



Mark Scheme (Results)

Summer 2026

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01A

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at:

www.pearson.com/uk

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN:

- bod – benefit of doubt
 - ft – follow through
 - the symbol $\checkmark$ will be used for correct ft
 - cao – correct answer only
 - cso – correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC – special case
 - oe – or equivalent (and appropriate)
 - d... or dep – dependent
 - indep – independent
 - dp – decimal places
 - sf – significant figures
 - * – The answer is printed on the paper or ag- answer given
 - $\square$ or d... – The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a

misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:
 - a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(NB specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a, b and c)

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$ leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1 ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1 ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$(8+32x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+4x)^{\frac{1}{3}}$	B1
	$(1+4x)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)(4x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(4x)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(4x)^3 + \dots$	M1A1
	$(8+32x)^{\frac{1}{3}} = 2 + \frac{8}{3}x - \frac{32}{9}x^2 + \frac{640}{81}x^3 + \dots$	A1
		(4)
		Total 4

Notes

B1: For taking out a factor of $8^{\frac{1}{3}}$ or e.g. 2 to obtain e.g. $8^{\frac{1}{3}}(1+\dots)$. Seen anywhere in their work

M1: Correct attempt at the 3rd or 4th term of a binomial expansion of $(1 \pm ax)^{\frac{1}{3}}$

Look for $\frac{1\left(\frac{1}{3}-1\right)}{2}(\pm ax)^2$ or $\frac{1\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(\pm ax)^3$ where a could be 1

For this mark we are looking for the correct binomial coefficient paired with the correct power of x

A1: Correct unsimplified (or simplified) expansion of the **bracket** e.g.:

$$(1+4x)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)(4x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(4x)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(4x)^3 + \dots$$

$$\text{NB simplified is: } 1 + \frac{4}{3}x - \frac{16}{9}x^2 + \frac{320}{81}x^3 + \dots$$

or for 2 correct simplified terms for the final expansion from $\frac{8}{3}x - \frac{32}{9}x^2 + \frac{640}{81}x^3$

Alcao: $2 + \frac{8}{3}x - \frac{32}{9}x^2 + \frac{640}{81}x^3 + \dots$

All coefficients must be simplified, accept as a list and ignore any extra terms

Alternative by direct expansion:

$$(8+32x)^{\frac{1}{3}} = 8^{\frac{1}{3}} + \frac{1\left(-\frac{2}{3}\right)}{3}\left(8^{\frac{2}{3}}\right)(32x) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{2}\left(8^{\frac{5}{3}}\right)(32x)^2 + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)}{6}\left(8^{\frac{8}{3}}\right)(32x)^3 + \dots$$

B1: For $8^{\frac{1}{3}}$ or 2 seen as the constant term.

M1: For the correct term 3 or term 4 of the expansion.

A1: For two correct and simplified terms.

A1: Correct expansion from correct work. Accept as a list and ignore any extra terms.

NOTE: Isw any further 'simplification' once the correct response has been seen.

