Pearson Edexcel

Mark Scheme (Results)

October 2022

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>‘M’ marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>‘A’ marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>‘B’ marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ The answer is printed on the paper
- ⎿ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to *x* = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to *x* = …

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$, leading to *x* = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	Attempts to get t in terms of x or y. E.g. $x=\dfrac{t}{t-3}\Rightarrow t=\dfrac{3x}{x-1}$	M1
	Forms a Cartesian equation and makes progress to making y the subject E.g. $y=\dfrac{1}{t}+2\Rightarrow y=\dfrac{x-1}{3x}+2$	dM1
	$\Rightarrow y=\dfrac{7x-1}{3x}$	A1
		(3) **(3 marks)**

M1: For an attempt to get t or $\dfrac{1}{t}$ in terms of x or y.

Condone poor attempts here. It is the intention that is important

dM1: For a complete attempt to form an equation of the form $y=\mathrm{f}(x)$

Look for

- getting t or $\dfrac{1}{t}$ in terms of x or y
- using this to form a Cartesian equation and makes some progress towards making y the subject.

E.g.I. $x=\dfrac{t}{t-3}\Rightarrow t=\dfrac{3x}{x-1}$ **followed by** $y=\dfrac{1}{t}+2\Rightarrow y=\dfrac{1}{\frac{3x}{x-1}}+2$

E.g.II. $y=\dfrac{1}{t}+2\Rightarrow t=\dfrac{1}{y-2}$ **followed by**

$$x=\frac{\frac{1}{y-2}}{\frac{1}{y-2}-3}\Rightarrow x=\frac{1}{1-3\left(y-2\right)}\Rightarrow\left(7-3y\right)x=1$$

Condone slips, for example in the signs and miscopies/misreads, but the overall mechanics of the attempt should be sound.

In E.g.II. an attempt must be made to make progress towards $y = \mathrm{f}(x)$, so allow for example when an intermediate line such as $\mathrm{g}(y)\times\mathrm{h}(x)=b$ or $\mathrm{g}(y)=\mathrm{h}(x)$ where $\mathrm{g}(y)$ is a linear expression in y

A1: Correct equation $y=\dfrac{7x-1}{3x}$ but allow $y=\dfrac{-1+7x}{3x}$

Alt Method:

M1: Substitutes both $y=\dfrac{1}{t}+2$ and $x=\dfrac{t}{t-3}$ into $y=\dfrac{ax-1}{bx}$ and makes progress to a point in which sides can be compared

$$\frac{1}{t}+2=\frac{a\times\frac{t}{t-3}-1}{b\times\frac{t}{t-3}}=\left(\frac{a-1}{b}\right)+\frac{3}{bt} \qquad \text{or} \qquad b+2bt=\left(a-1\right)t+3$$

dM1: Compares terms to set up and solve simultaneous equations in both a and b.

FYI $\dfrac{3}{b}=1,\ \dfrac{a-1}{b}=2$ or $b=3,\quad 2b=a-1$

A1: $y=\dfrac{7x-1}{3x}$ NOT just for correct values of a and b

Question Number	Scheme	Marks
2 (a)	$\dfrac{3x}{(2x-1)(x-2)} \equiv \dfrac{A}{2x-1}+\dfrac{B}{x-2} \Rightarrow$ Values for A and B	M1
	One correct value, either $A=-1$ or $B=2$	A1
	Correct PF form $\dfrac{-1}{2x-1}+\dfrac{2}{x-2}$	A1
		(3)
(b)	$\displaystyle\int \frac{-1}{2x-1}+\frac{2}{x-2}\,dx = -\frac{1}{2}\ln(2x-1)+2\ln(x-2)+(c)$	M1, A1ft
	$\displaystyle\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \left[-\frac{1}{2}\ln(2x-1)+2\ln(x-2)\right]_5^{25}$	
	$=\left(-\frac{1}{2}\ln 49+2\ln 23\right)-\left(-\frac{1}{2}\ln 9+2\ln 3\right)$	dM1
	$\ln\dfrac{529}{21}$	A1
		(4) **(7 marks)**

(a)

M1: Attempts a correct PF form leading to values for A and B.

Condone $\dfrac{3x}{(2x-1)(x-2)} \equiv C+\dfrac{A}{2x-1}+\dfrac{B}{x-2} \Rightarrow$ Values for at least A and B

A1: One correct value for A or B or one correct term

A1: Correct PF form $\dfrac{-1}{2x-1}+\dfrac{2}{x-2}$.

