Mark Scheme (Results)

Summer 2026

Pearson Edexcel International Advanced Level

In Further Pure Mathematics F2 (WFM02) Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2026

Question Paper Log Number P79504A

Publication Code WFM02_01A_2606_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)

  Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
- ft – follow through
  - the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso – correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ – The answer is printed on the paper or ag- answer given
- └ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
  - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = ...$
  - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = ...$
- Formula
  - Attempt to use the correct formula (with values for *a*, *b* and *c*).
- Completing the square
  - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = ...$

Method marks for differentiation and integration:

- Differentiation
  - Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
  - Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$$\frac{2}{x+3}<1-\frac{3}{x-2}$$	
	CV's $x=-3, x=2$ seen anywhere (perhaps on a diagram)	B1
	$2(x+3)(x-2)^2<(x+3)^2(x-2)^2-3(x+3)^2(x-2)$ **OR** $\frac{2}{x+3}<\frac{x-5}{x-2}\Rightarrow 2(x+3)(x-2)^2<(x-5)(x+3)^2(x-2)$ **OR** $\frac{2}{x+3}<\frac{x-5}{x-2}\Rightarrow\frac{2}{x+3}-\frac{x-5}{x-2}\Rightarrow\frac{\mathrm{f}(x)}{(x+3)(x-2)}<0$	M1
	$(x+3)(x-2)\left[2(x-2)-(x+3)(x-2)+3(x+3)\right]$ $\Rightarrow(x+3)(x-2)\left(-x^2+4x+11\right)\ (<0)$ **OR** $(x+3)(x-2)\left[2(x-2)-(x-5)(x+3)\right]$ $\Rightarrow(x+3)(x-2)\left(-x^2+4x+11\right)\ (<0)$ **OR** $\frac{2(x-2)-(x+3)(x-5)}{(x+3)(x-2)}\Rightarrow\frac{-x^2+4x+11}{(x+3)(x-2)}\ (<0)$ **OR** $x^4-3x^3-21x^2+13x+66\ (<0)$	M1
	$-x^2+4x+11=0\Rightarrow x=\ldots.(\text{usual rules})$	M1
	$x=2\pm\sqrt{15}$	A1
	$x<-3,\ \ 2-\sqrt{15}<x<2,\ x>2+\sqrt{15}$ **OR** $x\in(-\infty,-3)\cup\left(2-\sqrt{15},2\right)\cup\left(2+\sqrt{15},\infty\right)$	M1A1
		(7)
		Total 7

Notes
B1: CV's -3 **and** 2 seen anywhere, on a diagram or stated **M1:** Attempts to multiply both sides by $(x+3)^2(x-2)^2$ - allow if one side correct **OR** combine terms on RHS with common denominator and multiply by $(x+3)^2(x-2)^2$ allow if one side correct **OR** combine fractions to a single fraction with common denominator. Allow with >, <, = etc between the terms. **M1:** Factorises out $(x+3)(x-2)$ and makes progress towards a 3-term quadratic **OR** combines terms on the numerator and makes progress towards a 3-term quadratic **OR** multiplies out and combines all terms to obtain a 4-term quartic (terms must be collected) Allow sign errors only if they progress to a quartic. Allow with >, <, = etc between the terms. **M1**: Correct attempt at solving **their** three-term quadratic factor (usual rules) or their quartic- may be solved on a calculator, but if incorrect solutions for their equation, then method needed to gain this mark. Allow with >, <, = etc between the terms. **A1:** Correct solutions obtained from correct quadratic/quartic and condone decimal equivalents for this mark (awrt 5.87 and awrt -1.87) **M1:** Obtains the correct form for the solution using their CV's (condoning strict/non-strict inequalities) $x<a$, $b<x<c$, $x>d$ with $a<b<c<d$ **A1:** All three correct in exact form (not decimals) and no extras and must have all inequality signs correct. Allow equivalent notations e.g. soft brackets for intervals. Ignore any word between the regions but withhold this last mark if $\cap$ used with set notation. **Allow non-strict inequalities to score for all but the final mark**. **Allow recovery from non-strict to strict if the final solution set is fully correct.** **Note** Multiplication of the inequality by $(x+3)(x-2)$ only can gain max B1M0M0M1A1M1A0 BUT setting the equations equal and then multiplying by $(x+3)(x-2)$ to find the POIs can score full marks if the correct values and inequality solution are found.

Question Number	Scheme	Marks
2(a)	$\frac{1}{(r+1)!}-\frac{1}{(r+2)!}=\frac{(r+2)!-(r+1)!}{(r+1)!(r+2)!}=\left\{\frac{(r+1)!((r+2)-1)}{(r+1)!(r+2)!}\right\}$ OR $\frac{1}{(r+1)!}-\frac{1}{(r+2)!}=\frac{r+2-1}{(r+2)!}$	M1
	$\frac{\cancel{(r+1)!}(r+1)}{\cancel{(r+1)!}(r+2)!}=\frac{r+1}{(r+2)!}$	A1*
		(2)
(b)	$r=1: \quad \left(\frac{1}{2!}-\cancel{\frac{1}{3!}}\right).$ $r=2: \quad \left(\cancel{\frac{1}{3!}}-\cancel{\frac{1}{4!}}\right)$ $\vdots \qquad \vdots$ $\vdots \qquad \vdots$ $r=n-1: \quad \left(\cancel{\frac{1}{n!}}-\cancel{\frac{1}{(n+1)!}}\right)$ $r=n: \quad \left(\cancel{\frac{1}{(n+1)!}}-\frac{1}{(n+2)!}\right)$	M1
	$\left[\sum_{r=1}^{n}\frac{r+1}{(r+2)!}=\right]\ \frac{1}{2}-\frac{1}{(n+2)!}=\frac{(n+2)!-2}{2(n+2)!}$	A1A1
		(3)
(c)	$\left[\sum_{r=4}^{10}\frac{3r+3}{(r+2)!}=\right]3\left[\mathrm{f}(10)-\mathrm{f}(3)\right]=3\left[\left(\frac{1}{2}-\frac{1}{12!}\right)-\left(\frac{1}{2}-\frac{1}{5!}\right)\right]$ $=3\left[\frac{(12)!-2}{2(12)!}-\frac{(5)!-2}{2(5)!}\right]$	M1
	$=3\left[\frac{1}{5!}-\frac{1}{12!}\right]=0.024999\ldots=0.0250$	A1
		(2)
		Total 7