Question Number	Scheme	Marks
2(a)	$\frac{2x^2 - x + 11}{(x+2)(2x-3)} = 1 + \frac{-2x+17}{(x+2)(2x-3)}$	M1A1
	$\frac{-2x+17}{(x+2)(2x-3)} = \frac{B}{x+2} + \frac{C}{2x-3} \Rightarrow B = \dots, C = \dots$	M1
	$\frac{2x^2 - x + 11}{(x+2)(2x-3)} = 1 - \frac{3}{x+2} + \frac{4}{2x-3}$	A1
	ALT 1	
	$2x^2 - x + 11 = A(x+2)(2x-3) + B(2x-3) + C(x+2)$ Let e.g., $x = -2$ or $x = \frac{3}{2}$ $\Rightarrow$ eliminates either B or $C \Rightarrow B = -3$ or $C = 4$	M1A1
	$2x^2 - x + 11 = A(x+2)(2x-3) + B(2x-3) + C(x+2)$ Let e.g., $x = -2$ and $x = \frac{3}{2} \Rightarrow$ finds B, C and A	M1
	$\frac{2x^2 - x + 11}{(x+2)(2x-3)} = 1 - \frac{3}{x+2} + \frac{4}{2x-3}$	A1
	ALT 2	
	$2x^2 - x + 11 = 2Ax^2 + x(A+2B+C) + (-6A-3B+2C)$ $\Rightarrow$ equates coefficients to find $A \Rightarrow 2 = 2A \Rightarrow A = 1$	M1A1
	$A+2B+C = -1 \Rightarrow 1+2B+C = -1$ $-6A-3B+2C = 11 \Rightarrow -6-3B+2C = 11$ $\Rightarrow B = \dots, C = \dots$	M1
	$\frac{2x^2 - x + 11}{(x+2)(2x-3)} = 1 - \frac{3}{x+2} + \frac{4}{2x-3}$	A1
		(4)
(b)	$\int \left(1 - \frac{3}{x+2} + \frac{4}{2x-3} \right) dx = x - 3 \ln x+2 + 2 \ln 2x-3 (+c)$	M1A1ftA1ft
		(3)
		Total 7

Notes

(a)

M1: Uses e.g. long division or inspection to express $f(x)$ as a constant with a linear remainder of the

form $1 + \frac{\pm \alpha x \pm \beta}{(x+2)(2x-3)}$ where α, β are non zero constants.

A1: Correct expression. i.e., $1 + \frac{-2x+17}{(x+2)(2x-3)} \Rightarrow A=1$

M1: Uses an appropriate method to both B and C from an expression of the form $\frac{\alpha x + \beta}{(x+2)(2x-3)}$. E.g.

substituting values or comparing coefficients to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1, B = -3, C = 4$

ALT 1

M1: Multiplies through by the denominator to obtain the shown equation, substitutes an appropriate value of x to find a value for B or C Allow sign errors for this mark.

Some candidates are deducing that $A = 1$ in this method. Award this mark for the equation and for B or C .

A1: Either B or C correct. **following fully correct processing**

M1: For **complete** processing to find both A and either B or C to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1, B = -3, C = 4$ o.e

ALT 2

M1: Multiplies through by the denominator to obtain the shown equation [can be deduced], multiplies out the RHS,

equates coefficients and finds a value for A . Allow sign errors for this mark.

A1: A correct **following fully correct processing**.

M1: For **complete** processing to find both B and C to result in a three term expression.

A1: Fully correct expression. .Accept either embedded or listed values. E.g., $A = 1, B = -3, C = 4$ o.e

isw further simplification

(b)

M1: $\int \frac{\dots}{x+2} dx = \dots \ln(x+2)$ or $\int \frac{\dots}{2x-3} dx = \dots \ln(2x-3)$ Allow modulus or round brackets.

Do not allow errors on $(x+2)$ or $(2x-3)$

A1ft: For any 2 terms correctly integrated. Follow through their stated A, B and C .

The brackets must be present but allow e.g. $-3 \ln(x+2)$ or e.g. $2 \ln(2x-3)$

A1ft: All **3** of their terms fully simplified. Follow through their stated A, B and C . The “+ c ” is not required. If their $A = 0$ this mark is not awarded.

Allow $x - 3 \ln(x+2) + 2 \ln(2x-3)(+c)$

isw further simplification

Question Number	Scheme	Marks
3	$V = x^3 \Rightarrow \left(\frac{dV}{dx}\right) = 3x^2$	B1
	$V = 100 \Rightarrow x^3 = 100 \Rightarrow x = \sqrt[3]{100}$	B1
	$k = \frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{1}{3 \times 100^{\frac{2}{3}}} \times 0.085$	M1
	$= 0.00132$	A1
		(4)
		Total 4

Notes

$$\left[\text{Note: } 0.085 = \frac{17}{200} \right]$$

Allow other variables in place of x but not t .