This is not just for correct values for A and B but it may be awarded from work seen in (b)

(b)

M1: $\displaystyle\int \frac{A}{2x-1}+\frac{B}{x-2}\,dx = C\ln(2x-1)+D\ln(x-2)$ o.e. Condone missing brackets here

A1ft: Correct ft on their values for A and B $\displaystyle\int \frac{A}{2x-1}+\frac{B}{x-2}\,dx = \frac{A}{2}\ln(2x-1)+B\ln(x-2)+(c)$

Do not condone missing brackets but the brackets can be implied by subsequent working, E.g. ln 49

dM1: Attempts to substitute the limits 5 and 25 (no misreads on the limits but condone slips, so don't accept a consistent lower limit of 0 for instance) into an expression of the correct form (with brackets which may be implied) and subtracts, either way around. Condone attempts where the substitutions were made into incorrectly combined ln terms.

A1: cso $\ln\dfrac{529}{21}$ Also allow in mixed number form $\ln 25\frac{4}{21}$

Question Number	Scheme	Marks
3 (a)	Attempts $(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})-(8\mathbf{i}-5\mathbf{j}+3\mathbf{k})$	M1
	$\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$	A1
		(2)
(b)	Attempts $\overrightarrow{PQ}\cdot\overrightarrow{RQ}=2\times-6+-3\times2+4\times1$	M1
	Full attempt to find $\cos PQR$ E.g. $2\times-6+-3\times2+4\times1=\sqrt{29}\sqrt{41}\cos PQR$	dM1
	Angle PQR = 114°	A1
		(3)
		(5 marks)

(a)

M1: Attempts to subtract vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ either way around. Look for $\overrightarrow{PQ}-\overrightarrow{PR}$ or $\overrightarrow{PR}-\overrightarrow{PQ}$. If a method is not shown it can be implied by two correct components of $\pm6\mathbf{i}\pm2\mathbf{j}\pm\mathbf{k}$ Note that an attempt such as $\overrightarrow{PR}-\overrightarrow{QP}$ is M0

A1: $\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$ o.e. such as $\begin{pmatrix}-6\\2\\1\end{pmatrix}$ Do not accept coordinates or indeed $\begin{pmatrix}-6\mathbf{i}\\2\mathbf{j}\\\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts scalar product of $\overrightarrow{PQ}$ and their $\overrightarrow{RQ}$. Look for an attempt at multiplying together the components and adding. There will be some confusion over direction so allow for sight of

$(\pm2\times"\pm6")+(\pm3\times"\pm2")+(\pm4\times"\pm1")$

This cannot be scored if they attempt a scalar product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ for instance.

dM1: Full attempt to find $\cos PQR$ using $\mathbf{a.b}=|\mathbf{a}|\,|\mathbf{b}|\cos\theta$ using vectors $\pm\overrightarrow{PQ}$ and their $\pm\overrightarrow{RQ}$.

There must be an attempt at both moduli with at least one correct (which may be unsimplified)

but you should ft on their $\overrightarrow{RQ}$. Don't be concerned whether the angle is acute or obtuse.

A1: Angle PQR = awrt 114° ISW after sight of this, e.g followed by 66°

Alt (b)

M1: Attempts all three lengths or all three lengths^2 using Pythagoras' Theorem. Look for an attempt to square and add with at least one modulus or modulus^2 correct.

dM1: Attempts to use the cosine rule with the lengths in the correct positions in order to find angle PQR

Look for $\cos PQR=\dfrac{PQ^2+QR^2-PR^2}{2\times PQ\times QR}$

There are more round about methods including finding other angles first and then using the sine rule

but this method mark can only be awarded when an attempt is made at angle PQR

A1: Angle PQR = awrt 114°

It must be found using "correct" vectors, e.g . $\pm(-6\mathbf{i}+2\mathbf{j}+\mathbf{k})$ and $\pm(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$

If, for example $\overrightarrow{RQ}$ is incorrect, e.g. $6\mathbf{i}-2\mathbf{j}+\mathbf{k}$, A0 will be awarded even if 114° is stated

Question Number	Scheme	Marks
4(a)	$\dfrac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\left(1-...\right)$	B1
	$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$	M1, A1
	$\dfrac{1}{\sqrt{4-x^2}}=\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$	A1, A1
		(5)
(b)	$\lvert x\rvert<2$	B1
		(1)
(c)	Substitutes an appropriate value of *x* in both sides with LHS in terms of $\sqrt{3}$	M1
	E.g. with $x=1$ $\sqrt{3}=\dfrac{2048}{1181}$ or $\dfrac{3543}{2048}$	A1
		(2) **(8 marks)**

(a) Note that the first mark in (a) is M1 on epen. We are now marking it as B1

B1: Correct constant term/ factor.

Accept as $4^{-\frac{1}{2}}$ or $\frac{1}{2}$ as the constant term or as the factor with the bracket starting (1....