Notes
(a) **M1:** Indicates intention to combine over a common denominator, with only sign errors condoned on the numerator. This mark may be awarded by sight of two separate correct terms with a common denominator. May work in reverse from right hand side to left. This mark can be scored with two separate fractions with the same denominator **A1*:** Fully correct proof. Must see evidence of cancellation (which can be implied by a correct factorised form) or one line of correct intermediate working before the stated printed result. (b) **M1:** Correct process of differences evidenced in their work, e.g. attempts first two and last or first and last two etc. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at $r = 0$ if they start earlier than $r = 1$. **A1:** Correct non-cancelling terms. Accept 2 or 2! Sight of the correct non-cancelling terms is M1A1 **A1:** Correct simplified expression obtained. Accept 2 or 2! Note that A0A1 is not possible. (c) **M1:** Attempts $3\left[\mathrm{f}(10)-\mathrm{f}(3)\right]$ using **their answer to (b)**. Must be indication of subtraction and the 3 must be included. The '$-\frac{1}{2}$'s may be omitted and allow a slip in signs/brackets. They may use (a) again to obtain $\mathrm{f}(3)$ but their answer to (b) must be used for $\mathrm{f}(10)$. Allow from attempts starting at $r=0$ in their summations. $\mathrm{f}(10)-\mathrm{f}(4)$ is M0 and subtraction of decimals is M0 unless supported by evidence of using (b) **A1:** Correct answer only. Note: 0.025 or 1/40 is A0

Question Number	Scheme	Marks
3(a)	$y=(1-2x)^2\ln(1-2x)$ $\Rightarrow \dfrac{dy}{dx}=-4(1-2x)\ln(1-2x)+(1-2x)^2\left(\dfrac{-2}{1-2x}\right)$ $=-2(1-2x)\big(2\ln(1-2x)+1\big)$ o.e.	M1A1
	$\Rightarrow \dfrac{d^2y}{dx^2}=4\big(2\ln(1-2x)+1\big)-2(1-2x)\left(\dfrac{-4}{1-2x}\right)$ oe $\{=8\ln(1-2x)+12\}$	**d**M1 A1
	$\dfrac{d^3y}{dx^3}=\dfrac{-16}{1-2x}$	A1
		(5)
(b)	$\dfrac{d^4y}{dx^4}=16(1-2x)^{-2}\times -2=-32(1-2x)^{-2}$	M1 A1ft
	$y_0=0\quad y_0'=-2\quad y_0''=12\quad y_0'''=-16\quad y_0^{(4)}=-32$	M1
	$y=y_0+y_0'x+y_0''\dfrac{x^2}{2!}+y_0'''\dfrac{x^3}{3!}+y_0^{(4)}\dfrac{x^4}{4!}\ldots$ $\Rightarrow y=-2x+12\dfrac{x^2}{2!}-16\dfrac{x^3}{3!}-32\dfrac{x^4}{4!}\ldots$	M1
	$y=-2x+6x^2-\dfrac{8}{3}x^3-\dfrac{4}{3}x^4\ldots$	A1
		(5)
		Total 10

Notes
(a) **M1:** Differentiates using product rule, allowing for sign/coefficient errors only **A1:** Fully correct first derivative in any equivalent form. Award when first seen correct and ISW if they simplify incorrectly. **dM1: Dependent on the previous M mark.** Uses product rule again to obtain an expression for the 2nd derivative, allowing for sign/coefficient errors only **A1:** Fully correct expression in any form. Award when first seen correct and ISW if they simplify incorrectly. **A1:** Fully correct expression for the third derivative, which must come from correct work (b) **M1:** Correct form for the 4th derivative $\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = \alpha(1-2x)^{-2}$ **A1ft:** Follow through their 3rd derivative. ISW if they then simplify a correct expression Incorrectly- award when first seen. **M1:** Attempt at calculating values for y and their 1st, 2nd 3rd **and** 4th derivatives at $x = 0$ No need to check all values are correct for their derivatives, but award if the intention to evaluate all at $x = 0$ is clear and values are actually found for each. **M1:** Correct method for finding the Maclaurin expansion **with all five** of their values (condone the missing 0 at the start). May have to check that their series follows from their values if formula not quoted. Series must be correct for their values but allow slips if a correct formula is quoted. **A1:** Correct series obtained. Allow = or ≈ etc but must be $y = \ldots$ Allow $\mathrm{f}(x) = \cdots$ etc if $y = \mathrm{f}(x)$ has already been defined.