B1: $\left(\frac{dV}{dx}\right) = 3x^2$ seen anywhere in the working

B1: M1 in Epen Finds x when $V = 100 \Rightarrow x = \sqrt[3]{100}$ or awrt 4.64

M1: Applies $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ with their $\frac{dV}{dx} = 3x^2$ provided it is of the form $\frac{dV}{dx} = \mu x^2$ and their numerical value for x substituted correctly

This must lead to $\frac{dx}{dt} = \dots$

Do not award this mark for $x = 100$ substituted here.

A1: Awrt 0.00132 (oe e.g. awrt 1.32×10^{-3}) Ignore units given.

Question Number	Scheme	Marks
4(a)	$x = \frac{3}{5t-2} \Rightarrow 5tx - 2x = 3 \Rightarrow t = \frac{2x+3}{5x} \Rightarrow y = \frac{\frac{2x+3}{5x}}{4 - \frac{2x+3}{5x}} \text{ o.e.}$	M1A1
	$y = \frac{2x+3}{18x-3} \text{ oe}$	A1
		(3)
(b)	Assume that C and l do intersect	B1
	At intersection $\frac{2x+3}{18x-3} = -x+1 \Rightarrow 2x+3 = -18x^2 + 21x - 3 \Rightarrow 18x^2 - 19x + 6 (=0)$ or $y = -x+1 \Rightarrow \frac{t}{4-t} = \frac{3}{2-5t} + 1 \Rightarrow 2t - 5t^2 = 3(4-t) + (2-5t)(4-t)$ $\Rightarrow 10t^2 - 27t + 20 (=0)$	M1
	$b^2 - 4ac = 19^2 - 4 \times 18 \times 6 (= -71)$ or $b^2 - 4ac = 27^2 - 4 \times 10 \times 20 (= -71)$	M1
	$b^2 - 4ac < 0$ so there are no real (roots) So C and l do not intersect	A1
		(4)
		Total 7

Notes

(a)

M1: Finds t in terms of x **and** substitutes into y

A1: Any correct equation connecting y and x which need not be simplified for this mark.

A1: Correct equation in the required form with no errors. Accept equivalent correct expressions.

(b)

B1: Sets up a correct assumption statement. This step **must** be seen.

M1: Sets their $\frac{2x+3}{18x-3} = -x+1$ and multiplies up to obtain a 3 term quadratic in x

or

Uses the parametric form and sets $\frac{t}{4-t} = -\left(\frac{3}{5t-2}\right) + 1$ oe and multiplies up to obtain a 3 term quadratic in

t

M1: Any correct strategy for establishing the contradiction e.g. attempts the discriminant on their 3TQ.
This can be part of the quadratic formula.

Complex roots are: $\frac{19 \pm \sqrt{71}i}{36}$

A1: Fully correct work including -71 seen evaluated, **and** conclusion stated that **roots are not (real), so C and l do not intersect**. Mark the intention to write these statements and mark positively.

This is a formal proof. Only award the final A mark if B1M1M1 has been scored.

Question Number	Scheme	Marks
5	$u = \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 \Rightarrow \left(\frac{du}{dx} = \frac{1}{2u} \times 2 \right) \Rightarrow (dx = u du)$ or e.g. $u^2 = 2x-1 \Rightarrow 2u \frac{du}{dx} = 2 \Rightarrow (dx = u du)$ or e.g. $u = \sqrt{2x-1} \Rightarrow x = \frac{u^2+1}{2} \Rightarrow \frac{dx}{du} = u \Rightarrow (dx = u du)$	B1
	$\int \frac{x}{\sqrt{2x-1}} dx = \int \frac{1}{2}(u^2+1) \frac{1}{u} \times u du$	M1
	$\int \frac{1}{2}(u^2+1) \frac{1}{u} \times u du = \int \frac{1}{2}(u^2+1) du = \frac{u^3}{6} + \frac{u}{2} (+k)$	M1
	$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{(2x-1)^{\frac{1}{2}}}{2} (+k)$	ddM1
	$= \frac{1}{3}(2x-1)^{\frac{1}{2}}(x+1) + k$	A1
		(5)
		Total 5

Notes

B1: Any correct equation involving $\frac{du}{dx}$ or $\frac{dx}{du}$ See the alternatives above.