M1: Correct attempt at the third or the fourth term of binomial expansion of form $\left(1+ax^2\right)^{-\frac{1}{2}}$

Look for a correct binomial coefficient with a correct power of *x*

E.g. $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(\pm ax^2\right)^2$ or $\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(\pm ax^2\right)^3$ where *a* could be 1

A1: Correct unsimplified

For example

$$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)}{2}\left(-\frac{1}{4}x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}-1\right)\times\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{1}{4}x^2\right)^3$$

A1: Two correct and simplified terms of $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$

A1: $\frac{1}{2}+\frac{1}{16}x^2+\frac{3}{256}x^4+\frac{5}{2048}x^6$. Accept as a list, ignore any terms with indices greater than these.

(b)

B1: States a correct range. E.g $\lvert x\rvert<2$ or $-2<x<2$ o.e.

(c) For example: See **

M1: Substitutes an exact value of *x* into both sides of their expansion that enables $\sqrt{3}$ to be found

.

E.g. $x=1$ $\dfrac{1}{\sqrt{4-1^2}}=\dfrac{1}{2}+\dfrac{1}{16}\times1^2+\dfrac{3}{256}\times1^4+\dfrac{5}{2048}\times1^6$

Note that $x=-1$ and $x^2=1$ all work with the same results

Alternatively correctly solves $\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$ o.e. and substitutes this into their expansion

A1: With $x=1$ $\sqrt{3}=\dfrac{2048}{1181}$ or $\dfrac{3543}{2048}$. This can only be scored from a correct expansion

**There are lots of values of *x* that work, all more difficult than the one above.

E.g $x^2=\dfrac{8}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\dfrac{\sqrt{3}}{2}$ and $x^2=\dfrac{11}{3}\Rightarrow\dfrac{1}{\sqrt{4-x^2}}=\sqrt{3}$

In both these cases the fractions become much more difficult to find.

For $x^2=\dfrac{8}{3}\Rightarrow\sqrt{3}=\dfrac{43}{27}$ and for $x^2=\dfrac{11}{3}\Rightarrow\sqrt{3}=\dfrac{55687}{55296}$

Alt I (a) By direct expansion

$$\frac{1}{\sqrt{4-x^2}}=\left(4-x^2\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\times4^{-\frac{3}{2}}\left(-x^2\right)^1+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times4^{-\frac{5}{2}}\left(-x^2\right)^2+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\times4^{-\frac{7}{2}}\left(-x^2\right)^3$$

B1: For $4^{-\frac{1}{2}}+$

M1: A correct attempt at the third or fourth terms condoning sign slips on the $-x^2$

A1: Correct and unsimplified expansion. See above

A1: Two correct and simplified terms of $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$

A1: $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$. Accept as a list

Alt II (a) By difference of two squares

$$\frac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+......$$

B1: For $4^{-\frac{1}{2}}+$

M1: For correctly writing $\dfrac{1}{\sqrt{4-x^2}}=\left(2-x\right)^{-\frac{1}{2}}\times\left(2+x\right)^{-\frac{1}{2}}=...\left(1-\dfrac{x}{2}\right)^{-\frac{1}{2}}\times\left(1+\dfrac{x}{2}\right)^{-\frac{1}{2}}=$

with a correct attempt at the third of fourth term in either expansion, followed by an attempt to combine both the expansions.

A1: One correct term in x^2, x^4 or x^6

A1: Two correct and simplified terms of $\dfrac{1}{2}+\dfrac{1}{16}x^2+\dfrac{3}{256}x^4+\dfrac{5}{2048}x^6$ with no terms in x, x^3 or x^5

A1: Fully correct

Question Number	Scheme	Marks
5.	States or implies Volume = $\int_{\frac{1}{\sqrt{2}}}^{k} \pi \left(\dfrac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 \mathrm{d}x$	B1
	Attempts $\int \left(\dfrac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 \mathrm{d}x = \int \dfrac{144x}{(2x^2+3)^3}\,\mathrm{d}x = -18(2x^2+3)^{-2}$	M1A1
	Sets $-\dfrac{18\pi}{(2k^2+3)^2} + \dfrac{18\pi}{(2\times\frac{1}{2}+3)^2} = \dfrac{713}{648}\pi$	M1
	$\Rightarrow -\dfrac{18}{(2k^2+3)^2} + \dfrac{9}{8} = \dfrac{713}{648} \Rightarrow (2k^2+3)^2 = 729 \Rightarrow k^2 = ...$	ddM1
	$k = 2\sqrt{3}$	A1
		(6 marks)

B1: States or implies Volume = $\int_{\frac{1}{\sqrt{2}}}^{k} \pi \left(\dfrac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 \mathrm{d}x$ o.e. including limits and π.

You can condone a missing dx and the limits the wrong way around.
You may not see the limits until the integration has been attempted which is fine.

M1: Attempts $\int \left(\dfrac{\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 \mathrm{d}x$ to achieve $A(2x^2+3)^{-2}$ o.e.

