Mark Scheme (Results)
Summer 2026

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03) Paper 01A

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)

Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
- ft – follow through
  - the symbol √ will be used for correct ft
- cao – correct answer only
- cso – correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ – The answer is printed on the paper or ag- answer given
- └ or d… – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

- Factorisation
  - $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$
  - $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$
- Formula
  - Attempt to use the correct formula (with values for *a*, *b* and *c*).
- Completing the square
  - Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

- Differentiation
  - Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)
- Integration
  - Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first. Normal marking procedure is as follows:

- Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Notes	Marks
1(a)	$x = 2\sec\theta,\ y = \sqrt{3}\tan\theta \Rightarrow \dfrac{dy}{dx} = \dfrac{\sqrt{3}\sec^2\theta}{2\sec\theta\tan\theta}$ $\dfrac{x^2}{4} - \dfrac{y^2}{3} = 1 \Rightarrow \dfrac{2x}{4} - \dfrac{2y}{3}\dfrac{dy}{dx} = 0$ $y = \left(\dfrac{3x^2}{4} - 3\right)^{\frac{1}{2}}$, $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{3x^2}{4} - 3\right)^{-\frac{1}{2}}\left(\dfrac{6x}{4}\right)$	Any correct equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ and award when first seen	**B1**
	$= \dfrac{\sqrt{3}\sec\theta}{2\tan\theta}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{3x}{4y} = \dfrac{\sqrt{3}\sec\theta}{2\tan\theta}$ $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{12\sec^2\theta}{4} - 3\right)^{-\frac{1}{2}}(3\sec\theta) = \dfrac{3\sec\theta}{2\sqrt{3\sec^2\theta - 3}} = \dfrac{\sqrt{3}\sec\theta}{2\tan\theta}$		
	$\Rightarrow m_N = -\dfrac{2\tan\theta}{\sqrt{3}\sec\theta}$ oe e.g. $\Rightarrow m_N = -\dfrac{2\sqrt{3}}{3}\sin\theta$	Correct perpendicular gradient rule to obtain gradient of the normal in terms of θ Sight of this normal gradient scores B1M1 if it is clearly identified/used as m_N	**M1**
	$y - \sqrt{3}\tan\theta = -\dfrac{2\tan\theta}{\sqrt{3}\sec\theta}(x - 2\sec\theta)$ or $\sqrt{3}\tan\theta = -\dfrac{2\tan\theta}{\sqrt{3}\sec\theta}2\sec\theta + c \Rightarrow c = \dfrac{7\sqrt{3}}{3}\tan\theta$	Correct straight-line method with their changed gradient in terms of θ	**M1**
	$\sqrt{3}y\sec\theta - 3\tan\theta\sec\theta = -2x\tan\theta + 4\tan\theta\sec\theta \Rightarrow 2x\tan\theta + \sqrt{3}y\sec\theta = 7\tan\theta\sec\theta$ or $y = -\dfrac{2\tan\theta}{\sqrt{3}\sec\theta}x + \dfrac{7\sqrt{3}}{3}\tan\theta \Rightarrow 2x\tan\theta + \sqrt{3}y\sec\theta = 7\tan\theta\sec\theta$ Obtains correct equation with $k = 7$ with at least one line of intermediate working.		**A1**
			(4)
(b)	$2\sec\theta = 4,\ \sqrt{3}\tan\theta = 3$ $\Rightarrow \cos\theta = \dfrac{1}{2},\ \tan\theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$	Correct value for θ Allow 60° **OR** States or uses $\sec\theta = 2,\ \tan\theta = \sqrt{3}$	**B1**
	$2\sqrt{3}x + 2\sqrt{3}y = 14\sqrt{3}\ \{\Rightarrow y = 7 - x\}$	Substitutes their value for θ into their normal of the correct form **OR** Substitutes $\sec\theta = 2,\ \tan\theta = \sqrt{3}$ into their normal of the correct form (scores B1M1)	**M1**
ALT for B1M1	$\dfrac{dy}{dx} = \dfrac{3x}{4y} = \dfrac{3\times 4}{4\times 3} = 1 \Rightarrow m_N = -1$	Correct <u>value</u> for the gradient of the normal	**B1**
	$y - 3 = -(x - 4)\ \{\Rightarrow y = 7 - x\}$	Uses a correct line equation method to find the equation of the normal Sight of this equation scores B1M1	**M1**

