

OXFORD

INTERNATIONAL
AQA EXAMINATIONS

INTERNATIONAL A-LEVEL MATHEMATICS

MA04

(9660/MA04) Unit S2 Statistics

Mark scheme

January 2022

Version: 1.0 Final



2 2 1 X M A 0 4 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from oxfordaqaexams.org.uk

Copyright information

OxfordAQA retains the copyright on all its publications. However, registered schools/colleges for OxfordAQA are permitted to copy material from this booklet for their own internal use, with the following important exception: OxfordAQA cannot give permission to schools/colleges to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Copyright © 2022 Oxford International AQA Examinations and its licensors. All rights reserved.

Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
-x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)

Q	Answer	Marks	Comments
1 (a)	$H_0 : \mu = 27$ $H_1 : \mu \neq 27$ $\bar{X} \sim N\left(27, \frac{9.2^2}{50}\right)$ $z = \frac{25.3 - 27}{\left(\frac{9.2}{\sqrt{50}}\right)}$ $z = -1.31$ $z_{\text{critical}} = -1.96$ Do not reject H_0 as $z_{\text{critical}} < z$ or $-1.96 < -1.31$ or $ z < 1.96$ Evidence to support the claim that the mean age of subscribers to the course is 27 [at the 5% level of significance]	B1 B1 M1 A1 B1 A1ft E1	Accept 27 as 27.0 throughout Must be seen $z = \frac{25.3 - 27}{\text{their } \left(\frac{S}{\sqrt{n}}\right)}$ for $n \neq 1$ PI AWRT or exact value $-\frac{85\sqrt{2}}{92}$ AWRT or $P(z < -1.31) = 0.0951$ to 0.0957 or comparison of probability to 2.5% Allow 'accept H_0 ' Comment about H_0 and 0.0951 to $0.0957 > 0.025$ Correct conclusion based upon their z Correct statement based on their comparison and given in context Must have been awarded at least M1 A0 A1ft
		7	

Q	Answer	Marks	Comments
1(b)	The central limit theorem states that when the sample size is large enough [e.g. $n \geq 30$], the sample mean will be (approximately) normally distributed	B2	B1 for mention of the CLT B1 for comment on the size of the sample
		2	

	Question 1 Total	9	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
2(a)(i)	$[= 3^2 =]$ 9	B1	
		1	

Q	Answer	Marks	Comments
2(a)(ii)	$P(X < 8) = P(X \leq 7)$ 0.324	M1 A1	PI SC1 for $ = 3 \Rightarrow 0.988$ Condone 0.3239 (from use of tables)
		2	

Q	Answer	Marks	Comments
2(a)(iii)	$E(X) = 9$ $[P(X = 9) =] \frac{e^{-9} \times 9^9}{9!}$ 0.132	B1 M1 A1	PI Allow ft with $ = 3$ Using tables: $= 0.5874 - 0.4557$ SC2 for $ = 3 \Rightarrow 0.224$ Condone 0.1317 (from use of tables)
		3	

Q	Answer	Marks	Comments
2(b)(i)	Exponential [$\lambda =$] 9	B1	$T \sim \text{Exp}(9)$ is B2
		B1	PI by subsequent working
		2	

Q	Answer	Marks	Comments
2(b)(ii)	Mean = $\frac{1}{9}$	B1	SC1 only for $\lambda = 3$ in part (b)(i) leading to a mean of $\frac{1}{3}$
		1	

Q	Answer	Marks	Comments
2(b)(iii)	$P(T > 0.5) = 1 - F(0.5)$ $= 1 - (1 - e^{-9 \cdot 0.5})$ 0.0111	M1	oe PI
		m1	Attempts to find correct probability using cdf of exponential or integration of pdf
		A1	AWRT NMS 3/3 SC2 for 0.2231 from $\lambda = 3$
		3	

Q	Answer	Marks	Comments
2(b)(iv)	$1 - e^{-9t} = 0.9$ $t = 0.256$ [hours]	M1	Forms equation using a cdf of exponential or integration of pdf with their λ
		A1	SC1 for 0.768 from $\lambda = 3$
		2	

	Question 2 Total	14	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
3(a)	Correctly identifying Stage 2 and Stage 5	B1	oe, such as critical region should not include 9
	Stage 2 $X \sim B(14, 0.4)$	B1	
	Stage 5 [critical region is] $\{0, 1, 2, 10, 11, 12, 13, 14\}$	B1	
		3	

Q	Answer	Marks	Comments
3(b)	As 8 is not in the critical region	B1	Comment about the test statistic and critical region (allow a comparison of probabilities)
	Do not reject H_0 as we have evidence to suggest the germination rate is 40% for a cabbage seed [at the 10% level of significance]	B1	General conclusion Allow accept H_0
		2	

