Mark Scheme (FINAL)

Summer 2016

Pearson Edexcel International A Level
in Decision Mathematics 1
(WDM01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
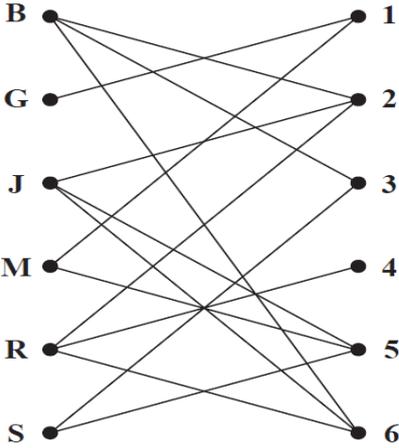   - ⌐ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    *   If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    *   If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)	Bin 1: <u>4.2</u> <u>1.8</u> <u>1.3</u> Bin 2: <u>3.1</u> 4.0 Bin 3: 4.1 3.7 Bin 4: 2.3 2.7	<u>M1</u> A1 **(2)**
(b)	$\dfrac{27.2}{7.8} \approx 3.487$ so yes 4 bins is optimal	M1 A1 **(2)**
(c)	e.g. middle right 4.2 1.8 3.1 1.3 |4.0| 4.1 3.7 2.3 2.7 Pivot 4.0 1.8 3.1 1.3 |3.7| 2.3 2.7 |4.0| 4.2 |4.1| Pivot 3.7 4.1 1.8 3.1 |1.3| 2.3 2.7 |3.7| |4.0| |4.1| 4.2 Pivot 1.3 (4.2) |1.3| 1.8 3.1 |2.3| 2.7 |3.7| |4.0| |4.1| 4.2 Pivot 2.3 |1.3| 1.8 |2.3| 3.1 |2.7| |3.7| |4.0| |4.1| 4.2 Pivot (1.8) 2.7 |1.3| 1.8 |2.3| |2.7| 3.1 |3.7| |4.0| |4.1| 4.2 Sort complete	M1 (quick) A1 (2 passes + choice of pivot for the 3rd) A1ft (3rd and 4th passes correct) A1 (CSO) **(4)**
(d)	Kruskal: AC, AD, CE, reject DE, reject CD, AB, reject BE, EF	M1 A1 **(2)**
(e)	e.g. Prim cannot be used since with Prim the tree 'grows' in a connected fashion e.g. AB and DE have no vertex in common and since Prim adds arcs which introduce new vertices to the tree, they will never be connected	B1 **(1)** **11 marks**

Notes for Question 1

a1M1: First four (underlined) items placed correctly

a1A1: CSO – all correct

b1M1: Attempt to find lower bound $(27.2 \pm 4.2) / 7.8$ (awrt 3.49 or 3.48 with no working can imply this mark)

b1A1:: CSO – correct calculation seen or awrt 3.49 or 3.48 **and** a conclusion – accept 'yes' as a minimum conclusion – however, '4 is the optimal number of bins' (or equivalent) with no reference to the solution in (a) is A0

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). **If only choosing one pivot per iteration then M1 only** – Bubble sort is not a MR and scores M1 only for 1.8 3.1 1.3 4.0 4.1 3.7 2.3 2.7 4.2 (for left to right) or 1.3 4.2 1.8 3.1 2.3 4.0 4.1 3.7 2.7 (for right to left)

c1A1: First two passes correct **and** next pivots chosen correctly for third pass (but third pass does not need to be correct) – so they must be choosing (if middle right) a pivot value of 1.3 for the third pass **or** (if middle left) a pivot value 3.7

c2A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots). They do not need to be choosing a pivot for the fifth pass for this mark

c3A1: CSO – including a fifth pass (the 27) **and** a 'sort complete' statement if middle right **or** a fifth pass (the 23) if middle left

d1M1: Kruskal: first three arcs correctly chosen (AC(1.3), AD(1.8), CE(2.3)) and arc DE(2.7) rejected at the correct time – no follow through from an incorrect list – condone the use of the arc weights for this mark

d1A1: CSO – all selections and rejections correct (in the correct order at the correct time) – arcs not weights

e1B1: CAO (an indication that AB and DE are not connected is sufficient for this mark)

Question Number	Scheme	Marks

For (c) using middle left as pivot

4.2 1.8 3.1 1.3 [4.0] 4.1 3.7 2.3 2.7	Pivot 4.0	
1.8 3.1 [1.3] 3.7 2.3 2.7 [4.0] [4.2] 4.1	Pivot 4.2 1.3	
[1.3] 1.8 3.1 [3.7] 2.3 2.7 [4.0] 4.1 [4.2]	Pivot 3.7 (4.1)	
[1.3] 1.8 [3.1] 2.3 2.7 [3.7] [4.0] 4.1 [4.2]	Pivot 3.1	
[1.3] 1.8 [2.3] 2.7 [3.1] [3.7] [4.0] 4.1 [4.2]	Pivot 2.3	
[1.3] 1.8 [2.3] 2.7 [3.1] [3.7] [4.0] 4.1 [4.2]	(Sort complete)	

