4x^2 - 1) = 1$		
	$4(x^2)^2 - x^2 - 1 = 0$	M1	Obtains a quadratic in x^2
	$x^2 = \dfrac{1-\sqrt{17}}{8}$ or $x^2 = \dfrac{1+\sqrt{17}}{8}$	A1	Correct exact roots
	$x^2 = \dfrac{1-\sqrt{17}}{8} < 0$ reject as $x \in \mathbb{R}$	E1	Correct rejection of $x^2 < 0$ roots
	$x^2 = \dfrac{1+\sqrt{17}}{8}$ $x = -\sqrt{\dfrac{1+\sqrt{17}}{8}}$ reject as $x > \dfrac{1}{2}$	E1	Correct rejection of $x = -\sqrt{\dfrac{1+\sqrt{17}}{8}}$ root. Must reference $x > \dfrac{1}{2}$ and not just $x > 0$
	$\therefore x = \sqrt{\dfrac{1+\sqrt{17}}{8}}$ Hence $f(x)$ has a single stationary point	A1	**ACF** **A1** with correct conclusion **CSO**
		6	

	Question 11 Total	**12**	

Q	Answer	Marks	Comments
12(a)	$\mathbf{p} = \overrightarrow{AB} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{q} = \overrightarrow{AC} = \begin{bmatrix} -2 \\ -3 \\ -3 \end{bmatrix}$	**M1 A1**	**M1**: clear attempt to calculate two directional vectors **A1**: two correct directional vectors
	$\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} -2 \\ -3 \\ -3 \end{bmatrix}$	**A1**	**oe** In correct form
		3	

Q	Answer	Marks	Comments
12(b)	$\mathbf{n}_1 = \begin{bmatrix}1\\2\\4\end{bmatrix} \times \begin{bmatrix}-2\\-3\\-3\end{bmatrix} = \begin{bmatrix}6\\-5\\1\end{bmatrix}$	M1	Calculates the cross product of their directional vectors from **part (a)**
	$d_1 = \begin{bmatrix}1\\1\\2\end{bmatrix} \bullet \begin{bmatrix}6\\-5\\1\end{bmatrix} = 3$	M1	Calculates the scalar product of their $\mathbf{n}_1$ and the position vector of A, B or C
	$6x - 5y + z = 3$	A1ft	Correct Cartesian form of Π_1 for their $\mathbf{n}_1$ and their d_1 **PI** Later working
	$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{bmatrix}6\\-5\\1\end{bmatrix} \times \begin{bmatrix}2\\-1\\-3\end{bmatrix}$	M1	Calculates cross product of their normal vectors
	$\mathbf{v} = \alpha\begin{bmatrix}4\\5\\1\end{bmatrix}$	A1	Correct **v** (accept any scalar multiple)
	$z = 0$ $6x - 5y = 3$ $2x - y = 7$ $x = 8, y = 9$	M1	Sets $x, y,$ or $z = \text{constant}\ (\text{eg } 0)$ and attempts to solve the resultant simultaneous equations
	$\mathbf{u} = \begin{bmatrix}8\\9\\0\end{bmatrix}$	A1	A correct value of **u**, eg $\begin{bmatrix}0\\-1\\-2\end{bmatrix}$ or $\begin{bmatrix}\frac{4}{5}\\0\\\frac{-9}{5}\end{bmatrix}$
	$\left(\mathbf{r} - \begin{bmatrix}8\\9\\0\end{bmatrix}\right) \times \begin{bmatrix}4\\5\\1\end{bmatrix} = \mathbf{0}$	A1ft	Correct for their **v** and their **u** Must have $= \mathbf{0}$
		8	