Q	Answer	Marks	Comments
3(c)(i)	$[0.0398 + 0.0175 =] 0.0573$	B1	
		1	

Q	Answer	Marks	Comments
3(c)(ii)	Accepting that the germination rate is not 0.4 when it is.	B1	oe
		1	

Q	Answer	Marks	Comments
3(d)(i)	The acceptance region may decrease [or may not change if k is close to 10]	E1	oe Condone a definitive statement
		1	

Q	Answer	Marks	Comments
3(d)(ii)	The acceptance region may increase [or may not change if k is close to 10]	E1	oe Cannot be definitive
		1	

	Question 3 Total	9	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
4(a)	$\int_0^1 ct^2 dt + \int_1^2 \frac{1}{2}t dt = 1$	M1	Correct integrals set equal to 1 at some point in working oe
	$\left[\frac{ct^3}{3}\right]_0^1 + \left[\frac{1}{4}t^2\right]_1^2 = 1$	M1	Correct integration
	$\frac{c}{3} + \left(1 - \frac{1}{4}\right) = 1$ $c = \frac{3}{4}$	A1	oe eg $\frac{c}{3} = \frac{1}{4}$ AG
		3	

Q	Answer	Marks	Comments
4(b)(i)	$\int_0^1 \frac{3}{4}t^3 dt + \int_1^2 \frac{1}{2}t^2 dt$	M1	At least one correct integral with correct limits PI
	$= \left[\frac{3t^4}{16}\right]_0^1 + \left[\frac{1}{6}t^3\right]_1^2$	A1	Sum of two correct integrations PI by correct answer
	$= \left(\frac{3}{16} - 0\right) + \left(\frac{8}{6} - \frac{1}{6}\right)$ $= \frac{65}{48}$	A1	oe eg 1.35416
		3	

Q	Answer	Marks	Comments
4(b)(ii)	$\text{Var}(T) = E(T^2) - (E(T))^2 = \frac{81}{40} - \left(\frac{65}{48}\right)^2$	M1	Allow their $E(T)$ from part (b)(i) PI
	$\frac{2203}{11520}$	A1	CAO
		2	

Q	Answer	Marks	Comments
4(c)(i)	$= 3E(T) + 1$ $= 3 \cdot \frac{65}{48} + 1$ $\frac{81}{16}$	M1 A1ft	Allow their $E(T)$ from part (b)(i) PI 5.0625
		2	

Q	Answer	Marks	Comments
4(c)(ii)	$= 9\text{Var}(T)$ $= 9 \cdot \frac{2203}{11520}$ $\frac{2203}{1280}$	M1 A1ft	Allow their $\text{Var}(T)$ from part (b)(ii) PI 1.72109375
		2	

	Question 4 Total	12	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
5(a)	$P(X < 6) = P\left(z < \frac{6 - 8}{1.3}\right)$	M1	Standardises
	$P(z < -1.54)$	m1	Use of 1 – their p , PI
	$[= 1 - F(1.54)] = 1 - 0.93822$ [using tables]	A1	Accept 0.0620 Condone 0.062 (i.e. omitting trailing zero)
	0.0618 [using tables]	3	

Q	Answer	Marks	Comments
5(b)	$[F^{-1}(0.8) = z =]$ 0.8416	B1	Condone a z -value of 0.84 or 0.842 Seen or used
	$\frac{26.3 - m}{1.5} = 0.8416$	M1	oe Standardising with their z -value
	$[m =]$ $26.3 - 1.5 \times 0.8416 = 25.037\dots$ $= 25$ to 2 sf	A1	CSO Be convinced
		3	

Q	Answer	Marks	Comments
5(c)(i)	$X_M + X_B + X_D \sim N(53, 10.19)$	B2	B1 for $m = 53$ seen or used B1 for $S^2 = 10.19$ seen or used
	$P(z < 2.19)$	M1	PI standardises with their m and S^2
	$= 0.986$	A1	AWRT z -value is 2.18... and answer is 0.985 if $m = 25.037\dots$ is used
		4	