Sorting list into descending order in (c)

- If the candidate sorts the list into descending order and reverses the list in this part then this can score full marks in (c)
- If the list is not reversed in (c) then mark as a misread (so remove the last two A marks earned in (c). If the candidate says that the list needs reversing in (c) but does not actually show the reversed list in (c) then remove the final A mark

Descending (middle right)

4.2 1.8 3.1 1.3 [4.0] 4.1 3.7 2.3 2.7	Pivot 4.0	
4.2 [4.1] [4.0] 1.8 3.1 1.3 [3.7] 2.3 2.7	Pivot 4.1 3.7	
4.2 [4.1] [4.0] [3.7] 1.8 3.1 [1.3] 2.3 2.7	Pivot (4.2) 1.3	
4.2 [4.1] [4.0] [3.7] 1.8 3.1 [2.3] 2.7 [1.3]	Pivot 2.3	
4.2 [4.1] [4.0] [3.7] 3.1 [2.7] [2.3] 1.8 [1.3]	Pivot 2.7 (1.8)	
4.2 [4.1] [4.0] [3.7] 3.1 [2.7] [2.3] 1.8 [1.3]	(Sort complete)	

Descending (middle left)

4.2 1.8 3.1 1.3 [4.0] 4.1 3.7 2.3 2.7	Pivot 4.0	
[4.2] 4.1 [4.0] 1.8 3.1 [1.3] 3.7 2.3 2.7	Pivot 4.2 1.3	
[4.2] 4.1 [4.0] 1.8 3.1 [3.7] 2.3 2.7 [1.3]	Pivot (4.1) 3.7	
[4.2] 4.1 [4.0] [3.7] 1.8 [3.1] 2.3 2.7 [1.3]	Pivot 3.1	
[4.2] 4.1 [4.0] [3.7] [3.1] 1.8 [2.3] 2.7 [1.3]	Pivot 2.3	
[4.2] 4.1 [4.0] [3.7] [3.1] 2.7 [2.3] 1.8 [1.3]	Sort complete	

Question Number	Scheme	Marks
2. (a)		B1 **(1)**
(b)	Alternating path G – 1 = M – 5 = J – 2 = B – 3 or G – 1 = M – 5 = J – 6 = R – 2 = B – 3 Change status G = 1 – M = 5 – J = 2 – B = 3 or G = 1 – M = 5 – J = 6 – R = 2 – B = 3 Improved matching 1. B = 3, G = 1, J = 2, M = 5, R = 6, (S unmatched) or 2. B = 3, G = 1, J = 6, M = 5, R = 2, (S unmatched)	M1 A1 A1 **(3)**
(c)	**Alternating path from IM 1**: S – 3 = B – 6 = R – 4 **or** S – 3 = B – 2 = J – 6 = R – 4 With change status leading to Complete matching B = 6, G = 1, J = 2, M = 5, R = 4, S = 3 or B = 2, G = 1, J = 6, M = 5, R = 4, S = 3 **Alternating path from IM 2**: S – 3 = B – 2 = R – 4 or S – 3 = B – 6 = J – 2 = R – 4 With change status leading to Complete matching B = 2, G = 1, J = 6, M = 5, R = 4, S = 3 or B = 6, G = 1, J = 2, M = 5, R = 4, S = 3	M1 A1 A1 **(3)** **7 marks**
Notes for Question 2	

a1B1: CAO

b1M1: An alternating path from G to 3 (or vice versa)
b1A1: CAO – a correct path including change status **either** stated **or** shown. Chosen path clear
b2A1: CAO – must follow from the correct stated path. Accept on a **clear** diagram (with five arcs only)

c1M1: An alternating path from S to 4 (or vice versa)
c1A1: CAO – a correct path including change status **either** stated **or** shown. Chosen path clear.
c2A1: CAO – must follow from **two** correct stated paths (so both previous M marks must have been awarded). Accept on a clear diagram (with six arcs only)

Question Number	Scheme	Marks
3. (a)	$2y \le x+12$ $5y \ge 2x$ $5x+2y \le 60$ $x \ge 0$	B1, B1, B1, B1 **(4)**
(b)	$(0,0),\ (0,6),\ (8,10),\ \left(\dfrac{300}{29},\dfrac{120}{29}\right)$	B1 M1 A1 **(3)**
(c)	At $(0, 0)$ $P = 0$ At $(0, 6)$ $P = 6$ At $(8, 10)$ $P = 26$ At $\left(\dfrac{300}{29},\dfrac{120}{29}\right)$ $P = \dfrac{720}{29}$ therefore (8,10) is the optimal vertex	M1 A1 A1 **(3)**