Question Number	Scheme	Marks
4	$4\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}-4\dfrac{\mathrm{d}y}{\mathrm{d}x}+y=2\mathrm{e}^{\frac{1}{2}x}$	
(a)	$(\text{AE:})\ 4m^2-4m+1=0$ $\left[\Rightarrow(2m-1)^2=0\right]$ $\Rightarrow m=\dfrac{1}{2}$	M1
	$(\text{CF:}y=)\ \ (A+Bx)\mathrm{e}^{\frac{1}{2}x}$	A1
	$(\text{PI:}y=)\ \ \lambda x^2\mathrm{e}^{\frac{1}{2}x}$	B1
	$y'=2\lambda x\mathrm{e}^{\frac{1}{2}x}+\dfrac{\lambda}{2}x^2\mathrm{e}^{\frac{1}{2}x}=\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{2}x^2+2\lambda x\right)$	M1
	$y''=\dfrac{1}{2}\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{2}x^2+2\lambda x\right)+\mathrm{e}^{\frac{1}{2}x}(\lambda x+2\lambda)=\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{4}x^2+2\lambda x+2\lambda\right)$	dM1
	Substituting into given equation to get $\mathrm{e}^{\frac{1}{2}x}\left[\left(\lambda x^2+8\lambda x+8\lambda\right)-\left(2\lambda x^2+8\lambda x\right)+\lambda x^2\right]=2\mathrm{e}^{\frac{1}{2}x}$ $\Rightarrow 8\lambda=2\Rightarrow\lambda=\ldots$	ddM1
	$y=(A+Bx)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{4}x^2\mathrm{e}^{\frac{1}{2}x}$	A1ft
		(7)
(b)	Sub $(0,1)\Rightarrow\cdots(1=A)$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=B\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{2}(A+Bx)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{2}x\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{8}x^2\mathrm{e}^{\frac{1}{2}x}$	M1
	$\mathrm{e}^{\frac{1}{2}}=B+\dfrac{A}{2}\Rightarrow B=\mathrm{e}^{\frac{1}{2}}-\dfrac{1}{2}$ $[y]=\left(1+\left(\mathrm{e}^{\frac{1}{2}}-\dfrac{1}{2}\right)x\right)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{4}x^2\mathrm{e}^{\frac{1}{2}x}$ oe	M1A1
		(4)
		Total 11

Notes
Allow recovery from mixed/incorrect variables etc but if not recovered then A and B marks may be lost (a) **M1:** Forms the auxiliary equation (condone one slip/copying error) and solves 3TQ (usual rules). One consistent solution if no working (could be complex). Implied by a correct CF if no incorrect working shown. **A1:** Correct complementary function $y = ...$ not required. May only be seen in final answer. **B1:** Correct form for particular integral $y = ...$ not required. **M1:** Differentiates their PI using the product rule- allowing for sign/coefficient errors only **dM1:** Differentiates their PI twice using the product rule- allowing for sign/coefficient errors only. **Dependent on the previous M mark.** **ddM1: Dependent on both previous M marks** Substitutes into the given equation, equates coefficients and obtains a value for λ. Must be comparing like terms. **A1ft:** A correct general solution following through on their CF only - **the PI must be correct**. Must have " $y =$ " e.g., not " GS = " **Note:** If a candidate uses a PI of the form $\lambda x e^{\frac{1}{2}x}$ then they can score a max of M1A1B0M1M1M0A0 If a candidate uses a PI of the form $\lambda e^{\frac{1}{2}x}$ then they can score a max of M1A1B0M0M0M0A0 (b) **B1:** Uses (0, 1) in their answer to (a) and forms an equation in one or both of their constants. This mark could be implied by a correct value for 'A' from a correct part (a). **M1:** Differentiates their GS, which must contain a term "Bxe^{kx}" and a non-zero PI, to obtain an expression of the correct form for the derivative- product rule must be used, allowing for sign/coefficient errors only **M1:** Substitutes $x = 0, \frac{dy}{dx} = e^{\frac{1}{2}}$ into their equation where their derivative is a changed function and finds values for their B (and their A if not found already). **A1:** Any correct expression. Condone e.g., " PS =... "

Question Number	Scheme	Marks
5(a)	$w=2-\frac{3}{z+\mathrm{i}} \Rightarrow w=\frac{2z+2\mathrm{i}-3}{z+\mathrm{i}} \Rightarrow wz+w\mathrm{i}=2z+2\mathrm{i}-3$ $\Rightarrow z(w-2)=2\mathrm{i}-3-w\mathrm{i} \Rightarrow z=\frac{2\mathrm{i}-3-w\mathrm{i}}{w-2}$ oe e.g. $z=\frac{3}{2-w}-\mathrm{i}$	M1 A1
	$(z=)\frac{(v-3)+\mathrm{i}(2-u)}{(u-2)+\mathrm{i}v}\times\frac{(u-2)-\mathrm{i}v}{(u-2)-\mathrm{i}v}=\ldots$	dM1
	$(z=)\frac{(v-3)(u-2)-\mathrm{i}v(v-3)-\mathrm{i}(u-2)^2+v(2-u)}{(u-2)^2+v^2}$ $\Rightarrow x=\frac{6-3u}{(u-2)^2+v^2}$ and $y=\frac{(-u^2-v^2+4u+3v-4)}{(u-2)^2+v^2}$	ddM1
	Half-line $\arg z=\frac{\pi}{4}\Rightarrow y=x$	B1
	$6-3u=-u^2-v^2+4u+3v-4$ $\Rightarrow u^2+v^2-7u-3v+10=0$ $\Rightarrow\left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2-\frac{49}{4}-\frac{9}{4}+10=0$	dddM1
	$\Rightarrow\left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2=\frac{9}{2}$	A1
		(7)
(b)(i)	radius is $\frac{3}{\sqrt{2}}$ or $\left(\frac{3\sqrt{2}}{2}\right)$	B1ft
(ii)	Centre is $\left(\frac{7}{2},\frac{3}{2}\right)$	B1ft
		(2)
		Total 9

Notes(a)
M1: Completes an attempt to make z the subject.
A1: Correct expression. Any equivalent correct expression for z. Award when first seen and ISW if they then simplify incorrectly.
dM1: **Dependent on previous M mark.** Replaces w with $u + \mathrm{i}v$ and shows an appropriate attempt to rationalise the denominator. Accept if $x + \mathrm{i}y$ is used instead for this mark.
ddM1: **Dependent on both previous M marks.** Extracts the correct real and imaginary parts for their expression, allowing for sign errors only, but allow the 'i' coefficients in the imaginary parts.
B1: Any valid statement that $y = x$ on the half-line. This could be implied by setting their real and imaginary parts equal, after rationalisation, or equivalent work.
dddM1: **Dependent on all previous M marks** Attempts to equate their real and imaginary parts and proceed to the equation of a circle via a valid completing the square attempt. Must be a valid circle equation with equal coefficients for the square terms
A1: Correct circle equation obtained from correct work
(b)
In this part mark parts (i) and (ii) together but if the radius is clearly stated as a coordinate etc then B0
B1 ft: For correct radius ft their circle equation. Can only be scored if they have a valid circle equation
B1ft: For the centre ft their circle equation. Can only be scored if they have a valid circle equation

Note: Alternative methods are possible (see below). If in doubt send to review.