	$\frac{x^2}{4}-\frac{(7-x)^2}{3}=1 \Rightarrow 3x^2-4\left(49-14x+x^2\right)=12$ $\Rightarrow x^2-56x+208=0$ $(x-52)(x-4)=0 \Rightarrow x=52$	Substitutes their normal equation into H and solves 3TQ (usual rules - no incorrect root if no working). Eliminating x leads to $y^2+42y-135=(y+45)(y-3)=0$	**M1**
	$(52,\ -45)$	Correct coordinates or values for x and y	**A1**
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
Note: Attempts that don't integrate, or that use the given standard integral in the formula book will generally score max B1M0A0			
2(a)	$x=\tanh u \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{sech}^2\,u$	Sets $x=\tanh u$ and differentiates correctly. Condone different or interchanged variables as long as the differentiation is correct	**B1**
	$\displaystyle\int \frac{1}{1-x^2}\,\mathrm{d}x=\int \frac{\mathrm{sech}^2\,u}{1-\tanh^2 u}\,\mathrm{d}u=\ldots$	A full substitution, correct for their x and $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ and integrates. Allow $\mathrm{sech}^2\,u=\pm1\pm\tanh^2 u$ If there are clear sign errors but the method is otherwise correct, allow the M mark but deduct the A1* mark	**M1**
	$\displaystyle=\int 1\,\mathrm{d}u=u+c=\mathrm{artanh}\,x+c$	Correct proof with one intermediate step. Penalise notational errors e.g. missing 'h' or constant 'c' etc but allow equivalent notation for $\mathrm{ar}\tanh x$ such as $\tanh^{-1} x$ but not $\mathrm{arc}\tanh x$	**A1***
			(3)
Alt	$\dfrac{1}{1-x^2}=\dfrac{1}{2}\left(\dfrac{1}{1+x}+\dfrac{1}{1-x}\right)$	Correct partial fractions	**B1**
	$\displaystyle\int\frac{1}{1-x^2}\,\mathrm{d}x=\frac{1}{2}\int\left(\frac{1}{1+x}+\frac{1}{1-x}\right)\mathrm{d}x=\frac{1}{2}\left(\ln(1+x)-\ln(1-x)\right)\quad[+c]$ Correctly integrates partial fractions of the correct form		**M1**
	$=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)+c=\mathrm{artanh}\,x+c$	Correct proof with one intermediate step. Condone the omission of brackets. Penalise notational errors e.g. missing 'h' or constant 'c' etc but allow equivalent notation for $\mathrm{ar}\tanh x$ such as $\tanh^{-1} x$ but not $\mathrm{arc}\tanh x$	**A1***
			(3)
SC	$y=\mathrm{ar}\tanh x\Rightarrow x=\tanh y\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{sech}^2\,y$ $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{sech}^2\,y=1-\tanh^2 y\Rightarrow y=\displaystyle\int\frac{1}{1-\tanh^2 y}\,\mathrm{d}x$ $\Rightarrow y=\displaystyle\int\frac{1}{1-x^2}\,\mathrm{d}x=\mathrm{ar}\tanh x$	Gains B1M0A0 (No integration undertaken)	
(b) (i) & (ii)	$f(x)=\mathrm{artanh}2x-8\arctan 4x$ $\Rightarrow \mathrm{f}'(x)=\dfrac{2}{1-4x^2}-\dfrac{32}{1+16x^2}$	<u>One term</u> correctly differentiated. Could be unsimplified	**B1**
	$1+16x^2=16-64x^2\Rightarrow x^2=\dfrac{3}{16}\Rightarrow x=\ldots$	Obtains a value for x from an equation of the form $\dfrac{a}{1-bx^2}-\dfrac{c}{1\pm dx^2}=0,\ a,b,c,d>0$ i.e. accept sign errors with d only but allow recovery if recovered correctly	**M1**

	$x=\frac{\sqrt{3}}{4}$	$x=\frac{\sqrt{3}}{4}$ or exact equivalent. Not $\pm$ but no need to specifically reject the negative value	**A1**
	$y=\frac{1}{2}\ln\frac{\left(1+\frac{\sqrt{3}}{2}\right)}{\left(1-\frac{\sqrt{3}}{2}\right)}\ \left\{-8\arctan\sqrt{3}\right\}$	Uses correct logarithmic form of artanh with their value of x where $0<x<\frac{1}{2}$	**M1**
	$y=\left\{\frac{1}{2}\ln\frac{\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)}-8\arctan\sqrt{3}\right\}=\frac{1}{2}\ln\left(7+4\sqrt{3}\right)-\frac{8\pi}{3}$	Correct answer in the correct form	**A1**
			(5)
			Total 8