With substitution of $u = 2x^2+3$ look for Au^{-2}

A1: Correct integration of $\int \left(\dfrac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 \mathrm{d}x$ to $-18(2x^2+3)^{-2}$ or $-18u^{-2}$ which may be left unsimplified.

Note that this can be scored with or without the π and may even have an incorrect coefficient of 2π

M1: Uses the limits (subtracts either way around) within their attempted integration of y^2 NOT y and sets equal to $\dfrac{713}{648}\pi$. Condone poor attempts at integrating the y^2 here.

Note that if they change limits to u where $u = 2x^2+3$ look for limits of $2k^2+3$ and 4

ddM1: Dependent upon both previous M's. It is for proceeding to

either $(2k^2+3)^{\pm 2} = B$ where $B > 0$ leading to a value for k^2 or k

or setting up a quadratic equation in k^2 leading to a value for k^2 or k

.

FYI

$$k^4 + 3k^2 - 180 = 0 \Rightarrow \left(k^2 - 12\right)\left(k^2 + 15\right) = 0 \Rightarrow k^2 = 12$$

A1: $k = 2\sqrt{3}$ or exact equivalent such as $\sqrt{12}$ Condone if $k = \pm 2\sqrt{3}$ o.e.

Question Number	Scheme	Marks
6 (a)	For correct parameter $t=\dfrac{\pi}{4}$ or x coordinate at P $x=4$	B1
	$\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{-4\sin 2t}{3\sec^2 t}$	M1 A1
	Equation of tangent $y-0=-\dfrac{2}{3}(x-4)\Rightarrow y=-\dfrac{2}{3}x+\dfrac{8}{3}$	dM1 A1
		(5)
(b)	$k=1-\sqrt{3}$	B1
		(1)
(c)	-1,, f ,, 2	M1 A1
		(2) **(8 marks)**

(a)

B1: Correct parameter or x coordinate at P.

The correct x coordinate $x=4$ (can even be scored following $x=45°$)

M1: Attempts to find $\dfrac{dy}{dx}$ using $\dfrac{dy/dt}{dx/dt}$. Condone poor differentiation.

A1: Correct $\dfrac{dy}{dx}=\dfrac{-4\sin 2t}{3\sec^2 t}$. You may see other forms for this following use of identities.

If you see $\dfrac{dy}{dx}=\dfrac{-4\sin 2t}{3\sec^2 \underline{\underline{x}}}$ then only allow if they subsequently use this as $\dfrac{dy}{dx}=\dfrac{-4\sin 2t}{3\sec^2 t}$

Other possible correct gradients are $\dfrac{-8\sin t\cos t}{3\sec^2 t}$ and $\dfrac{-8\sin t\cos^3 t}{3}$

ISW after you see a correct answer. Quite often we see candidates incorrectly adapting a correct answer.

It is also possible for candidates to find $\dfrac{dy}{dt}$ at $t="\dfrac{\pi}{4}"$, $\dfrac{dx}{dt}$ at $t="\dfrac{\pi}{4}"$ and divide the two values.

dM1: Attempts to find the equation of the tangent at $t=\dfrac{\pi}{4}$

It is dependent upon the previous M and use of $(4,0)$ and $t=\dfrac{\pi}{4}$

Allow substitution into their adapted $\dfrac{dy}{dx}$

A1: $y=-\dfrac{2}{3}x+\dfrac{8}{3}$

(b)

B1: $k=1-\sqrt{3}$

(c)

M1: Either end correct. Allow strict inequalities here. E.g f < 2 scores M1 but f > 2 is M0

A1: -1,, f ,, 2 o.e such as $[-1,2]$ and -1,, y ,, 2

Question Number	Scheme	Marks
7 (i)	$u = e^x - 3 \Rightarrow \dfrac{du}{dx} = e^x$	B1
	$\displaystyle\int \frac{4e^{3x}}{e^x - 3}\,dx = \int \frac{4(u+3)^3}{u(u+3)}\,du = \int \frac{4(u+3)^2}{u}\,du$	M1 A1
	$\displaystyle = \int \frac{4u^2 + 24u + 36}{u}\,du$	
	$\displaystyle = \int 4u + 24 + \frac{36}{u}\,du = 2u^2 + 24u + 36\ln u$	dM1 A1
	$= \left[2u^2 + 24u + 36\ln u\right]_{u=2}^{u=4}$	M1
	$= 72 + 36\ln 2$	A1
		(7)
(ii)	$\displaystyle\int 3e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x - \int \frac{3}{2}e^x \sin 2x\,dx$	M1 A1
	$\displaystyle = \frac{3}{2}e^x \sin 2x - \left(-\frac{3}{4}e^x \cos 2x + \int \frac{3}{4}e^x \cos 2x\,dx\right)$	dM1
	Collect terms $\displaystyle \frac{15}{4}\int e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x$	ddM1
	$\displaystyle\int 3e^x \cos 2x\,dx = \frac{6}{5}e^x \sin 2x + \frac{3}{5}e^x \cos 2x\,(+c)$	A1
		(5) **(12 marks)**

(i) The 4 must be present to score the A marks. No misreads

B1: States or uses $\dfrac{du}{dx} = e^x$ o.e.such as $\dfrac{dx}{du} = \dfrac{1}{u+3}$ This may be seen within the integrand.