(d)	e.g. At $(0, 0)$ $P = 0$
	      At $(0, 6)$ $P = 6$
	      At $(8, 10)$ $P = 30$
	      At $\left(\dfrac{300}{29},\dfrac{120}{29}\right)$ $P = 30$ therefore the set of points on the line
	
	 $5x+2y = 60$ for which $8 \le x \le \dfrac{300}{29}$ gives P its maximum value
		M1 A1 A1
		 (3)

		13 marks

Notes for Question 3

a1B1: Either $5y \ge 2x$ or $5x+2y \le 60$ (accept strict inequality for this mark)

a2B1: Both $5y \ge 2x$ and $5x+2y \le 60$ correct

a3B1: Correct equation of the line $2y = x+12$ (aef) – accept any inequality for this mark

a4B1: Both $2y \le x+12$ **and** $x \ge 0$

b1B1: The three coordinates (0, 0), (0, 6) and (8, 10) correct

b1M1: Using simultaneous equations to find fourth vertex – must be a correct method to solve simultaneous equations and must arrive at $x = \ldots$ and $y = \ldots$ but allow slips/errors. This mark can be awarded for the correct exact coordinates stated with no working

b1A1: CAO for $\left(\dfrac{300}{29},\dfrac{120}{29}\right)$ or stated in terms of x and y

c1M1: Point testing at least two of their vertices using the correct objective function (objective line is M0)

c1A1: Point testing three of the correct vertices correctly

c2A1: All four correct vertices tested correctly and correct conclusion that (8, 10) is the optimal vertex

d1M1: Either point testing (at least two vertices) using the correct objective function **or** using the objective line method (by considering an objective line with the correct gradient)

d1A1: Recognising that the points on the line $5x+2y = 60$ **or** points in the interval $8 \le x \le \dfrac{300}{29}$

$\left(\text{or } \dfrac{120}{29} \le y \le 10\right)$ give P its maximum value. Stating/recognising that the gradient of the objective is the

same as the gradient of $5x+2y = 60$ scores M1A1. Just stating the two points for which $P = 30$ is A0A0

d2A1: Stating that all points on $5x+2y = 60$ when $8 \le x \le \dfrac{300}{29}$ $\left(\text{or } \dfrac{120}{29} \le y \le 10\right)$ give P its maximum value

Question Number	Scheme	Marks
4. (a)		M1 A1 A1 A1 A1 **(5)**
(b)	I = 9	B1 **(1)**
(c)	$7 + 4 + $ (their I) $ + K < 30$ or $K < J + L$ $0 < K < 10$ (accept $0 \le K < 10$) or $1 \le K \le 9$	M1 A1 **(2)** **8 marks**

<div align="center">

Notes for Question 4

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Condone lack of, or incorrect, numbered events throughout and arcs which cross one another. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'H dealt with correctly' requires the correct precedences for this activity, i.e. E, F and G labelled correctly and leading into the same node and H starting from that node but not H need not end in the correct place. **Activity on node is M0**

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: 7 activities (labelled on arc), one start and one dummy placed
a1A1: Activities A, B, C, D, E and G dealt with correctly
a2A1: Activities F, H, I and the two dummies (+ arrows) at the end of activities A and C leading into the end of activity B dealt with correctly
a3A1: Activities J, K, L and the dummy (+ arrow) at the end of activities H and I dealt with correctly
a4A1: CSO – all arrows present and correctly placed with one finish (with no extra dummies)

b1B1: CAO

c1M1: $K < 30 - 7 - 4 - $ (their I) **or** $K < 10$ **or** $K \le 9$ **or** $K \le 30 - 7 - 4 - ($their I$) - 1$
c1A1: CAO (see main scheme for the three acceptable answers)

Question Number	Scheme	Marks
5. (a)	A path is a (i) (finite) sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2, 1, 0 **(2)**
(b)	Shortest path: A – B – C – F – E – J Shortest length: 16 (km)	M1 A1 (BCF) A1 (HEG) A1ft (DJ) A1 A1ft **(6)**
(c)	Shortest path from A to G is A – B – C – F – H – G Shortest route from G to J via A is G – H – F – C – B – A – B – C – F – E – J Length: 16 + 12 = 28 (km)	B1 B1 B1ft **(3)**
(d)	A(BCFE)D + EF = 13 + 3 = 16 A(BCF)E + D(E)F = 11 + 5 = 16 A(BC)F + DE = 8 + 2 = 10* Arcs AB, BC, CF, DE will be traversed twice Route length = 88 − 7 + 10 = 91 (km)	M1 A1ft A1ft A1 A1 **(5)**
(e)	Route e.g. BABCBGHFJEDACDEFCFB	B1 **(1)** **17 marks**