Question Number	Scheme	Notes	Marks
3(a)	$\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}-2-\lambda & 1 & -3\\ 0 & 3-\lambda & k\\ -2 & -2 & 3-\lambda\end{vmatrix}=(-2-\lambda)\left[(3-\lambda)(3-\lambda)+2k\right]-1(0+2k)-3\left[2(3-\lambda)\right]$ Any recognisable attempt at $\det(\mathbf{M}-\lambda\mathbf{I})$. Allow recovery from invisible brackets May see e.g., $(-2-\lambda)\left[(3-\lambda)(3-\lambda)+2k\right]-0-2\left[k+3(3-\lambda)\right]$ (1st column) Sarrus leads to $(-2-\lambda)(3-\lambda)(3-\lambda)-2k+0-\left[6(3-\lambda)-2k(-2-\lambda)+0\right]$		**M1**
	$=(-2-\lambda)\left(9-6\lambda+\lambda^2+2k\right)-2k-18+6\lambda$ $=-\lambda^3+4\lambda^2+3\lambda-2k\lambda-4k-18-2k-18+6\lambda$ $\Rightarrow \lambda^3-4\lambda^2-3\lambda+2k\lambda+4k+18+2k+18-6\lambda=0$ $\Rightarrow \lambda^3-4\lambda^2+(2k-9)\lambda+6k+36=0$ **d**M1: Expands to a cubic and collects terms. No requirement for "$=0$" A1: Correct equation. Allow $+(2k-9)\lambda$ written as e.g., $-(9-2k)\lambda$ or $2k\lambda-9\lambda$		**dM1** **A1**
			(3)
(b)	$2^3-4\times 2^2+2(2k-9)+6k+36=0\Rightarrow k=...$	Substitutes $\lambda=2$ into their equation and solves for k or uses long division to divide their equation in (a) by $\lambda-2$ and makes progress to achieve an equation in k (finding the remainder and setting equal to 0) Must use part (a) and any attempt not using (a) scores M0A0	**M1**
	$\Rightarrow 8-16-18+4k+6k+36=0$ $\Rightarrow 10+10k=0\Rightarrow k=-1$	Obtains $k=-1$ with no errors. Accept a clear verification using both values with minimal conclusion	**A1***
			(2)
(c)	$\{(-2)\times 3\times(-\lambda)=6(-1)+36\Rightarrow 6\lambda=30\}$ $\{\text{or } \lambda^3-4\lambda^2-11\lambda+30=(\lambda-2)(\lambda+3)(\lambda-5)=0\}$ $\{\text{or trace = sum}\Rightarrow -2+3+3=2-3+\lambda\Rightarrow 4=-1+\lambda\}$ $\{\text{or } 2\times(-3)\times\lambda=\det\mathbf{M}=-30\Rightarrow 6\lambda=30\}$ $\lambda=5$ Correct eigenvalue		**B1**
			(1)
(d)	$\mathbf{Mx}=\lambda\mathbf{x}\Rightarrow \begin{matrix}-2x+y-3z=-3x\\ 3y-z=-3y\\ -2x-2y+3z=-3z\end{matrix}$ or $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=\mathbf{0}\Rightarrow \begin{matrix}x+y-3z=0\\ 6y-z=0\\ -2x-2y+6z=0\end{matrix}\Rightarrow x=...,\ y=...,\ z=....$ Correct method to obtain simultaneous equations and solves for all variables to find values for each (at least one non-zero) Other approaches are possible, such as finding the cross product between any two rows of the $\mathbf{M}-\lambda\mathbf{I}$ matrix. If in doubt sent to review.		**M1**
	$\pm\begin{pmatrix}17\\1\\6\end{pmatrix}$	Correct eigenvector. Any integer multiple/either direction	**A1**
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
	Note: any attempts that begin with the stated result in the question score 0		
4(a)	$y=\text{arsinh}\,x \Rightarrow \sinh y = x \Rightarrow \dfrac{e^y - e^{-y}}{2} = x$	Correct expression for x in exponentials	**B1**
	$\Rightarrow e^{2y} - 2xe^y - 1 = 0 \Rightarrow e^y = \dfrac{2x \pm \sqrt{4x^2+4}}{2}$ or $\left(e^y - x\right)^2 - x^2 - 1 = 0 \Rightarrow e^y = x \pm \sqrt{x^2+1}$	Obtains a 3TQ in e^y and attempts to solve by formula or CTS "= 0 " not required here. Must see either of these elements of working shown. Do not accept going straight to $e^y = x \pm \sqrt{x^2+1}$	**M1**
	$\left\{\ln\left(x - \sqrt{x^2+1}\right) \text{is undefined since } x - \sqrt{x^2+1} < 0\ \forall x\right\}$ $y = \ln\left(x + \sqrt{x^2+1}\right)$	Fully correct proof. Not $\pm$. May have just used + throughout. Reason for rejection of $-$ is not required and can be ignored.	**A1***
			(3)
ALT	$\underline{x = \text{arsinh}\,y} \Rightarrow \sinh x = y \Rightarrow \dfrac{e^x - e^{-x}}{2} = y$	The underlined statement must be seen (perhaps at the end) using this method to interchange the x and y. Correct expression for y in exponentials	**B1**
	$\Rightarrow e^{2x} - 2ye^x - 1 = 0 \Rightarrow e^x = \dfrac{2y \pm \sqrt{4y^2+4}}{2}$ or $\left(e^x - y\right)^2 - y^2 - 1 = 0 \Rightarrow e^x = y \pm \sqrt{y^2+1}$	Obtains a 3TQ in e^x and attempts to solve by formula or CTS (usual rules) "= 0 " not required here. Must see either of these elements of working shown. Do not accept going straight to $e^x = y \pm \sqrt{y^2+1}$	**M1**
	$\left\{\ln\left(y - \sqrt{y^2+1}\right) \text{is undefined since } y - \sqrt{y^2+1} < 0\ \forall y\right\}$ $\underline{x = \text{ar}\sinh y = \ln\left(y + \sqrt{y^2+1}\right)}$ and so $\underline{\text{ar}\sinh x = \ln\left(x + \sqrt{x^2+1}\right)}$	Fully correct proof. Not $\pm$. May have just used + throughout. Reason for rejection of $-$ is not required and can be ignored. There needs to be a clear connection between x and arsinh$y$ and a statement of the printed result, so the underlined statements are required	**A1***
			(3)
(b)	$y = \left(\sinh x - 3\right)\cosh x - 6x$ $\Rightarrow \dfrac{dy}{dx} = \sinh x\left(\sinh x - 3\right) + \cosh^2 x - 6$ or $\Rightarrow y = \dfrac{1}{2}\sinh 2x - 3\cosh x - 6x$ $\Rightarrow \dfrac{dy}{dx} = \cosh 2x - 3\sinh x - 6$	Differentiates to the correct form (condone coefficient errors). Allow if -6 is missing. If expanded first allow with $\sinh x \cosh x$ replaced with $k\sinh 2x$	**M1**
	$= \sinh^2 x - 3\sinh x + 1 + \sinh^2 x - 6$ or $= 2\sinh^2 x + 1 - 3\sinh x - 6$ $\left\{\Rightarrow 2\sinh^2 x - 3\sinh x - 5\right\}$	Uses $\cosh^2 x = \pm 1 \pm \sinh^2 x$ or $\cosh 2x = \pm 2\sinh^2 x \pm 1$ and obtains a quadratic expression in $\sinh x$	**M1**
	$2\sinh^2 x - 3\sinh x - 5 = 0$	Correct 3TQ = 0 (could be implied by correct values after solving, if working with inequalities etc throughout)	**A1**
	$\left(2\sinh x - 5\right)\left(\sinh x + 1\right) = 0$	Solves 3TQ by usual rules - no incorrect root if no working (could be implied by	**M1**