M1: A "correct" attempt to write the integrand in terms of u.

Look for $\displaystyle\int \frac{4e^{3x}}{e^x - 3}\,dx = \int \frac{P(u \pm 3)^3}{u(u \pm 3)}\,du$ which does not need to be simplified.

Condone a missing du but dx is M0

A1: Correct $\displaystyle\int \frac{4(u+3)^2}{u}\,du$. Condone a missing d$u$ (cannot be dx) .

dM1: Integrates by attempting to multiply out and integrating each term.

Look for two "correct" terms, one of which must be $\displaystyle\int \frac{P}{u}\,du = P\ln u$.

It is dependent upon having achieved $\displaystyle\int \frac{P(u \pm 3)^2}{u}\,du$

A1: Correct $2u^2 + 24u + 36\ln u$ o.e.

M1: Uses limits 2 and 4 within their attempted integral and subtracts . Condone poor attempts at the integration

Alternatively converts their answer in u back to x's using the correct substitution and uses the given limits.

There must have been some attempt to use the substitution $u = e^x - 3$ at some point.

A1: $72+36\ln 2$ or $36\ln 2+72$

(ii) Condone missing dx's here

M1: Attempts to integrate by parts $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = A\mathrm{e}^{x}\sin 2x \pm \int B\mathrm{e}^{x}\sin 2x\,\mathrm{d}x$

A1: $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = \frac{3}{2}\mathrm{e}^{x}\sin 2x - \int \frac{3}{2}\mathrm{e}^{x}\sin 2x\,\mathrm{d}x$ Condone missing dx's

The 3 may have been "removed"/factorised out and replaced at the final answer.
If so you can this would be awarded at the sight of a correct answer.

dM1: Attempts to integrate by parts again. See below of what to look for

Look for $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = A\mathrm{e}^{x}\sin 2x \pm B\mathrm{e}^{x}\cos 2x \pm \int C\,\mathrm{e}^{x}\cos 2x\,\mathrm{d}x$

ddM1: Dependent upon both previous M's

It is awarded for attempting to collect both terms in $\int \mathrm{e}^{x}\cos 2x\,\mathrm{d}x$

A1: $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = \frac{6}{5}\mathrm{e}^{x}\sin 2x + \frac{3}{5}\mathrm{e}^{x}\cos 2x\,(+c)$ condone the omission of the + c.

Alternative method for (ii) . This can be scored in a similar way to the above.

(ii)	$\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = 3\mathrm{e}^{x}\cos 2x + \int 6\mathrm{e}^{x}\sin 2x\,\mathrm{d}x$	M1 A1
	$= 3\mathrm{e}^{x}\cos 2x + 6\mathrm{e}^{x}\sin 2x - \int 12\mathrm{e}^{x}\cos 2x\,\mathrm{d}x$	dM1
	$15\int \mathrm{e}^{x}\cos 2x\,\mathrm{d}x = 6\mathrm{e}^{x}\sin 2x + 3\mathrm{e}^{x}\cos 2x$	ddM1
	$\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = \frac{6}{5}\mathrm{e}^{x}\sin 2x + \frac{3}{5}\mathrm{e}^{x}\cos 2x\,(+c)$	A1
		(5)

1stM1: $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = P\mathrm{e}^{x}\cos 2x \pm Q\int \mathrm{e}^{x}\sin 2x\,\mathrm{d}x$

2ndM1: $\int 3\mathrm{e}^{x}\cos 2x\,\mathrm{d}x = P\mathrm{e}^{x}\cos 2x \pm Q\mathrm{e}^{x}\sin 2x \pm R\int \mathrm{e}^{x}\cos 2x\,\mathrm{d}x$

Question Number	Scheme	Marks
8	Solves one of $\left(3x-y\right)=25$ and $\left(x+y\right)=1$ or $\left(3x-y\right)=5$ and $\left(x+y\right)=5$	M1
	Correct solution of one. Either $\left.\begin{array}{r}3x-y=25\\x+y=1\end{array}\right\}\Rightarrow 4x=26\Rightarrow x=6.5,\left(y=-5.5\right)$ Or $\left.\begin{array}{r}3x-y=5\\x+y=5\end{array}\right\}\Rightarrow 4x=10\Rightarrow x=2.5,\left(y=2.5\right)$	A1
	Solves both equations	dM1
	Both solved correctly with a minimal reason given for the contradiction e.g "not integers" with conclusion "hence there are no integers x and y such that $3x^2+2xy-y^2=25$"	A1
		(4)

Notes

M1: Attempts to solve one of the two possible cases.