M1: Makes a complete substitution to obtain an integral in terms of u including their dx in terms of $u..du$

M1: Integrates to the form $...u^3 + ...u(+k)$

ddM1: Reverses the substitution **correctly** into their integrated expression in the form $au^3 + bu + k$ where a and b are rational numbers. The minimum form required is $= \alpha(2x-1)^{\frac{3}{2}} + \beta(2x-1)^{\frac{1}{2}}(+k)$

Note: This mark is dependent on **both** previous M marks.

A1: Correct answer in the required form including the “+ k ” allow + C for this mark.

Question Number	Scheme	Marks
6(a)	$x^2 - xy + y^2 + 4x = 12$ $\Rightarrow 2x - x \frac{dy}{dx} - y + 2y \frac{dy}{dx} + 4 = 0$	M1A1
	$\frac{dy}{dx}(2y - x) = y - 2x - 4 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\Rightarrow \left(\frac{dy}{dx} \right) = \frac{y - 2x - 4}{2y - x} \text{ oe}$	A1
		(4)
(b)	$(-4, -6) \rightarrow \frac{dy}{dx} = \frac{-6 + 8 - 4}{-12 + 4} = \frac{1}{4} \Rightarrow m_N = -4$	M1
	$y + 6 = -4(x + 4)$	M1
	$y = -4x - 22$	A1
		(3)
(c)	$y = -4x - 22 \Rightarrow 21x^2 + 202x + 472 = 0$ or $x = \frac{y + 22}{-4} \Rightarrow 21y^2 + 116y - 60 = 0$	M1
	$21x^2 + 202x + 472 = 0 \Rightarrow x = -\frac{118}{21}, (-4)$ or $21y^2 + 116y - 60 = 0 \Rightarrow y = \frac{10}{21}, (-6)$	dM1
	$x = -\frac{118}{21}, \text{ o.e.e. } y = \frac{10}{21} \text{ o.e.e.}$	A1
		(3)
		Total 10

(a)

M1: For $xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$ **or** $y^2 \rightarrow \alpha y \frac{dy}{dx}$

A1: Fully correct differentiation on the whole equation unsimplified or simplified.

M1: Makes $\frac{dy}{dx}$ the subject from **2** different terms in $\frac{dy}{dx}$ coming from the correct terms only

A1: Any correct expression

(b)

M1: Correct method for the gradient of the normal using their numerical value for $\frac{dy}{dx}$

M1: Correct method for finding the equation of the normal. Ft their normal gradient which must not be their tangent gradient.

A1: Correct equation in the required form

(c)

M1: Substitutes for x or y into the given equation to obtain a 3TQ in x or y .

dM1: Solves 3TQ using any method [including using a calculator] to obtain either the x or y coordinate of Q
This mark is dependent on the first M mark awarded.

A1: Both correct **exact** coordinates. Accept values given separately. Ignore any reference to $(-4, -6)$

Question Number	Scheme	Marks
7(a)	$\pm(5\mathbf{i} - 3\mathbf{j} + \mathbf{k} - (3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}))$	M1
	e.g. $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	A1
		(2)
(b)	e.g. $1 - 2\lambda = -5 \Rightarrow \lambda = 3$ $\lambda = 3 \Rightarrow \mathbf{r} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k} + 3(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	M1
	$\Rightarrow \alpha = 11, \beta = -6$	A1
		(2)
(c)	$\overrightarrow{OC} \cdot \overrightarrow{AB} = (11\mathbf{i} - 6\mathbf{j} - 5\mathbf{k}) \cdot (-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) = -22 - 6 - 10 = (-38)$	M1
	$-38 = \sqrt{11^2 + 6^2 + 5^2} \sqrt{2^2 + 1^2 + 2^2} \cos \theta \Rightarrow \theta = \dots$	dM1
	$\theta = 20.1^\circ$	A1
		(3)
(d)	e.g., $\overrightarrow{OD} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 2\lambda + 5 \\ -\lambda - 3 \\ -2\lambda + 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \Rightarrow \lambda = \dots \left(-\frac{11}{9}\right)$ or e.g. $\overrightarrow{OD} \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 3 - 2\lambda \\ -2 + \lambda \\ 3 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \lambda = \dots \left(\frac{2}{9}\right)$	M1
	e.g. $\lambda = -\frac{11}{9} \Rightarrow \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix} - \frac{11}{9} \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ or e.g. $\lambda = \frac{2}{9} \Rightarrow \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + \frac{2}{9} \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	M1