	$\sinh x = -1,\ \frac{5}{2}$	correct values after solving, if working with inequalities etc throughout)	
	$x = \ln\left(-1+\sqrt{(-1)^2+1}\right),\ x = \ln\left(\frac{5}{2}+\sqrt{\left(\frac{5}{2}\right)^2+1}\right)$	<u>Either</u> correct numerical expression	**A1**
	$\ln\left(\sqrt{2}-1\right) \leqslant x \leqslant \ln\left(\frac{5+\sqrt{29}}{2}\right)$ or $\ln\left(\sqrt{2}-1\right) < x < \ln\left(\frac{5+\sqrt{29}}{2}\right)$	Correct range in any suitable notation with simplified lns but allow $\ln\left(\frac{5}{2}+\frac{\sqrt{29}}{2}\right)$ e.g. $x \in \left(\ln\left(\sqrt{2}-1\right), \ln\left(\frac{5+\sqrt{29}}{2}\right)\right)$ Allow with strict or non-strict inequalities, or soft/hard brackets	**A1**
	Working in exponentials: $\left(\frac{e^x+e^{-x}}{2}\right)\left(\frac{e^x-e^{-x}}{2}-3\right)-6x$ $\Rightarrow \frac{\left(e^x+e^{-x}\right)\left(e^x-e^{-x}-6\right)}{4}-6x \Rightarrow \frac{e^{2x}-6e^x-6e^{-x}-e^{-2x}}{4}-6x \Rightarrow \frac{2e^{2x}-6e^x+6e^{-x}+2e^{-2x}}{4}-6$ $\Rightarrow e^{2x}-3e^x-12+3e^{-x}+e^{-2x}=0 \Rightarrow e^{4x}-3e^{3x}-12e^{2x}+3e^x+1=0$ Score second M1 for at least one hyperbolic function replaced correctly then 1st M1 for differentiating to a correct form ($e^{kx} \to ke^{kx}$ for each term) allowing the -6 to be missing. 1st A1 for a correct 5 term quartic equation in e^x Further marks require working with exact values so would require solving one of $e^{2x}-5e^x-1=0$ or $e^{2x}+2e^x-1=0$ by usual rules for the next M then last 2 marks as above		
			(6)
			Total 9