Take as a minimum, one correct pair of equations followed by a value for x or a value for y

A1: Correctly solves one of the two possible cases.

To solve you need only find a value for x or y. Once a correct value is found you can ISW

dM1: Attempts to solve both possible cases

A1: Correctly solves the two possible cases and makes a concluding argument.

To score this mark (i) **all** calculations must be correct.

(ii) reason(s) for the contradiction must be written down. Allow "not integers", "×"

and (iii) gives a concluding statement must be given. Allow for example " hence proven"

Ignore any possible cases which would give rise to negative numbers but satisfy $\left(3x-y\right)\left(x+y\right)=25$

E.g $\left(3x-y\right)=-5,\left(x+y\right)=-5$

Withhold the final mark only if they include cases which do not satisfy $\left(3x-y\right)\left(x+y\right)=25$

E.g $\left(3x-y\right)=20,\left(x+y\right)=5$

Question Number	Scheme	Marks
9	$\begin{pmatrix} 2-\lambda \\ 8+2\lambda \\ 10+3\lambda \end{pmatrix} = \begin{pmatrix} -4+5\mu \\ -1+4\mu \\ 2+8\mu \end{pmatrix}$ Attempts to solve any two of the three equations Either (1) and (2) $\left.\begin{array}{r} 2-\lambda=-4+5\mu \\ 8+2\lambda=-1+4\mu \end{array}\right\} \Rightarrow \lambda=-\frac{3}{2}, \mu=\frac{3}{2}$ (1) and (3) $\left.\begin{array}{r} 2-\lambda=-4+5\mu \\ 10+3\lambda=2+8\mu \end{array}\right\} \Rightarrow \lambda=\frac{8}{23}, \mu=\frac{26}{23}$ (2) and (3) $\left.\begin{array}{r} 8+2\lambda=-1+4\mu \\ 10+3\lambda=2+8\mu \end{array}\right\} \Rightarrow \lambda=-10, \mu=-\frac{11}{4}$	M1, A1
	Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda=-\frac{3}{2}$ into $10+3\lambda=\frac{11}{2}$ and $\mu=\frac{3}{2}$ into $2+8\mu=14$	dM1
	Concludes that lines don't intersect with correct calculations and minimal reason	A1
	Additionally states that $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$ with a minimal reason So lines are skew CSO *	A1*
		(5) **(5 marks)**

Notes:
Main method seen
M1: Attempts to solve two of the three equations.
Accept as an attempt, writing down two of the three equations (condoning slips) followed by values for both λ and μ
A1: Solves two of the three equations to find correct values for both λ and μ, Allow equivalent fractions
dM1: Either: Substitutes their values of λ and μ into both sides of the third equation....or into the equations of both lines to find both coordinates
A1: Having achieved correct values for λ and μ, the values for the third equation are found to enable a comparison to be made. E.g. solving equations (1) and (2) and using equation (3) stating $10+3\times-\frac{3}{2}\neq 2+8\times\frac{3}{2}$ is sufficient. If the values are found they must be correct.
Important: Additionally, to score this mark, a minimal statement must be made that states that the lines do not intersect /cross. Condone statements such as $l_1 \neq l_2$
Stating that the lines are skew at this point is not sufficient to score this mark
In the alternative stating that "as the values are not the same, the lines cannot intersect" is sufficient.

A1*: CSO . Hence all previous marks must have been scored.
In addition to not intersecting there must be a statement, with a minimal reason, that the lines are not parallel and hence skew. Accept statements like, not intersecting, not parallel (with reason), hence proven.

Reasons could be $\begin{pmatrix}5\\4\\8\end{pmatrix} \neq k\begin{pmatrix}-1\\2\\3\end{pmatrix}$ o.e such as $5 = -5\times-1$ but $4 = 2\times 2$ so they are not parallel.

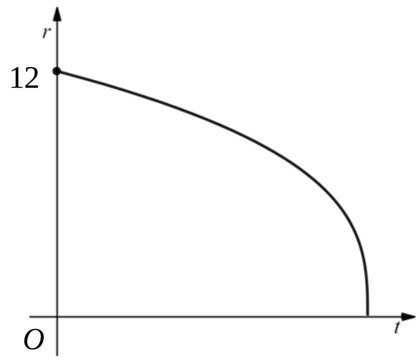
Accept an argument based around the scalar product of the direction vectors. If parallel $\cos\theta = 1$

A reason for the lines not being parallel cannot be $\begin{pmatrix}5\\4\\8\end{pmatrix} \neq \begin{pmatrix}-1\\2\\3\end{pmatrix}$