5(a) Alt 1	$z = x + \mathrm{i}y, y = x \Rightarrow w = \dfrac{2z+2\mathrm{i}-3}{z+\mathrm{i}} = \dfrac{2x-3+\mathrm{i}(2x+2)}{x+(x+1)\mathrm{i}}$	B1
	$\Rightarrow (u+\mathrm{i}v)(x+(x+1)\mathrm{i}) = 2x-3+\mathrm{i}(2x+2) \Rightarrow \ldots + \ldots\mathrm{i} = \ldots$	M1
	$ux - v(x+1) + \mathrm{i}(u(x+1)+vx) = (2x-3)+\mathrm{i}(2x+2)$	A1
	$ux - v(x+1) = (2x-3) \Rightarrow x = \dfrac{v-3}{u-v-2}$ $u(x+1)+vx = (2x+2) \Rightarrow x = \dfrac{2-u}{u+v-2}$	dM1
	$\Rightarrow \dfrac{v-3}{u-v-2} = \dfrac{2-u}{u+v-2}\ (v-3)(u+v-2) = (2-u)(u-v-2)$ $\Rightarrow uv + v^2 - 2v - 3u - 3v + 6 = 2u - 2v - 4 - u^2 + uv + 2u(=0)$ $\Rightarrow u^2 + v^2 - 7u - 3v + 10 = 0$	ddM1
	$\Rightarrow \left(u-\dfrac{7}{2}\right)^2 + \left(v-\dfrac{3}{2}\right)^2 - \dfrac{49}{4} - \dfrac{9}{4} + 10 = 0$	dddM1
	$\Rightarrow \left(u-\dfrac{7}{2}\right)^2 + \left(v-\dfrac{3}{2}\right)^2 = \dfrac{9}{2}$	A1
		(7)

5(a) ALT 2	$w=2-\frac{3}{z+\mathrm{i}} \Rightarrow w=\frac{2z+2\mathrm{i}-3}{z+\mathrm{i}} \Rightarrow wz+w\mathrm{i}=2z+2\mathrm{i}-3$ $\Rightarrow z(w-2)=2\mathrm{i}-3-w\mathrm{i} \Rightarrow z=\frac{2\mathrm{i}-3-w\mathrm{i}}{w-2}$ oe e.g. $z=\frac{3}{2-w}-\mathrm{i}$	M1A1
	$y=x \Rightarrow \|z-1\|=\|z-\mathrm{i}\|$	B1
	$\frac{\|2\mathrm{i}-3-w\mathrm{i}-w+2\|}{\|w-2\|}=\frac{\|2\mathrm{i}-3-w\mathrm{i}-\mathrm{i}(w-2)\|}{\|w-2\|}$ $\Rightarrow \|2\mathrm{i}-3-(u+\mathrm{i}v)\mathrm{i}-(u+\mathrm{i}v)+2\|=\|2\mathrm{i}-3-(u+\mathrm{i}v)\mathrm{i}-\mathrm{i}(u+\mathrm{i}v-2)\|$ $\Rightarrow \|(-1-u+v)+\mathrm{i}(2-u-v)\|=\|(-3+2v)+\mathrm{i}(4-2u)\|$	dM1
	$(-1-u+v)^2+(2-u-v)^2=(-3+2v)^2+(4-2u)^2$ $\Rightarrow \cdots$	ddM1
	$1-2(v-u)+u^2+v^2-2uv+4-4u-4v+u^2+v^2+2uv=9-12v+4v^2+16-16u+4u^2$ $\Rightarrow \cdots$ $\Rightarrow u^2+v^2-7u-3v+10=0$ $\Rightarrow \left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2-\frac{49}{4}-\frac{9}{4}+10=0$	dddM1
	$\Rightarrow \left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2=\frac{9}{2}$	A1
		(7)

Notes
(a) **Alt 1** **B1**: Use of $y = x$ in the expression for z **M1**: Uses $y = x$ and $z = x + \mathrm{i}y$ in the expression for w and equates to $u + \mathrm{i}v$, cross multiples and expands. **A1**: Correct expression with the i^2 terms simplified. **dM1**: Equates real and imaginary parts and proceeds to find two expressions for x. **Dependent on previous M mark.** **ddM1**: Eliminate x by solving simultaneously then proceed to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.** **dddM1: Dependent on all previous M marks** proceeds to the equation of a circle via a valid competing the square attempt. Must be a valid circle equation with equal coefficients for the square terms **A1:** Correct circle equation obtained from correct work
(a) **Alt 2** **M1:** Completes an attempt to make z the subject. **A1:** Correct expression. Any equivalent correct expression for z. Award when first seen and ISW if they then simplify incorrectly. **B1:** Any valid statement that $y = x$ on the half-line. This could be implied by using $\lvert z-1\rvert = \lvert z-\mathrm{i}\rvert$ **dM1**: **Dependent on previous M mark.** Replaces w with $u + \mathrm{i}v$ in their modulus equation, collects real and imaginary parts and equates the numerators **ddM1**: **Dependent on both previous M marks.** Applies a correct method to find the modulus of both sides (with no i terms) etc and $\mathrm{i}^2 = -1$ etc and proceeds to collect terms **dddM1**: **Dependent on all previous M marks** Proceeds to the equation of a circle via a valid completing the square attempt. Must be a valid circle equation with equal coefficients for the square terms **A1:** Correct circle equation obtained from correct work