Question Number	Scheme	Marks

Notes for Question 5

a1B1: One of the three points made clearly or two suggested. Arcs(edges)/vertices(nodes) must be referred to correctly. Do not condone incorrect technical language e.g. point for vertex

a2B1: All three points made clearly

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 20 17 16 in that order (20 16 17 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

b1M1: A larger value replaced by a smaller value at least once at C or D or F or G or J

b1A1: All values in B, C and F correct and the working values in the correct order

b2A1: All values in H, E and G correct and the working values in the correct order. Penalise order of labelling only once per question (H, E and G must be labelled in that order and H must be labelled after A, B, C and F)

b3A1ft: All values in D and J correct on the follow through and the working values in the correct order. To follow through D check that all the working values at D follow from the candidate's final values from nodes A, C and E (in the order that the candidate has labelled these three nodes) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will have working values from F, H and E)

b4A1: CAO for the path (from either A to J or J to A)

b5A1ft: If their answer is not 16 follow through their final value at J (condone lack of units)

c1B1: Shortest path from A to G stated correctly (most probably stated implicitly as part of their shortest route from G to J via A)

c2B1: Shortest path from G to J via A stated correctly

c3B1ft: Shortest route length correct on the follow through (their final value at G + their final value at J)

d1M1: Three distinct pairings of the **correct** four odd nodes

d1A1ft: Any two rows correct including pairing **and** totals (the ft on this and the next A mark in (d) is for using their final values at D, E and F from (b) for the lengths of AD, AE and AF only)

d2A1ft: All three rows correct including pairing and total (using their final values at D, E and F from (b))

d3A1: CAO correct arcs clearly stated (no ft on this mark)

d4A1: CAO (no ft on this mark)

e1B1: Any correct route – checks: starts and finishes at B, 19 vertices, AB, BC, CF, DE repeated and A appears 2, B(4), C(3), D(2), E(2), F(3), G(1), H(1), J(1)

Question Number	Scheme	Marks
6. (a)		M1 A1 M1 A1 **(4)**
(b)	Critical activities: C, D, H, K, M and N	B1 **(1)**
(c)	Float on E = 20 − 13 − 4 = 3 (days)	M1 A1 **(2)**
(d)	$\dfrac{88}{35} \approx 2.514 = 3$ (workers)	M1 A1 **(2)**
(e)		M1 A1 M1 A1 **(4)**
		13 marks

<div align="center">Notes for Question 6</div>

a1M1: **All** top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value – condone a missing 0 in the first box for the M mark only
a1A1: CAO (top boxes)
a2M1: **All** bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one 'rogue' value – condone a missing 0 in the first box for the M mark only
a2A1: CAO (bottom boxes)

b1B1: CAO on the critical activities (C, D, H, K, M, N)

c1M1: Correct calculation seen for activity E – all three numbers correct (following through the candidates completed diagram), float ≥ 0
c1A1: Float correct (no ft on this mark) – correct answer with no working scores M0A0

Question Number	Scheme	Marks

d1M1: Attempt to find lower bound: (a value in the interval [79 – 97] / their finish time) **or** (sum of the activities / their finish time) **or** as a minimum an awrt 2.51

d1A1: CSO – either a **correct** calculation seen or awrt 2.51 **then** 3. An answer of 3 with no working scores M0A0

e1M1: At least 10 activities including 6 floats. Scheduling diagram scores M0

e1A1: Critical activites dealt with correctly and five other non-critical activities dealt with correctly

e2M1: Exactly 16 activities (just once) including all 10 floats (on the correct non-critical activities) – this mark is not dependent on the previous A mark

e2A1: CAO (all activities correct and present just once)

Question Number	Scheme	Marks
7.	Maximise $(P =) 5x + 8y$ Subject to: $x + y \leq 450$ $\frac{17}{20}(x + y) \geq x$, simplifies to $3x \leq 17y$ $x \geq 3y$ $(x, y \geq 0)$	B1 B1 M1 A1 M1 A1 **6 marks**
Notes for Question 7		

1B1: CAO – expression correct and 'maximise' (accept $500x + 800y$)

2B1: CAO ($x + y \leq 450$)

1M1: Correct method – must see $\frac{85}{100}(x + y) \blacksquare x$ (or equivalent) where $\blacksquare$ is any inequality or equals. The bracket must be present or implied by later working

1A1: CAO – simplified – answer must have integer coefficients ($3x \leq 17y$)

2M1: Correct method – one of $x \blacksquare 3y$ or $3x \blacksquare y$ (or equivalent) where $\blacksquare$ is any inequality or equals

2A1: CAO – answer must have integer coefficients ($x \geq 3y$)