	Question 12 Total	**11**	

Q	Answer	Marks	Comments
13	Auxiliary equation $m^2 - 2m - 8 = 0$ $m = -2, 4$	M1	Solves correct auxiliary equation
	$\left[y_{\text{CF}} =\right]$ $A\mathrm{e}^{-2x} + B\mathrm{e}^{4x}$	A1	Correct CF
	Particular Integral $\left[y_{\text{PI}} =\right]$ $Cx\mathrm{e}^{-2x} + D$	M1	Uses correct form for particular integral
	$\left[y'_{\text{PI}} =\right]$ $C\mathrm{e}^{-2x} - 2Cx\mathrm{e}^{-2x}$ $\left[y''_{\text{PI}} =\right]$ $-4C\mathrm{e}^{-2x} + 4Cx\mathrm{e}^{-2x}$	A1	Correct first and second derivatives of correct y_{PI}
	$\left(-4C\mathrm{e}^{-2x} + 4Cx\mathrm{e}^{-2x}\right)$ $-2\left(C\mathrm{e}^{-2x} - 2Cx\mathrm{e}^{-2x}\right)$ $-8\left(Cx\mathrm{e}^{-2x} + D\right)$ $= 12\mathrm{e}^{-2x} + 8$	M1	Substitutes their first and second derivatives of y_{PI} and into DE and attempts to compare their coefficients
	e^{-2x} $-6C = 12$ $C = -2$	A1	Correct coefficient of e^{-2x}
	constant $-8D = 8$ $D = -1$	B1	Correct constant term
	General Solution $\left[\mathrm{f}(x) =\right] A\mathrm{e}^{-2x} + B\mathrm{e}^{4x} - 2x\mathrm{e}^{-2x} - 1$ As $x \to \infty$ $\lim_{x\to\infty}(y) = -1$ $= A\lim_{x\to\infty}\left(\mathrm{e}^{-2x}\right) + B\lim_{x\to\infty}\left(\mathrm{e}^{4x}\right) - 2\lim_{x\to\infty}\left(x\mathrm{e}^{-2x}\right) - 1$ $= B\lim_{x\to\infty}\left(\mathrm{e}^{4x}\right) - 1$ $0 = B\lim_{x\to\infty}\left(\mathrm{e}^{4x}\right)$ $B = 0$	M1	Correctly uses limits to obtain a value for B
	As $\lim_{x\to\infty}\left(x\mathrm{e}^{-2x}\right) = 0$	E1	States $\lim_{x\to\infty}\left(x\mathrm{e}^{-2x}\right) = 0$
	$x = 0$ $y = 3 = A + B - 1 = A - 1$ $A = 4$	A1	Correct values for A and B
	$y = 4\mathrm{e}^{-2x} - 2x\mathrm{e}^{-2x} - 1$	A1	Correct **ISW** Can obtain this mark even if E0

	Question 13 Total	**11**	

Q	Answer	Marks	Comments
14(a)(i)	$\frac{1}{5}\begin{bmatrix} a & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$	M1	Uses $\mathbf{Mv}_1 = \lambda_1\mathbf{v}_1$
	$\frac{1}{5}\begin{bmatrix} a-4 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$		
	$a = 14$	A1	Correct value of a
		2	

Q	Answer	Marks	Comments
14(a)(ii)	$\frac{1}{5}\begin{bmatrix} 14 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix} = \lambda_2\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$		
	$\frac{1}{5}\begin{bmatrix} -28-2b \\ 0 \\ 4+11b \end{bmatrix} = \lambda_2\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$		
	$-28 - 2b = -10\lambda_2$ $4 + 11b = 5b\lambda_2$	M1	Obtains simultaneous equations in b and λ_2
	$\frac{-28-2b}{-10} = \frac{4+11b}{5b}$		
	$10b^2 + 30b - 40 = 0$	M1	Obtains quadratic equation in b or λ_2 $25\lambda_2^2 - 125\lambda_2 + 150 = 0$
	$b = 1$ $\left[\text{and } b = -4 \ (\text{reject as } b > 0)\right]$	A1	
	$\lambda_2 = 3$	A1	
		4	

Q	Answer	Marks	Comments
14(a)(iii)	$\frac{1}{5}\begin{bmatrix}14 & 0 & -2\\ 0 & 5 & 0\\ -2 & 0 & 11\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\lambda_3\begin{bmatrix}0\\1\\0\end{bmatrix}$		
	$\lambda_3=1$	B1	Obtains correct value of λ_3
		1	