Question Number	Scheme	Marks
6(a)	$z=\cos\theta+\mathrm{i}\sin\theta$ **and** $\dfrac{1}{z}=\cos\theta-\mathrm{i}\sin\theta$ $\Rightarrow z+\dfrac{1}{z}=\ldots$ or $\Rightarrow z-\dfrac{1}{z}=\ldots$	M1
	$\dfrac{1}{2}\left(z+\dfrac{1}{z}\right)=\cos\theta$ and $\dfrac{1}{2\mathrm{i}}\left(z-\dfrac{1}{z}\right)=\sin\theta$	A1*
		(2)
(b)	$-\dfrac{1}{8\mathrm{i}}\left(z^3+3z^2\left(\dfrac{-1}{z}\right)+3z\left(\dfrac{-1}{z}\right)^2-\dfrac{1}{z^3}\right)$ **OR** $\dfrac{1}{16}\left(z^4+4z^3\left(\dfrac{1}{z}\right)+6z^2\left(\dfrac{1}{z}\right)^2+4z\left(\dfrac{1}{z}\right)^3+\left(\dfrac{1}{z}\right)^4\right)$	M1
	$\dfrac{\mathrm{i}}{128}\left(z^3-3z+\dfrac{3}{z}-\dfrac{1}{z^3}\right)\left(z^4+4z^2+6+\dfrac{4}{z^2}+\dfrac{1}{z^4}\right)=\ldots$	M1
ALT for first two M marks	$\left(z^2-\dfrac{1}{z^2}\right)^3=z^6+3z^4\left(-\dfrac{1}{z^2}\right)+3z^2\left(-\dfrac{1}{z^2}\right)^2+\left(-\dfrac{1}{z^2}\right)^3$	M1
	$=\dfrac{\mathrm{i}}{128}\left(z^6+3z^4\left(-\dfrac{1}{z^2}\right)+3z^2\left(-\dfrac{1}{z^2}\right)^2+\left(-\dfrac{1}{z^2}\right)^3\right)\left(z+\dfrac{1}{z}\right)=\cdots$	M1
	$=\dfrac{\mathrm{i}}{128}\left[\left(z^7-\dfrac{1}{z^7}\right)+\left(z^5-\dfrac{1}{z^5}\right)-3\left(z^3-\dfrac{1}{z^3}\right)-3\left(z-\dfrac{1}{z}\right)\right]$	M1
	$=\dfrac{\mathrm{i}}{128}\left(2\mathrm{i}\sin7\theta+2\mathrm{i}\sin5\theta-6\mathrm{i}\sin3\theta-6\mathrm{i}\sin\theta\right)$	M1
	$=-\dfrac{1}{64}\sin7\theta-\dfrac{1}{64}\sin5\theta+\dfrac{3}{64}\sin3\theta+\dfrac{3}{64}\sin\theta$ o.e.	A1
		(5)

ALT	Use of $\sin^3\theta\cos^4\theta = \sin^3\theta\left(1-\sin^2\theta\right)^2 = \sin^3\theta - 2\sin^5\theta + \sin^7\theta$	
	$\sin^3\theta = -\frac{1}{8i}\left(z^3 + 3z^2\left(-\frac{1}{z}\right) + 3z\left(-\frac{1}{z}\right)^2 - \frac{1}{z^3}\right) = \underline{-\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right)}$ **Or** $\sin^5\theta = \frac{1}{32i}\left(z^5 + 5z^4\left(-\frac{1}{z}\right) + 10z^3\left(-\frac{1}{z}\right)^2 + 10z^2\left(-\frac{1}{z}\right)^3 + 5z\left(-\frac{1}{z}\right)^4 - \frac{1}{z^5}\right)$ $= \underline{\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)}$ **Or** $\sin^7\theta = \frac{i}{128}\left(z^7 + 7z^6\left(-\frac{1}{z}\right) + 21z^5\left(-\frac{1}{z}\right)^2 + 35z^4\left(-\frac{1}{z}\right)^3 + 35z^3\left(-\frac{1}{z}\right)^4 + 21z^2\left(-\frac{1}{z}\right)^5 + 7z\left(-\frac{1}{z}\right)^6 - \frac{1}{z^7}\right)$ $= \underline{\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right)}$	M1
	$\sin^3\theta = -\frac{1}{8i}\left(z^3 + 3z^2\left(-\frac{1}{z}\right) + 3z\left(-\frac{1}{z}\right)^2 - \frac{1}{z^3}\right) = \underline{-\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right)}$ **and** $\sin^5\theta = \frac{1}{32i}\left(z^5 + 5z^4\left(-\frac{1}{z}\right) + 10z^3\left(-\frac{1}{z}\right)^2 + 10z^2\left(-\frac{1}{z}\right)^3 + 5z\left(-\frac{1}{z}\right)^4 - \frac{1}{z^5}\right)$ $= \underline{\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)}$ **and** $\sin^7\theta = \frac{i}{128}\left(z^7 + 7z^6\left(-\frac{1}{z}\right) + 21z^5\left(-\frac{1}{z}\right)^2 + 35z^4\left(-\frac{1}{z}\right)^3 + 35z^3\left(-\frac{1}{z}\right)^4 + 21z^2\left(-\frac{1}{z}\right)^5 + 7z\left(-\frac{1}{z}\right)^6 - \frac{1}{z^7}\right)$ $= \underline{\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right)}$	M1
	$\therefore \sin^3\theta\cos^4\theta = -\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right) - 2\left(\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)\right)$ $+\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right) = \cdots$	M1
	$= \frac{\mathrm{i}}{128}\left(2\mathrm{i}\sin 7\theta + 2\mathrm{i}\sin 5\theta - 6\mathrm{i}\sin 3\theta - 6\mathrm{i}\sin\theta\right)$	M1
	$= -\frac{1}{64}\sin 7\theta - \frac{1}{64}\sin 5\theta + \frac{3}{64}\sin 3\theta + \frac{3}{64}\sin\theta$	A1
		(5)