Question Number	Scheme	Notes	Marks
Note: accept **i, j, k** notation or column vector notation throughout but condone column vectors written as coordinates			
5(a) (i) & (ii)	$\overrightarrow{OP}\times\overrightarrow{OQ}=\begin{pmatrix}2\\5\\0\end{pmatrix}\times\begin{pmatrix}-2\\6\\3\end{pmatrix}=\begin{pmatrix}15\\-6\\22\end{pmatrix}$	Correct vector. Award when first seen	**B1**
	$\begin{pmatrix}1\\2\\-1\end{pmatrix}.\text{"}\begin{pmatrix}15\\-6\\22\end{pmatrix}\text{"}=\ldots\quad\{15-12-22=-19\}$	Correct method to find a value for the scalar product with their $\overrightarrow{OP}\times\overrightarrow{OQ}$. If just a value is given it must be correct, and could also be implied by the correct final answer	**M1**
	$\frac{1}{6}\left\lvert\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}15\\-6\\22\end{pmatrix}\right\rvert=\frac{19}{6}$	$\frac{19}{6}$ or any exact equivalent	**A1**
			(3)
(b)	$\overrightarrow{PQ}=\begin{pmatrix}-2\\6\\3\end{pmatrix}-\begin{pmatrix}2\\5\\0\end{pmatrix}=\ldots\left\{\begin{pmatrix}-4\\1\\3\end{pmatrix}\right\}$, $\overrightarrow{PR}=\begin{pmatrix}1\\2\\-1\end{pmatrix}-\begin{pmatrix}2\\5\\0\end{pmatrix}=\ldots\left\{\begin{pmatrix}-1\\-3\\-1\end{pmatrix}\right\}$, $\overrightarrow{QR}=\begin{pmatrix}1\\2\\-1\end{pmatrix}-\begin{pmatrix}-2\\6\\3\end{pmatrix}=\ldots\left\{\begin{pmatrix}3\\-4\\-4\end{pmatrix}\right\}$ Attempts two of $\pm\left(\overrightarrow{PR},\overrightarrow{QP},\overrightarrow{PQ}\right)$		**M1**
	$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{pmatrix}-4\\1\\3\end{pmatrix}\times\begin{pmatrix}-1\\-3\\-1\end{pmatrix}=\begin{pmatrix}8\\-7\\13\end{pmatrix}$, $\overrightarrow{PQ}\times\overrightarrow{QR}=\begin{pmatrix}-4\\1\\3\end{pmatrix}\times\begin{pmatrix}3\\-4\\-4\end{pmatrix}=\begin{pmatrix}8\\-7\\13\end{pmatrix}$, $\overrightarrow{PR}\times\overrightarrow{QR}=\begin{pmatrix}-1\\-3\\-1\end{pmatrix}\times\begin{pmatrix}3\\-4\\-4\end{pmatrix}=\begin{pmatrix}8\\-7\\13\end{pmatrix}$ Attempts a relevant vector product. This mark can be scored for sight of the correct 3 x 3 determinant with correct elements but allow one slip. If no working shown, then obtaining two correct components is sufficient to score this mark **Dependent on the first M mark**		**dM1**
	$\begin{pmatrix}8\\-7\\13\end{pmatrix}\cdot\begin{pmatrix}2\\5\\0\end{pmatrix}=\ldots\{16-35+0=-19\}$, $\begin{pmatrix}8\\-7\\13\end{pmatrix}\cdot\begin{pmatrix}-2\\6\\3\end{pmatrix}=\ldots\{-16-42+39=-19\}$ $\begin{pmatrix}8\\-7\\13\end{pmatrix}\cdot\begin{pmatrix}1\\2\\-1\end{pmatrix}=\ldots\{8-14-13=-19\}$ Calculates **a**.**n** with their normal. Requires 1st method mark only but normal cannot be $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{QR}$,$\overrightarrow{OP}$, $\overrightarrow{OQ}$ or $\overrightarrow{OR}$. If just a value is given it must be correct **Dependent on the first M mark**		**dM1**
	$8x-7y+13z+19=0$ oe	Correct equation. Any multiple is acceptable.	**A1**
			(4)
(c)	$\dfrac{\lvert(12)(-4)+(-1)(-3)+(10)(2)\rvert}{\sqrt{12^2+(-1)^2+10^2}}$	Correct numerical expression for the distance but condone missing modulus sign. Allow the numerator in dot product form. Allow if one component of the normal vector is misread or one sign error etc, but the method is otherwise correct. Accept 1^2 for $(-1)^2$	**M1**
	$\left\{=\dfrac{\lvert-25\rvert}{7\sqrt{5}}\right\}=\dfrac{5\sqrt{5}}{7}$	$\dfrac{5\sqrt{5}}{7}$ only	**A1**
			(2)

ALT	$\overrightarrow{QS}\bullet\frac{\mathbf{n}}{\lvert\mathbf{n}\rvert}=\frac{\lvert(12)(-2)+(-1)(-9)+(10)(-1)\rvert}{\sqrt{12^2+(-1)^2+10^2}}$ OR $\overrightarrow{RS}\bullet\frac{\mathbf{n}}{\lvert\mathbf{n}\rvert}=\frac{\lvert(12)(-5)+(-1)(-5)+(10)(3)\rvert}{\sqrt{12^2+(-1)^2+10^2}}$	Correct numerical expression for the distance using the projection of S to any of the other points on the plane, resolved in the direction of the normal vector, but condone missing modulus sign	**M1**
	$\left\{=\frac{\lvert-25\rvert}{7\sqrt{5}}\right\}=\frac{5\sqrt{5}}{7}$	$\frac{5\sqrt{5}}{7}$ only	**A1**
			Total 9