Q	Answer	Marks	Comments
14(b)(i)	$\begin{bmatrix}1\\0\\2\end{bmatrix}\bullet\begin{bmatrix}-2\\0\\1\end{bmatrix}$	M1	Calculates scalar product of two eigenvectors
	$=0$ so $\mathbf{v}_1$ and $\mathbf{v}_2$ are perpendicular	A1	Obtains 0 and interprets this as **perpendicular**
	$\begin{bmatrix}1\\0\\2\end{bmatrix}\times\begin{bmatrix}-2\\0\\1\end{bmatrix}$	M1	Calculates the vector product of the **same** two eigenvectors or calculates the scalar products of the other two pairs of eigenvectors
	$=\begin{bmatrix}0\\-5\\0\end{bmatrix}=-5\begin{bmatrix}0\\1\\0\end{bmatrix}$ so $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are all perpendicular to each other	A1	Shows that the vector product is a multiple of the third eigenvector and states that the three eigenvectors are **mutually** perpendicular or Shows that all three scalar products are zero and states that the three eigenvectors are **mutually** perpendicular
		4	

Q	Answer	Marks	Comments
14(b)(ii)	$x=z=0$	B1	Obtains correct Cartesian equation from $\mathbf{v}_3$
		1	

	Question 14 Total	**12**	

Q	Answer	Marks	Comments
15(a)	$7r^2 + 6(r\sin\theta)^2 + 6\sqrt{3}r\sin\theta r\cos\theta = 64$		
	$7(x^2 + y^2) + 6y^2 + 6\sqrt{3}\ yx = 64$	M1	Correct use of $x = r\cos\theta$ and $y = r\sin\theta$
		M1	Correct use of $r^2 = x^2 + y^2$
	$7x^2 + 6\sqrt{3}\ xy + 13y^2 = 64$	A1	**ACF** Condone $64 = 7x^2 + 6\sqrt{3}\ xy + 13y^2$ **NMS** = 0/3
		3	

Q	Answer	Marks	Comments
15(b)	$\tan\theta = 1/\sqrt{3}$ so $\theta = \pi/6$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(2\pi/6) & \sin(2\pi/6) \\ \sin(2\pi/6) & -\cos(2\pi/6) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $x = \dfrac{x' + \sqrt{3}\,y'}{2}$ $y = \dfrac{\sqrt{3}\,x' - y'}{2}$	B1	Obtains $x = \dfrac{x' + \sqrt{3}\,y'}{2}$ and $x = \dfrac{x' + \sqrt{3}\,y'}{2}$ Do not condone lack of ‘
	$64 = 7\left(\dfrac{x' + \sqrt{3}\,y'}{2}\right)^2$ $+6\sqrt{3}\left(\dfrac{x' + \sqrt{3}\,y'}{2}\right)\left(\dfrac{\sqrt{3}\,x' - y'}{2}\right)$ $+13\left(\dfrac{\sqrt{3}\,x' - y'}{2}\right)^2$	M1	Uses $x = \dfrac{\pm x' \pm \sqrt{3}\,y'}{2}$ and $y = \dfrac{\pm\sqrt{3}\,x' \pm y'}{2}$ Condone lack of ‘
	So equation of curve *E* is $64 = \dfrac{1}{4}\left(7x^2 + 14\sqrt{3}\,xy + 21y^2\right)$ (I) $+\dfrac{1}{4}\left(18x^2 + 12\sqrt{3}\,xy - 18y^2\right)$ (II)) $+\dfrac{1}{4}\left(39x^2 - 26\sqrt{3}\,xy + 13y^2\right)$ (III)	m1	Correctly expands to obtain at least one of their terms (I), (II) or (III) correct Condone presence of ‘
		A1ft	Obtains their correct equation of curve *E* as the unsimplified sum of x^2, xy and y^2 terms ie their (I), (II) and (III) correct Condone presence of ‘
	$64 = 16x^2 + 4y^2$ $16 = 4x^2 + y^2$	A1	**ACF** eg $1 = \dfrac{x^2}{4} + \dfrac{y^2}{16}$ Can score B0M1m1A1A1 Condone presence of ‘ **NMS** = 0/5
		5	

	Question 15 Total	**8**	