	$\left(\frac{23}{9}, -\frac{16}{9}, \frac{31}{9}\right)$	A1
		(3)
		Total 10

Notes

(a)

M1: Attempts the direction of l

Two out of three components correct in the direction implies the M mark.

A1: Any correct equation including ' $\mathbf{r} =$ ' Do not accept $\mathbf{l} = \dots$

Accept column vectors throughout, e.g., $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ but do **NOT** accept $\mathbf{r} = \begin{pmatrix} 3\mathbf{i} \\ -2\mathbf{j} \\ 3\mathbf{k} \end{pmatrix} + \lambda \begin{pmatrix} -2\mathbf{i} \\ 1\mathbf{j} \\ 2\mathbf{k} \end{pmatrix}$

(b)

M1: Uses the z component to find the value of the parameter for their equation and uses their parameter value to find the value of α and the value of β .

A1: Correct values or vector.

(c)

M1: Attempts the scalar product $\pm \vec{OC} \bullet \vec{AB}$ Ft their $\vec{OC}$ and $\vec{AB}$

Look for at least two correct products can be implied by the correct answer.

dM1: A complete method to find the angle (allow acute or obtuse, 20.13° or 159.87° respectively allow working in radians for the M marks)

A1: Awrt 20.1°

(d)

M1: Attempts the scalar product between their $\vec{OD}$ and the direction of l , sets $= 0$ and solves for λ for their equation.

M1: Uses their λ correctly to find D

A1: Correct exact coordinates (condone vector)

Do **NOT** accept the final answer in the form $\begin{pmatrix} \frac{23}{9}\mathbf{i} \\ -\frac{16}{9}\mathbf{j} \\ \frac{31}{9}\mathbf{k} \end{pmatrix}$

Question Number	Scheme	Marks
8(a)	$\int x \cos 2x \, dx = \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$	M1A1
	$= \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$	A1
		(3)
(b)	$V = (\pi) \int (4x + 3 \cos 2x)^2 \, dx$ $V = (\pi) \int (16x^2 + 24x \cos 2x + 9 \cos^2 2x) \, dx$	M1
	$\int 24x \cos 2x \, dx = 12x \sin 2x + 6 \cos 2x (+c)$	M1
	$\int 9 \cos^2 2x \, dx = \frac{9}{2} \int (1 + \cos 4x) \, dx$	M1
	$\left[(\pi) \int (16x^2 + 24x \cos 2x + 9 \cos^2 2x) \, dx \right]$ $= (\pi) \left(\frac{16x^3}{3} + 12x \sin 2x + 6 \cos 2x + \frac{9x}{2} + \frac{9}{8} \sin 4x (+c) \right)$	A1
	$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (4x + 3 \cos 2x)^2 \, dx = \pi \left(\frac{16\pi^3}{24} + 0 - 6 + \frac{9\pi}{4} + 0 - \left(\frac{16\pi^3}{192} + 3\pi + 0 + \frac{9\pi}{8} + 0 \right) \right)$	M1
	$\pi \left(\frac{7\pi^3}{12} - \frac{15\pi}{8} - 6 \right)$	A1
		(6)
		Total 9

Notes

(a)

M1: Applies integration by parts to obtain the form $\dots x \sin 2x \pm \dots \int \sin 2x \, dx$

A1: Correct expression

A1: Completes the integration to obtain the correct answer including $+ c$

If the “DI” method is used:

x	$\cos 2x$
1	$\frac{1}{2} \sin 2x$
0	$-\frac{1}{4} \cos 2x$

Award M1 for reaching the correct form of the answer e.g. $\dots x \sin 2x + \dots \cos 2x$ with A1 implied by their table and A1 as above.