Question Number	Scheme	Notes	Marks
6(a)	$\mathbf{A}=\begin{pmatrix}p&3&-1\\0&2&-1\\3&-1&1\end{pmatrix}\Rightarrow\mathbf{M}=\begin{pmatrix}1&3&-6\\2&p+3&-p-9\\-1&-p&2p\end{pmatrix}\Rightarrow\mathbf{C}=\begin{pmatrix}1&-3&-6\\-2&p+3&p+9\\-1&p&2p\end{pmatrix}$ Obtains the matrix of cofactors with **six** correct elements. This could be implied by a $\mathbf{C}^{\mathrm{T}}$ matrix with **six** correct elements, provided it is either labelled as $\mathbf{C}^{\mathrm{T}}$ or used correctly in $\dfrac{1}{\det\mathbf{A}}\mathbf{C}^{\mathrm{T}}$		**M1**
	$\det\mathbf{A}=p-3$	Correct simplified determinant	**B1**
	$\dfrac{1}{p-3}\begin{pmatrix}1&-2&-1\\-3&p+3&p\\-6&p+9&2p\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{1}{p-3}&\frac{-2}{p-3}&\frac{-1}{p-3}\\\frac{-3}{p-3}&\frac{p+3}{p-3}&\frac{p}{p-3}\\\frac{-6}{p-3}&\frac{p+9}{p-3}&\frac{2p}{p-3}\end{pmatrix}$ **d**M1: Applies $\dfrac{1}{\det\mathbf{A}}\mathbf{C}^{\mathrm{T}}$ with their det **A** and **C** (condone one slip) A1: Fully correct inverse. ISW once a correct matrix inverse is seen, if simplified incorrectly **Note:** the correct answer written down scores all four marks in part (a)		**dM1** **A1**
			(4)
(b)	$2p+3+5=16\Rightarrow 2p=8\Rightarrow p=4$ Or $\begin{pmatrix}2\\1\\-5\end{pmatrix}=\begin{pmatrix}\frac{1}{p-3}&\frac{-2}{p-3}&\frac{-1}{p-3}\\\frac{-3}{p-3}&\frac{p+3}{p-3}&\frac{p}{p-3}\\\frac{-6}{p-3}&\frac{p+9}{p-3}&\frac{2p}{p-3}\end{pmatrix}\begin{pmatrix}16\\7\\0\end{pmatrix}=\begin{pmatrix}\frac{2}{p-3}\\\frac{7p-27}{p-3}\\\frac{7p-33}{p-3}\end{pmatrix}$ $\Rightarrow 2=\dfrac{2}{p-3}\Rightarrow 2p-6=2\Rightarrow p=4$	$p=4$ following any other correct equation (for example, equating elements from this matrix product etc)	**B1***
			(1)
(c)	$\begin{pmatrix}1&-2&-1\\-3&7&4\\-6&13&8\end{pmatrix}\begin{pmatrix}2\\q\\-1\end{pmatrix}=\ldots$	Completes an attempt to multiply $\begin{pmatrix}2\\q\\-1\end{pmatrix}$ by their inverse using $p=4$	**M1**
	$(-2q+3,\ 7q-10,\ 13q-20)$ or $x=-2q+3,\quad y=7q-10,\quad z=13q-20$	<u>A1ft</u>: **One correct and simplified ft their inverse** A1: Fully correct solution. No incorrect work. Accept as a PV.	**A1ft** **A1**
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
7(a)(i)	$\frac{x^2}{45}+\frac{y^2}{20}=1\Rightarrow a^2=45,\ b^2=20$ $b^2=a^2\left(1-e^2\right)\Rightarrow 20=45\left(1-e^2\right)\Rightarrow e$ or $e^2=...$	Uses a correct eccentricity formula with $a^2=45$ and $b^2=20$ and finds a value for e or e^2	**M1**
	$e=\frac{\sqrt{5}}{3}$	Correct exact value. Any single fraction equivalent. Not $\pm$	**A1**
(a)(ii)	$\frac{a}{e}=\frac{3\sqrt{5}}{"\frac{\sqrt{5}}{3}"}$	For a <u>numerical</u> expression for $\frac{a}{e}$ with their e but allow this mark if 45 is used for a	**M1**
	$x=\pm 9$	**Both** correct equations. $x=\pm 9$ or $x=9,\ x=-9$ only	**A1**
			(4)
(b)	$x=2\cosh 2t,\ \ y=8\sinh t$ $\Rightarrow x=2\left(2\sinh^2 t+1\right)$	Replaces $\cosh 2t$ with $\pm 2\sinh^2 t\pm 1$ oe Missing 'h's is M0 but if recovered A0*	**M1**
	$\frac{y^2}{64}=\sinh^2 t\Rightarrow x=2\left(\frac{2y^2}{64}+1\right)$	Proceeds to eliminate t	**dM1**
	$x=\frac{y^2}{16}+2\Rightarrow y^2=16x-32$ *	Correct equation with no errors. Any notational slips will be penalised for this mark, such as missing 'h' or 'x' instead of 't' but condone siht for sinht etc	**A1***
			(3)
ALT 1	$y=8\sinh t=8\left(\frac{e^t-e^{-t}}{2}\right)$ $y^2=64\sinh^2 t=64\left(\frac{e^{2t}-2+e^{-2t}}{4}\right)$	Replaces y with the correct exponential form and squares	**M1**
	$=16\left(e^{2t}+e^{-2t}\right)-32$ $=16\left(2\cosh 2t\right)-32$	Separates showing the correct exponential form for $\cosh 2t$	**dM1**
	$y^2=16x-32$	Correct equation with no errors	**A1**
			(3)
ALT 2	$x=2\cosh 2t,\ \ y=8\sinh t$ $\Rightarrow x=2\left(2\cosh^2 t-1\right)$	Replaces $\cosh 2t$ with $\pm 2\cosh^2 t\pm 1$ oe Missing 'h's is M0 but if recovered A0*	**M1**
	$\cosh^2 t-\sinh^2 t=1$ $\Rightarrow\left(\frac{x+2}{4}\right)-\left(\frac{y}{8}\right)^2=1$	Proceeds to eliminate t	**dM1**
	$16\left(x+2\right)-y^2=64$ $\Rightarrow y^2=16x-32$	Correct equation with no errors. Any notational slips will be penalised for this mark, such as missing 'h' or 'x' instead of 't' but condone siht for sinht etc	**A1***
			(3)
(c)	$\frac{x^2}{45}+\frac{16x-32}{20}=1\Rightarrow 20x^2+45\left(16x-32\right)=900$ $20x^2+720x-2340=0\Rightarrow x^2+36x-117=0$	Substitutes $y^2=16x-32$ into E and solves a resulting 3TQ (usual rules - no incorrect root if no working)	**M1**