(c)	$\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^4\theta\,d\theta = -\frac{1}{64}\int_0^{\frac{\pi}{2}}\left(\sin 7\theta + \sin 5\theta - 3\sin 3\theta - 3\sin\theta\right)d\theta$ $= -\frac{1}{64}\left[\frac{-\cos 7\theta}{7}\frac{-\cos 5\theta}{5} + \cos 3\theta + 3\cos\theta\right]_0^{\frac{\pi}{2}}$	M1A1ft
	$-\frac{1}{64}\left[(0+0+0+0)-\left(-\frac{1}{7}-\frac{1}{5}+1+3\right)\right]$	M1
	$=\frac{2}{35}$	A1
		(4)
		Total 11

Notes

(a)

M1: Uses de Moivre correctly to obtain correct expression for $\frac{1}{z}$ and <u>either</u> adds or subtracts to z. Must connect LHS and RHS for the first or the second expression in some way for one of the expressions- a collection of trig terms with no connection to z and $\frac{1}{z}$ is M0

A1*: Both correct expressions obtained from correct work

(b)

Note: this is a 'hence' question so candidates must use the identities in (a)

M1: Attempts to expand <u>either</u> bracket using binomial expansion, coefficients must be correct but condone slips if the intention is clear

M1: Attempts to <u>multiply</u> <u>both expanded</u> brackets, in terms of z, together

<u>ALT for first two M marks</u>

M1: (Recognising the product can be written as a difference of two squares) Expands $\left(z^2-\frac{1}{z^2}\right)^3$. Coefficients must be correct but condone slips if the intention is clear

M1: Attempts to multiply their expansion of $\left(z^2-\frac{1}{z^2}\right)^3$ with $\left(z+\frac{1}{z}\right)$

M1: Groups terms in z correctly to form only sine terms after multiplication - could be implied by correct sine terms seen

M1: Use of general De Moivre $\left(z^n-\frac{1}{z^n}\right) \equiv 2\mathrm{i}\sin n\theta$ to obtain at least one correct term on the RHS- but the i's must be dealt with correctly- either consistently cancelled or remain in place

A1: Correct final result obtained from correct work.

(b) **ALT** **M1:** Any of these expansions in terms of z or in terms of sin terms. Coefficients must be correct but condone slips if the intention is clear **M1:** All three of these expansions in terms of z or in terms of sin terms. Coefficients must be correct but condone slips if the intention is clear **M1:** Substitutes into $\sin^3\theta - 2\sin^5\theta + \sin^7\theta$ and attempts to collect terms **M1:** Obtains at least one correct term on the RHS **A1:** Correct final result obtained from correct work
(c) **M1:** Integrates an expression of the form $a\sin 7\theta + b\sin 5\theta + c\sin 3\theta + d\sin\theta$ to obtain an expression of the form $...\cos 7\theta + ...\cos 5\theta + ...\cos 3\theta + ...\cos\theta$ but M0 if they are clearly differentiating. Accept if the candidate integrates with coefficients as *a, b, c, d* **A1ft:** Correct integration ft their values from (b) and accept in terms of *a, b, c, d* **M1:** Substitutes the correct limits and subtracts the correct way round. This can be implied by a correct final answer, once the first M1A1 is scored as no calculator warning given in this question. M0 if they have clearly differentiated. **A1:** Fully correct result, from correct integration

Question Number	Scheme		Marks
	$\sec x\dfrac{dy}{dx}-2y\sin x=e^{x+\sin^2 x}$		
7(a)	$\dfrac{dy}{dx}-2y\sin x\cos x=\cos x e^{x+\sin^2 x}\Rightarrow \text{I.F.}=e^{-\int 2\sin x\cos x dx}$ oe		M1
	$=e^{-\sin^2 x}$		A1
			(2)
(b)	$\dfrac{d}{dx}\left(ye^{-\sin^2 x}\right)=e^{-\sin^2 x}\cos x e^{x+\sin^2 x}$ $\Rightarrow ye^{-\sin^2 x}=\int e^x\cos x dx$ oe		M1A1
	$I=\int e^x\cos x dx=e^x\sin x-\int e^x\sin x dx$	$I=\int e^x\cos x dx=e^x\cos x+\int e^x\sin x dx$	M1
	$=e^x\sin x-\left[-e^x\cos x+\int e^x\cos x dx\right]$ $\Rightarrow I=\dfrac{1}{2}e^x(\sin x+\cos x)$	$=e^x\cos x+\left[e^x\sin x-\int e^x\cos x dx\right]$ $\Rightarrow I=\dfrac{1}{2}e^x(\sin x+\cos x)$	M1A1
	$ye^{-\sin^2 x}=\dfrac{1}{2}e^x(\sin x+\cos x)+C$ $\Rightarrow y=\dfrac{e^x(\sin x+\cos x)}{2e^{-\sin^2 x}}+\dfrac{C}{e^{-\sin^2 x}}$ oe		A1
			(6)
(c)	$x=0, y=2\Rightarrow 2=\dfrac{1}{2}+C$ $\Rightarrow C=\ldots\left(\dfrac{3}{2}\right)$		M1
	$\Rightarrow y=\dfrac{\frac{1}{2}e^x(\sin x+\cos x)}{e^{-\sin^2 x}}+\dfrac{3}{2e^{-\sin^2 x}}$ oe		A1
			(2)
			Total 10

Notes

Mark parts (a) and (b) together

(a)

M1: Dividing by sec x (or multiplying by cos x) and obtaining $\text{I.F.} = \mathrm{e}^{-\int 2\sin x\cos x\mathrm{d}x}$ oe e.g. $\text{I.F.} = \mathrm{e}^{-\int \sin 2x\mathrm{d}x}$

A1: Obtains $\text{I.F.} = \mathrm{e}^{-\sin^2 x}$

Note: if the candidate integrates $\mathrm{e}^{-\int \sin 2x\mathrm{d}x} = \mathrm{e}^{\frac{1}{2}\cos 2x} = \cdots \mathrm{e}^{\frac{1}{2}-\sin^2 x}$ then states that the $\text{I.F.} = \mathrm{e}^{-\sin^2 x}$ with justification (e.g. any multiple of the IF is also an IF) then full marks can be scored in (a)