(b)

Condone the missing π from $\pi \int \dots$ for the 4 marks.

M1: Attempts $\int (4x + 3 \cos 2x)^2 dx$ and multiplies out the bracket with 2 out of 3 terms correct.

M1: Uses part (a) to attempt their $\int 24x \cos 2x dx$ provided part (a) is in the form $\dots x \sin 2x \pm \dots \cos 2x$

An attempt is defined as using their part (a) correctly here with their '24'

M1: Attempts $\int \cos^2 2x dx$ using the correct identity $\cos 4x = 2 \cos^2 2x - 1$ and reaches

$$\int \cos^2 2x dx = \dots x + \dots \sin 4x (+c)$$

A1: Fully correct integration in any form.

M1: Complete method to find the volume with π and the given limits [accept substituted either way around] using their integral which must include at least two trigonometrical terms.

A1: Correct volume in the required form.

Question Number	Scheme	Marks
9(a)	$\frac{d\theta}{dt} = -k(\theta + 15) \Rightarrow \int \frac{d\theta}{\theta + 15} = \int -k dt \quad \text{OR} \quad \frac{d\theta}{d\theta} = \frac{1}{-k(\theta + 15)}$ $\Rightarrow \ln(\theta + 15) = -kt(+c) \quad \text{o.e.} \quad \Rightarrow t = -\frac{1}{k} \ln(\theta + 15) + c$	M1A1
	$t = 0, \theta = 21 \Rightarrow c = \dots\dots (\text{e.g., } c = \ln 36)$	M1
	$\ln(\theta + 15) = -kt + \ln 36 \quad \text{OR} \quad \ln(\theta + 15) = -kt + \ln 36$ $\Rightarrow \theta + 15 = e^{-kt + \ln 36} \quad \Rightarrow [\ln(\theta + 15) - \ln 36 = -kt]$ $\Rightarrow \ln\left(\frac{\theta + 15}{36}\right) = -kt \Rightarrow \theta = 36e^{-kt} - 15$	M1
	$\theta = 36e^{-kt} - 15$	A1
		(5)
(b)	$12.9 = 36e^{-20k} - 15 \Rightarrow e^{-20k} = \frac{31}{40} \Rightarrow -20k = \ln \frac{31}{40} \Rightarrow k = \dots$	M1
	$k = 0.0127$	A1
		(2)
(c)	$t = 60 \Rightarrow \theta = '36' e^{-60 \times '0.0127'} - '15'$	M1
	$= 1.8 (^{\circ} \text{C})$	A1
		(2)
		Total 9

Notes

(a)

M1: Separates the variables **and** integrates to obtain $\dots \ln(\theta + 15) = \dots kt(+c)$ (or equivalent) with or without a constant of integration.

A1: Correct equation with or without a constant of integration.

If they do not have a constant of integration the following 3 marks are not available

M1: States or uses $t = 0$ and $\theta = 21$ to find a value for “c”. Do not be concerned by the rearrangement as long as a value of c is found.

M1: Rearranges using correct application of log processing to make θ the subject

A1: Correct equation

(b)

M1: Sets their $\theta = 12.9$, substitutes $t = 20$ into their expression which is minimally of the form $\theta = \alpha e^{-kt} \pm \beta$ where α can equal 1 and rearranges **using correct explicit log work** to find k .

For the award of this mark their ' $\frac{31}{40}$ ', must be a positive number.

A1: Awrt 0.0127

(c)

M1: Substitutes $t = 60$ and their k into their $\theta = 36e^{-kt} - 15$ which must be minimally of the form $\theta = \alpha e^{-kt} - \beta$

Unless the final answer is correct coming from a correct equation, substitution of 60 must be explicit.

A1: Awrt 1.8(°C) A correct answer coming a correct expression implies A1
Units are not required, but if the units are incorrect score A0.