Note: Other methods are possible and it is important that you look at their complete attempt at proving that they don't intersect.
Alternative 1
For example it is possible to solve equations (1) and (2) to find just λ
then solve equations (1) and (3) to find just λ
and then conclude that "as the two values are not the same, the lines don't intersect"

M1 dM1 marks are scored together. Both aspects have to be attempted
Attempts to solve two of the three equations to find λ (or μ)
Attempts to solve a different pair of equations to find λ (or μ)
A1: Correct values for λ (or μ).
A1: conclude that "as the two values are not the same, the lines don't intersect"

If you see something that you feel deserves credit AND that you cannot mark, then please send to review

Question Number	Scheme	Marks
10 (a)	States or uses $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$	B1
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2} \Rightarrow \int r^2 \mathrm{d}r = \int \pm k \mathrm{d}t$	
	$\frac{1}{3}r^3 = \pm kt + c$	M1 A1
	Substitutes $t=0, r=12 \Rightarrow c = (576)$	
	AND $t=15, r=6$ and their $c = (576) \Rightarrow k = (\pm 33.6)$	M1
	$\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e.	A1
		(5)
(b)	Sets $r = 0 \Rightarrow -100.8t + 1728 = 0$	M1
	$\frac{120}{7}$ minutes or awrt 17.1 minutes	A1
		(2)
(c)		B1
		(1) **(8 marks)**

(a)

B1: States or uses $\frac{\mathrm{d}r}{\mathrm{d}t} \propto \pm\frac{1}{r^2}$, $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{k}{r^2}$ or $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{kr^2}$ Accept $k \leftrightarrow \lambda$ or other constant. Condone $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{1}{r^2}$

M1: Integrates to achieve an expression of the correct form $...r^3 = ...t(+c)$ o.e

There is no requirement for + c

A1: Correct integration with two different unknown constants. E.g. look for $\frac{1}{3}r^3 = \pm kt + c$ o.e

So if for example $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{1}{r^2} \Rightarrow \frac{1}{3}r^3 = -t + c$ the score would be B1, M1, A0 then dM0, A0

M1: Uses both boundary conditions to form an equation involving r and t. For this to be awarded there must be two different unknown constants that are initially correctly placed.

It is dependent upon having achieved $...r^n = \pm"k"t + "c"$ which may have been achieved from an

incorrect assumption. E.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \pm kr^2$

A1: Correct equation. E.g. $\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e. such as

$t = -\frac{5}{504}r^3 + \frac{120}{7}$

(b)

M1: Sets $r = 0 \Rightarrow -100.8t + 1728 = 0 \Rightarrow t = ...$ Follow through on their equation. Condone if this produces a negative value for t. Alt sets $V = 0$ to find $t = ...$

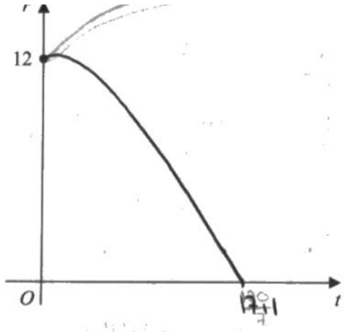
A1: Awrt 17.1 minutes. Must include the units. Also allow 17 minutes 8 seconds or 17 minutes 9 seconds

(c)

B1: For the correct shaped curve only in quadrant 1 starting at (0,12), ignoring value of t.

As $\frac{dr}{dt} = -\frac{k}{r^2}$ the gradient should appear to get increasingly steeper.

If there are two curves given on the axes, both need to be correct for this mark to be awarded
Do not be concerned if it is not infinite at the t axis.



The curve on the left would be at the limit of what we would allow
as the gradient on the left hand side does appear to increase.
We condone its appearance as it approaches the t- axis.
It does not get less steep

....
..

Note that it is possible to use $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$ with $\frac{dr}{dt} = \pm\frac{k}{r^2}$ to set up and solve

$\frac{dV}{dt} = \pm 4\pi k$ or $\frac{dV}{dt} = \pm\beta$ In this case the equation becomes $V = 2304\pi - \left(\frac{672\pi}{5}\right)t$

(a)

B1: States or uses either $\frac{dV}{dt} = \pm\beta$ or its exact value which is $\frac{dV}{dt} = -\frac{2016\pi}{15}$ o.e such as $\frac{dV}{dt} = -\frac{672\pi}{5}$.

M1: For $\frac{dV}{dt} = \pm\beta \Rightarrow V = \pm\beta t + c$ and attempts to find the values of β and c using the given conditions.