	$\Rightarrow x = ...$		
	$\{(x-3)(x+39)=0 \Rightarrow\}\ x=3$	Correct value	**A1**
	$2y\dfrac{dy}{dx}=16$ or $\dfrac{dy}{dx}=\dfrac{1}{2}(16x-32)^{-\frac{1}{2}}(16)$ **OR** $\dfrac{dx}{dt}=4\sinh 2t$ and $\dfrac{dy}{dt}=8\cosh t$	Correct expression involving $\dfrac{dy}{dx}$ **OR** Correct expressions for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ Allow if the derivatives are seen in (b)	**B1**

	$2\pi\int y\sqrt{1+\dfrac{64}{y^2}\dfrac{dx}{dy}}\,dy$ or $2\pi\int(16x-32)^{\frac{1}{2}}\sqrt{1+\dfrac{64}{16x-32}}\,dx$ Or $2\pi\int 8\sinh t\sqrt{16\sinh^2 2t+64\cosh^2 t}\,dt$ Uses $...\int y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\{dx\}$ OR $...\int y\sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2}\{dx\}$ There must be some attempt to apply this to the question with substitutions seen, not simply stated. The integral must be in terms of one variable but ignore the limits (if given) for this mark. Ignore error with (or missing) 2π Integral may be seen later		**M1**
	$=2\pi\int(16x-32+64)^{\frac{1}{2}}\,dx$ or e.g., $8\pi\int(x+2)^{\frac{1}{2}}\,dx$ Or $2\pi\int 8\sinh t\sqrt{64\cosh^2 t(1+\sinh^2 t)}\,dt=2\pi\int 8\sinh t\,8\cosh^2 t\,dt$ Or $=\dfrac{\pi}{4}\int y(y^2+64)^{\frac{1}{2}}\,dy$	Correct and in integrable form	**A1**
	$=\dfrac{16\pi}{3}(x+2)^{\frac{3}{2}}$ Or $=\dfrac{128\pi}{3}\cosh^3 t$ Or $=\dfrac{\pi}{12}(y^2+64)^{\frac{3}{2}}$	For $(Cx+D)^{\frac{1}{2}}\rightarrow\dfrac{(Cx+D)^{\frac{3}{2}}}{\frac{3}{2}}$ Or $a\sinh t\ b\cosh^2 t\,dt\rightarrow k\cosh^3 t$ Or For $y(y^2+C)^{\frac{1}{2}}\rightarrow\dfrac{(y^2+C)^{\frac{3}{2}}}{\frac{3}{2}}$	**M1**

	$\left[\frac{16\pi}{3}(x+2)^{\frac{3}{2}}\right]_3^9 = \frac{16\pi}{3}\left(11\sqrt{11}-5\sqrt{5}\right)$	Correct answer in correct form. Use of parametric form is unlikely to give an exact answer in the form required, as the limits are $t_1 = \frac{1}{2}\ln\left(\frac{3+\sqrt{5}}{2}\right)$ and $t_2 = \frac{1}{2}\ln\left(\frac{9+\sqrt{77}}{2}\right)$ If in doubt send to review	**A1**
			(7)
			Total 14

Question Number	Scheme	Notes	Marks
8(a)	$y=\sin^{n-1}x\cos x$ $\frac{dy}{dx}=(n-1)\sin^{n-2}x\cos^2 x-\sin x\sin^{n-1}x$	Achieves a derivative of the correct form (sign/coefficient errors only)	**M1**
	$=(n-1)\sin^{n-2}x(1-\sin^2 x)-\sin x\sin^{n-1}x$ $\{(n-1)\sin^{n-2}x-(n-1)\sin^{n}x-\sin^n x\}$ $=(n-1)\sin^{n-2}x-n\sin^n x$ *	Reaches given answer with no errors and replaces $\cos^2 x$ with $1-\sin^2 x$ <u>which must be shown</u>	**A1***
			(2)
(b)	**Note that candidates may split work over multiple lines due to the number of terms involved** $I_n=\int e^{6x}\sin^n x\,dx$ $u=\sin^n x\quad \frac{du}{dx}=n\sin^{n-1}x\cos x\quad \frac{dv}{dx}=e^{6x}\quad v=\frac{1}{6}e^{6x}$ $I_n=\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx=\frac{1}{6}e^{6x}\sin^n x-\frac{n}{6}\int e^{6x}\sin^{n-1}x\cos x\,dx$ M1: Achieves an expression of the form $Ae^{6x}\sin^n x\pm B\int e^{6x}\sin^{n-1}x\cos x\,\{dx\}$ A1: Any correct expression. "dx" is not required		**M1** **A1**
	$u=\sin^{n-1}x\cos x\quad \frac{du}{dx}=(n-1)\sin^{n-2}x-n\sin^n x\quad \frac{dv}{dx}=e^{6x}\quad v=\frac{1}{6}e^{6x}$ $I_n=\frac{1}{6}e^{6x}\sin^n x-\frac{n}{6}\left[\frac{1}{6}e^{6x}\sin^{n-1}x\cos x-\frac{1}{6}\int(n-1)e^{6x}\sin^{n-2}x\{dx\}+\frac{n}{6}\int e^{6x}\sin^n x\{dx\}\right]$ Applies parts again to obtain an expression of the correct form for I_n Some of the terms may be replaced with I_n and I_{n-2} in the intermediate working (condone sign and coefficient errors only)		**dM1**
	$I_n=\frac{1}{6}e^{6x}\sin^n x-\frac{n}{36}e^{6x}\sin^{n-1}x\cos x+\frac{n(n-1)}{36}I_{n-2}-\frac{n^2}{36}I_n$ Introduces I_{n-2} and I_n to an expression of the correct form		**ddM1**
	$(n^2+36)I_n=6e^{6x}\sin^n x-ne^{6x}\sin^{n-1}x\cos x+n(n-1)I_{n-2}$ $\Rightarrow(n^2+36)I_n=e^{6x}\sin^{n-1}x(6\sin x-n\cos x)+n(n-1)I_{n-2}$ * Obtains the printed answer <u>with intermediate step and no errors</u>. Poor bracketing can be condoned if it is recovered before the final answer. Condone missing "dx"s but there should be no missing or mixed variables		**A1***
			(5)