If candidates use a correct integration factor in parts (b) and (c) then all marks are available

(b)

M1: Obtains $y \times \text{their IF (or correct IF)} = \int \text{their IF (or correct IF)} \times \cos x\mathrm{e}^{x+\sin^2 x}$

A1: simplifies RHS to obtain $y \times \text{their IF (or correct IF)} = \int \mathrm{e}^x \cos x\mathrm{d}x$ or equivalent correct

expression: e.g. if their IF is $\mathrm{e}^{\frac{1}{2}-\sin^2 x}$ then they will obtain $y \times \mathrm{e}^{\frac{1}{2}-\sin^2 x} = \mathrm{e}^{\frac{1}{2}}\int \mathrm{e}^x \cos x\mathrm{d}x$

if their IF is $\mathrm{e}^{\frac{1}{2}\cos 2x}$ then they will obtain $y \times \mathrm{e}^{\frac{1}{2}\cos 2x} = \int \mathrm{e}^{\frac{1}{2}\cos 2x}\cos x\mathrm{e}^{x+\sin^2 x}\mathrm{d}x$

or $y \times \mathrm{e}^{\frac{1}{2}\cos 2x} = \int \mathrm{e}^{x+\frac{1}{2}}\cos x\mathrm{d}x$

M1: Uses IBP once on RHS (sign errors only)

M1: Uses IBP twice on RHS (sign errors only)

A1: Correct integral for $\int \mathrm{e}^x \cos x\mathrm{d}x$

A1: c.a.o. Constant introduced and treated correctly- correct final answer in any equivalent form

e.g. if their IF is $\mathrm{e}^{\frac{1}{2}-\sin^2 x}$ then they will obtain $y = \dfrac{\mathrm{e}^x\mathrm{e}^{\frac{1}{2}}(\sin x + \cos x)}{2\mathrm{e}^{\frac{1}{2}-\sin^2 x}} + \dfrac{C}{\mathrm{e}^{\frac{1}{2}-\sin^2 x}}$

e.g. if their IF is $\mathrm{e}^{\frac{1}{2}\cos 2x}$ then they will obtain $y = \dfrac{\mathrm{e}^x\mathrm{e}^{\frac{1}{2}}(\sin x + \cos x)}{2\mathrm{e}^{\frac{1}{2}\cos 2x}} + \dfrac{C}{\mathrm{e}^{\frac{1}{2}\cos 2x}}$

(c) **M1:** Substitutes the given boundary conditions and re-arranges to obtain a value for the constant from their GS

A1: Correct value and correct particular solution (in any equivalent form)

if their IF is $\mathrm{e}^{\frac{1}{2}-\sin^2 x}$ or $\mathrm{e}^{\frac{1}{2}\cos 2x}$ their constant will be $C = \ldots\left(\frac{3}{2}\mathrm{e}^{\frac{1}{2}}\right)$

Question Number	Scheme	Marks
8		
(a)	POI: $2\cos\left(\theta+\frac{\pi}{6}\right)=\sec\left(\theta-\frac{\pi}{3}\right)\Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta-\frac{\pi}{3}\right)=1$	B1
	$2\cos\left(\theta+\frac{\pi}{6}\right)\sin\left(\theta+\frac{\pi}{6}\right)=1\Rightarrow\sin\left(2\theta+\frac{\pi}{3}\right)=1\Rightarrow 2\theta=\cdots\Rightarrow\theta=\cdots$	M1
	$2\theta=\frac{\pi}{6},\ldots\Rightarrow\theta=\frac{\pi}{12}$ and $r=2\cos\frac{\pi}{4}=\sqrt{2}$	A1
		(3)
ALT	Use of $\cos\theta\equiv-\cos(\theta+\pi)$	
	$2\cos\left(\theta+\frac{\pi}{6}\right)=-\sec\left(\theta+\frac{2\pi}{3}\right)\Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta+\frac{2\pi}{3}\right)=-1$	B1
	$2\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{2\pi}{3}\right)=-1\Rightarrow\sin\left(2\theta+\frac{4\pi}{3}\right)=-1\Rightarrow 2\theta=\cdots\Rightarrow\theta=\cdots$	M1
	$2\theta=\frac{3\pi}{2}-\frac{4\pi}{3}=\frac{\pi}{6}\Rightarrow\theta=\frac{\pi}{12}$ and $r=2\cos\frac{\pi}{4}=\sqrt{2}$	A1
		(3)
(b) **Way 1**	**Way1** $\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta-\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta$ Overall strategy	M1
	$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **OR** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	For **both** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **and** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1

	For each correct integral $$\left(\frac{1}{2}\right)\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]-\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$$	A1 A1
	$$\frac{1}{2}\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{12}}-\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]_0^{\frac{\pi}{12}}$$ $$=\frac{1}{2}\left(\sqrt{3}-1\right)-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\ldots$$	dM1
	$$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$$	A1
		(7)
8(b) Way 2	$$\text{small triangle}-\left(\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta-\text{large triangle}\right)$$ Overall strategy mark	M1
	Area of small triangle $$=\frac{1}{2}\left[2-\sqrt{2}\cos\frac{\pi}{12}\right]\left[\sqrt{2}\sin\frac{\pi}{12}\right]=\ldots$$	M1
	$$=\left[\frac{2\sqrt{3}-3}{4}\right]$$	A1
	$$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$$	M1
	$$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$$	A1
	Area = $$=\left[\frac{2\sqrt{3}-3}{4}\right]-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]+\frac{1}{2}\sqrt{2}\cos\frac{\pi}{12}\sqrt{2}\sin\frac{\pi}{12}=\cdots$$	dM1
	$$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$$	A1
		(7)

8(b) Way 3	Largest triangle - $\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ Overall strategy mark	M1
	Largest triangle $\frac{1}{2}\times 2\times\sqrt{2}\sin\frac{\pi}{12} \quad = ...$	M1
	$=\frac{\sqrt{3}-1}{2}$	A1
	$\left(\frac{1}{2}\right)\int_{...}^{...}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta = k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1
	Area = $=\frac{\sqrt{3}-1}{2}-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\cdots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)
		Total 10