Question Number	Scheme	Marks
10(a)	$\theta = \frac{\pi}{3} \Rightarrow x = 8 \cos \frac{\pi}{3} = 4, y = 5 \sin \frac{2\pi}{3} = \frac{5\sqrt{3}}{2}$	B1
	$x = 8 \cos \theta \Rightarrow \frac{dx}{d\theta} = -8 \sin \theta \quad y = 5 \sin 2\theta \Rightarrow \frac{dy}{d\theta} = 10 \cos 2\theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{10 \cos 2\theta}{-8 \sin \theta} = \frac{10 \cos \frac{2\pi}{3}}{-8 \sin \frac{\pi}{3}} = \dots \left(\frac{5\sqrt{3}}{12} \right)$	M1
	$y - \frac{5\sqrt{3}}{2} = -\frac{4\sqrt{3}}{5}(x - 4)$	M1
	$y = -\frac{4\sqrt{3}}{5}x + \frac{57\sqrt{3}}{10}$	A1
		(4)
(b)	$\int y \, dx = \int y \frac{dx}{d\theta} (d\theta) = \int 5 \sin 2\theta \times -8 \sin \theta (d\theta)$	M1
	$\int y \, dx = \int y \frac{dx}{d\theta} (d\theta) = \int 5(2 \sin \theta \cos \theta) \times -8 \sin \theta (d\theta) = \pm 80 \int \sin^2 \theta \cos \theta (d\theta)$	M1
	$= \left(-\frac{80}{3} \right) \sin^3 \theta + c$	M1
	$0 = -\frac{4\sqrt{3}}{5}x + \frac{57\sqrt{3}}{10} \Rightarrow x = \dots \frac{57}{8} \text{ and } A_{\Delta} = \frac{1}{2} \left(\frac{57}{8} - 4 \right) \times \frac{5\sqrt{3}}{2} = \left(\frac{125\sqrt{3}}{32} \right)$	M1
	$\pm \frac{80}{3} \left[\sin^3 \theta \right]_{\frac{\pi}{2}}^{\frac{\pi}{3}} \text{ or } \pm \frac{80}{3} \left[\sin^3 \theta \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{80}{3} \left(1 - \frac{3\sqrt{3}}{8} \right)$	M1
	Total area = $\frac{80}{3} - \frac{195\sqrt{3}}{32}$	A1
		(6)
		Total 10

Notes

NB: If you see attempts to answer this question using a Cartesian equation please send to Review.

(a)

B1: Correct coordinates for P stated or implied by working.

M1: Attempts $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ at $\theta = \frac{\pi}{3}$

Minimally required differentiation is: $\cos \theta \Rightarrow \beta \sin \theta \quad \sin 2\theta \Rightarrow \alpha \cos 2\theta \quad \frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$

M1: Attempts to find the equation of the normal at $\theta = \frac{\pi}{3}$

A1: Correct equation in the required form.

(b)

Allow sign slips in this part for the M marks.

M1: Attempts $\int y \frac{dx}{d\theta} d\theta$ with the given y and their

and obtains $\frac{dx}{d\theta}$ which must be as a minimum $\pm 8 \sin \theta \Rightarrow \int 5 \sin 2\theta \times \pm 8 \sin \theta' (d\theta)$

M1: **This is an A mark in Epen** Uses $\sin 2\theta = 2 \sin \theta \cos \theta$ to obtain $\pm 80 \int \sin^2 \theta \cos \theta (d\theta)$ oe

Ignore limits for this mark.

M1: Integrates to the form $\alpha \sin^3 \theta (+c)$

M1: Attempts the x intercept of l and finds the area of the triangle using their equation of the line. You must check this. If their line provided the intercept on the x-axis > 4
This may be completed by integration

M1: Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{3}$ to evaluate the integral. Accept substitution of limits either way around
for this mark, but must include subtraction..

A1: Correct area of the required region R in the required form.

[Allow recovery from a negative integral].

Score this mark ONLY if all previous marks in part (b) have been awarded.