ALT	$I_n = \int e^{6x} \sin^n x \, dx$ $u = e^{6x}\sin^{n-1} x \quad \frac{du}{dx} = 6e^{6x}\sin^{n-1} x + e^{6x}\left[(n-1)\sin^{n-2} x\cos x\right] \quad \frac{dv}{dx} = \sin x \quad v = -\cos x$ $I_n = \int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ $= -e^{6x}\sin^{n-1} x\cos x + \int 6e^{6x}\sin^{n-1} x\cos x + (n-1)e^{6x}\sin^{n-2} x\cos^2 x \,\{dx\}$ M1: Achieves an expression of the form $Ae^{6x}\sin^{n-1} x\cos x \pm B\int e^{6x}\sin^{n-1} x\cos x + Ce^{6x}\sin^{n-2} x\cos^2 x\{dx\}$ A1: Any correct expression. "dx" is not required		**M1** **A1**
	$u = e^{6x} \quad \frac{du}{dx} = 6e^{6x} \quad \frac{dv}{dx} = \sin^{n-1} x\cos x \quad v = \frac{1}{n}\sin^n x$ $I_n = -e^{6x}\sin^{n-1} x\cos x + \frac{6}{n}e^{6x}\sin^n x - \frac{36}{n}\int e^{6x}\sin^n x\{dx\} + \int(n-1)e^{6x}\sin^{n-2} x\{dx\} - \int(n-1)e^{6x}\sin^n x\{dx\}$ Applies parts again to obtain an expression of the correct form for I_n. Some of the terms may be replaced with I_n and I_{n-2} in the intermediate working (condone sign and coefficient errors only)		**dM1**
	$I_n = -e^{6x}\sin^{n-1} x\cos x + \frac{6}{n}e^{6x}\sin^n x - \frac{36}{n}I_n + (n-1)I_{n-2} - (n-1)I_n$ Introduces I_{n-2} and I_n to an expression of the correct form		**ddM1**
	$I_n\left(n + \frac{36}{n}\right) = -e^{6x}\sin^{n-1} x\cos x + \frac{6}{n}e^{6x}\sin^n x + (n-1)I_{n-2}$ $\Rightarrow I_n\left(n^2 + 36\right) = -ne^{6x}\sin^{n-1} x\cos x + 6e^{6x}\sin^n x + n(n-1)I_{n-2}$ $\Rightarrow I_n\left(n^2 + 36\right) = e^{6x}\sin^{n-1} x\left(6\sin x - n\cos x\right) + n(n-1)I_{n-2}$ Obtains the printed answer <u>with intermediate step and no errors</u>. Poor bracketing can be condoned if it is recovered before the final answer. Condone missing "dx"s but there should be no missing or mixed variables		**A1***
			(5)
(c)	$40I_2 = \left[e^{6x}\sin x\left(6\sin x - 2\cos x\right)\right]_0^{\frac{\pi}{6}} + 2I_0$	Applies the reduction formula with $n = 2$ Limit does not need to applied or could be missing, but "n"s should be replaced. Condone '36 + 2^2' for '40' on LHS	**M1**
	$I_0 = \int_0^{\frac{\pi}{6}} e^{6x}dx = \frac{1}{6}\left[e^{6x}\right]_0^{\frac{\pi}{6}} = \frac{1}{6}\left(e^{\pi} - 1\right)$ Or embedded, such as $I_2 = \frac{1}{40}\left(e^{\pi}\frac{1}{2}\left(3 - \sqrt{3}\right) + \frac{1}{3}e^{\pi}\right) - \frac{1}{40}\left(\frac{1}{3}\right)$	Correct numerical expression for I_0 This may be seen embedded in a correct numerical expression for I_2	**B1**
	$I_2 = \frac{e^{\pi}}{80}\left(3 - \sqrt{3}\right) + \frac{1}{120}\left(e^{\pi} - 1\right)$	Uses the limit $\frac{\pi}{6}$ and their I_0 to obtain a numerical expression for I_2 (with trig functions evaluated etc) Sight of this expression scores the first three marks	**dM1**
	$I_2 = \frac{11e^{\pi}}{240} - \frac{\sqrt{3}e^{\pi}}{80} - \frac{1}{120}$ $= \frac{1}{240}\left(e^{\pi}\left(11 - 3\sqrt{3}\right) - 2\right)$	Correct answer in the correct form. Allow $\frac{1}{240}\left(11e^{\pi} - 3\sqrt{3}e^{\pi} - 2\right)$	**A1**
			(4)

Total 11
PAPER TOTAL: 75