Notes:

(a)**B1:** Sets both equations equal and replaces $\sec\left(\theta-\frac{\pi}{3}\right)$ with $\frac{1}{\cos\left(\theta-\frac{\pi}{3}\right)}$

M1: Uses the identity $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$ and simplifies using the identity $\sin 2\theta\equiv 2\sin\theta\cos\theta$ to obtain an expression of the form $\sin 2\theta = k$ then solves an equation of the form $\sin 2\theta = k$ to find a value for θ

A1: r and θ correctly obtained from correct work

ALT

B1: Sets both equations equal and replaces $\sec\left(\theta-\frac{\pi}{3}\right)$ with $-\frac{1}{\cos\left(\theta+\frac{2\pi}{3}\right)}$

M1: Uses the identity $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$ and simplifies using the identity $\sin 2\theta\equiv 2\sin\theta\cos\theta$ to obtain an expression of the form $\sin 2\theta = k$ then solves an equation of the form $\sin 2\theta = k$ to find a value for θ

A1: r and θ correctly obtained from correct work

(b) **Way 1**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct form for the integration of $\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **or** correct form for integration of $\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$

M1: Correct form for the integration of both $\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **and**

$\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$

NOTE: for $\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ they may expand and use identities e.g.

$$\left(\frac{1}{2}\right)\int_{\dots}^{\dots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=\int 1+\cos\left(2\theta+\frac{\pi}{3}\right)=\int 1+\cos 2\theta\cos\frac{\pi}{3}-\sin 2\theta\sin\frac{\pi}{3}$$
$$=\theta+\frac{1}{4}\sin 2\theta+\frac{\sqrt{3}}{4}\cos 2\theta$$

There may be variations on the above approach (too many to list on the MS) but if candidates choose this method, then it must be a complete method to score the M mark (use of correct identities condoning sign errors) and M0 if clear differentiation.

A1: For one correct integral (condone the omission of the $\frac{1}{2}$ for this mark)

A1: For both correct integrals (condone the omission of the $\frac{1}{2}$ for this mark)

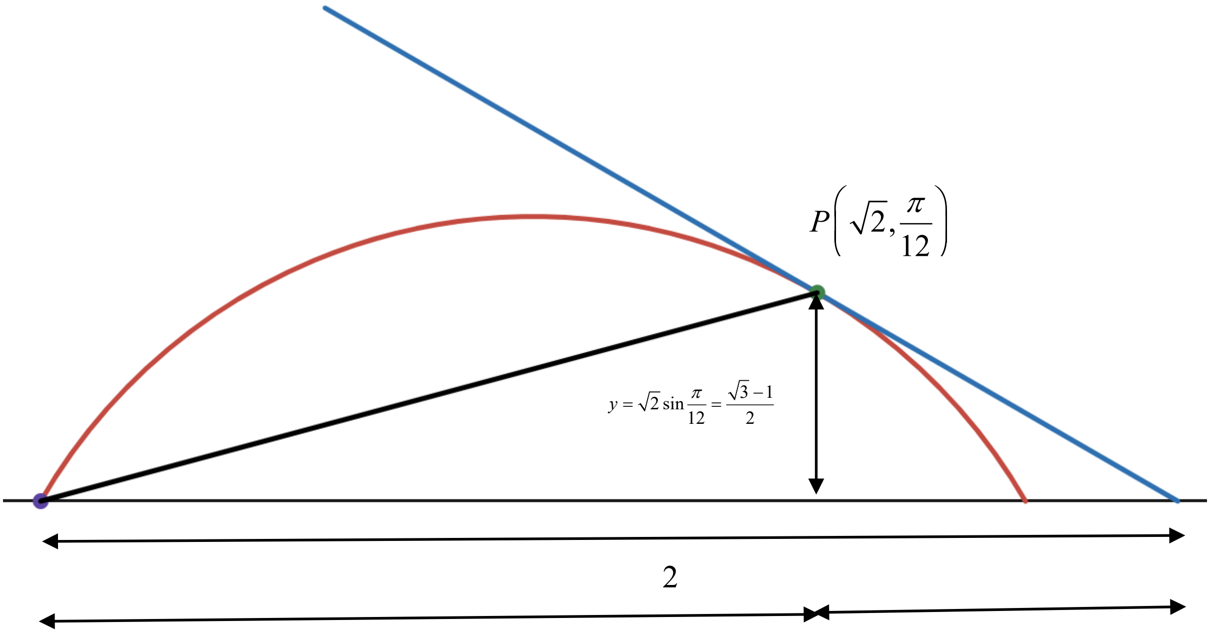
dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark.

A1: Correct final answer in exact equivalent form

(b) **Way 2** **M1:** Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark) **M1:** Correct attempt at the area of the small triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances **A1:** Correct exact area obtained **M1:** Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ **A1:** Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark **dM1: Dependent on the first M mark.** Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark **A1:** Correct final answer in exact equivalent form
(b) **Way 3** **M1:** Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark) **M1:** Correct attempt at the area of the largest triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances **A1:** Correct exact area obtained **M1:** Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ **A1:** Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark **dM1: Dependent on the first M mark.** Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark **A1:** Correct final answer in exact equivalent form

	TOTAL FOR PAPER: 75 MARKS

Useful diagram for Q8



$\sqrt{2}\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2}$ $\qquad$ $2-\sqrt{2}\cos\frac{\pi}{12}=\frac{3-\sqrt{3}}{2}$

$$\text{Area of small }\Delta=\frac{1}{2}\left(\frac{3-\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{2\sqrt{3}-3}{4}$$

$$\text{Area of large }\Delta=\frac{1}{2}\left(\frac{\sqrt{3}+1}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{1}{4}$$

$$\text{Area of largest }\Delta=\frac{1}{2}(2)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{\sqrt{3}-1}{2}$$