It is dependent upon knowing that $V = \alpha r^3$ and using this to find V at r = 6 and 12 with t = 15 and t = 0

A1: $V = 2304\pi - \left(\frac{672\pi}{5}\right)t$

M1: Substitutes $V = \alpha r^3$ to form an equation linking r^3 with t.
It is dependent on the previous M1 in this method

A1: Achieves $\frac{4}{3}\pi r^3 = 2304\pi - \left(\frac{672\pi}{5}\right)t$

(b)

M1: Sets $V = 0$ o.e and finds t

Question Number	Scheme	Marks
11 (a)	$(x+y)^3+10y^2=108x$	
	Differentiates $10y^2$ to $20y\frac{dy}{dx}$	B1
	Differentiates $(x+y)^3$ to $3(x+y)^2\left(1+\frac{dy}{dx}\right)$	M1
	$3(x+y)^2\left(1+\frac{dy}{dx}\right)+20y\frac{dy}{dx}=108$	A1
	$3(x+y)^2\frac{dy}{dx}+20y\frac{dy}{dx}=108-3(x+y)^2$	dM1
	$\frac{dy}{dx}=\frac{108-3(x+y)^2}{20y+3(x+y)^2}$ *	A1*
		(5)
(b)	Deduces that P and Q are where $108-3(x+y)^2=0\Rightarrow(x+y)^2=36$	M1
	Attempts to substitute $(x+y)=\pm6$ into $(x+y)^3+10y^2=108x$ to form an equation in y (or x)	dM1
	Solves $216+10y^2=108(6-y)$ and finds the negative root	ddM1
	Awrt 13900 metres	A1
		(4)
		(9 marks)

(a) Allow use of y' for $\frac{dy}{dx}$. This is a proof. Look carefully for evidence of candidates working backwards.

B1: Differentiates $10y^2$ to $20y\frac{dy}{dx}$

M1: Attempts to differentiate $(x+y)^3$ Via the chain rule look for $k(x+y)^2\left(1+\frac{dy}{dx}\right)$ See **

Alternatively you will see attempts where $(x+y)^3$ is multiplied out. Terms may not be collected but

look for 3 of $x^3\to ...x^2$, $...x^2y\to 2xy+x^2\frac{dy}{dx}$, $...xy^2\to y^2+2xy\frac{dy}{dx}$ and $y^3\to 3y^2\frac{dy}{dx}$

differentiated to the correct form.

A1: Correct differentiation $3(x+y)^2\left(1+\frac{dy}{dx}\right)+20y\frac{dy}{dx}=108$

$$\text{or } 3x^2+3\left(2xy+x^2\frac{dy}{dx}\right)+3\left(y^2+2xy\frac{dy}{dx}\right)+3y^2\frac{dy}{dx}+20y\frac{dy}{dx}=108$$

dM1: Collects the terms in $\frac{dy}{dx}$.

Look for either $\frac{dy}{dx}$ being factorised out of the relevant terms

or the terms in $\frac{dy}{dx}$ being set on one side of the equation and the other terms on the other side

It is dependent upon **BOTH t**he B and M marks.

A1*: Proceeds to the given answer.

In the approach $3x^2+3\left(2xy+x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)+3\left(y^2+2xy\frac{\mathrm{d}y}{\mathrm{d}x}\right)+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}+20y\frac{\mathrm{d}y}{\mathrm{d}x}=108$ it is acceptable

to go from $\left(3x^2+6xy+3y^2+20y\right)\frac{\mathrm{d}y}{\mathrm{d}x}=108-3x^2-6xy-3y^2$ straight to the given answer

....

Watch for candidates who use the given answer and work backwards **.

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{108-3(x+y)^2}{20y+3(x+y)^2}\Rightarrow 3(x+y)^2\frac{\mathrm{d}y}{\mathrm{d}x}+20y\frac{\mathrm{d}y}{\mathrm{d}x}=108-3(x+y)^2$$

Therefore candidates who start with

$3(x+y)^2\frac{\mathrm{d}y}{\mathrm{d}x}+20y\frac{\mathrm{d}y}{\mathrm{d}x}=108-3(x+y)^2$ or indeed $3(x+y)^2\frac{\mathrm{d}y}{\mathrm{d}x}+3(x+y)^2+20y\frac{\mathrm{d}y}{\mathrm{d}x}=108$

without any other working or evidence can only score B1 M0 A0 M0 A0

....

(b)

M1: Deduces that P and Q are where $108-3(x+y)^2=0\Rightarrow(x+y)^2=36$

This is implied if there is a statement that $(x+y)=\pm 6$

dM1: Attempts to solve $(x+y)=\pm 6$ and $(x+y)^3+10y^2=108x$ simultaneously to form a quadratic equation in y or x.

ddM1: Solves $216+10y^2=108(6-y)$ and finds the negative root. Allow if both roots are found

Note that $(x+y)=6$ must have be used.

If an equation was set up in x, then look for an attempt to find the larger root of $216+10(6-x)^2=108x$ which should then be used to find a negative y value

A1: Awrt 13900 metres or awrt 13.9 km with the correct units. Condone answers like -13.9 km

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