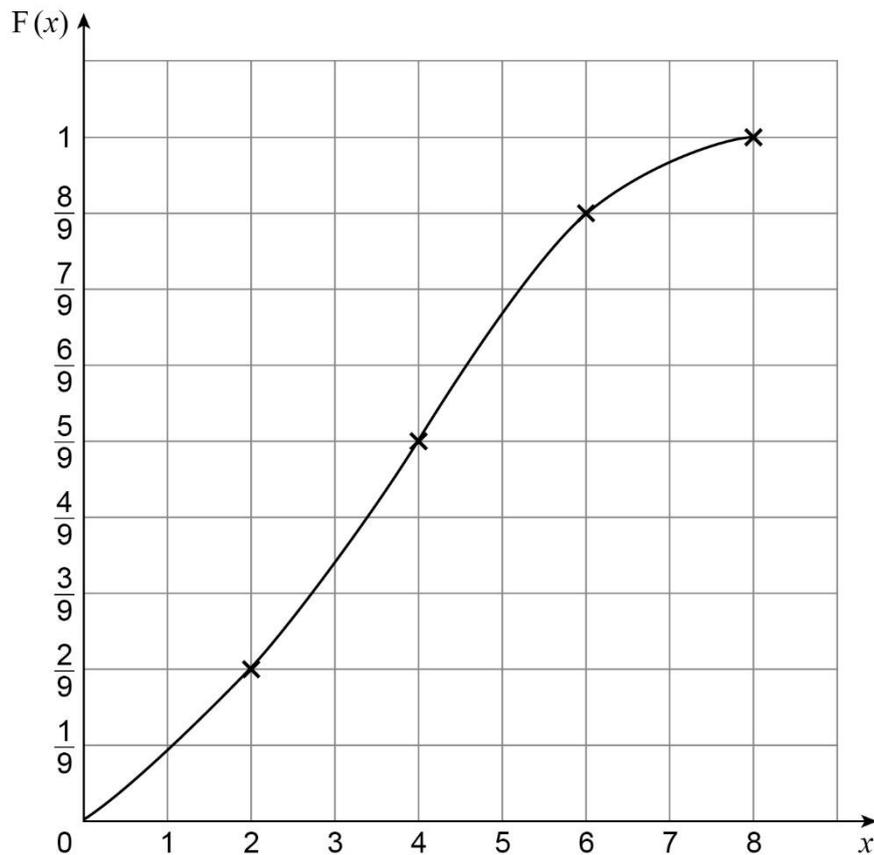
Q	Answer	Marks	Comments
5(c)(ii)	$X \sim B(8, 0.986)$	B1	PI
	$P(X = 8) = 0.986^8$	M1	ft their p from (c)(i) , $P(X = 8) = p^8$
	$= 0.89$	A1	AWRT, NMS 3/3
		3	

	Question 5 Total	13	
--	-------------------------	-----------	--

Q	Answer	Marks	Comments
6(a)	$2k + \frac{(k+2k)}{2} = 2 + \frac{2k-4}{2} = 1$ or $9k = 1$ so $k = \frac{1}{9}$	M1	Attempt to find the area and making equal to 1 Could be seen on the diagram
		A1	AG Be convinced
		2	

Q	Answer	Marks	Comments
6(b)	$P(X < 2) = \frac{2}{9} \text{ or } F(2) = \frac{2}{9}$ and $\left[P(X > 4) = \frac{4}{9} \text{ so } P(X < 4) = \frac{5}{9} \text{ or } F(4) = \frac{5}{9} \right]$ As $P(X < 2) < 0.5$ and $P(X < 4) > 0.5$ [and $P(X < c) = 0.5$] then $2 < c < 4$	M1	Finds two probabilities or finds two areas
		A1	AG Be convinced or correct working leading to $c = \sqrt{14}$
		2	

Q	Answer	Marks	Comments
6(c)	See image below	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>Any two of $\left(2, \frac{2}{9}\right), \left(4, \frac{5}{9}\right), (8, 1)$ plotted or stated</p> <p>Straight line for $0 \leq x \leq 2$</p> <p>(Increasing +ve gradient) Curve for $2 \leq x \leq 4$ where $x = 3, F(x) = \frac{3}{9}$</p> <p>(Decreasing +ve gradient) Curve for $4 \leq x \leq 8$ where $x = 6, F(x) = \frac{8}{9}$ and no curve for $x > 8$ or $F(x) > 1$</p>



		4	
--	--	----------	--

	Question 6 Total	8	
--	-------------------------	----------	--

Q	Answer	Marks	Comments
7(a)	$H_0 : m = 127$ $H_1 : m < 127$ d.o.f $n = 9$ $t_{\text{crit}} = -1.383$ $t = \frac{125 - 127}{\sqrt{\frac{12.9}{10}}}$ $t = -1.76$ Do not accept H_0 as $-1.76 < -1.38$ [3] or $t < t_{\text{crit}}$ or $ t > t_{\text{crit}}$ Evidence to suggest that Lottie's claim is true or evidence to suggest that the mean height is less than 127 cm [at the 10% level of significance]	B1 M1 A1 M1 A1 A1ft E1	Both hypotheses PI by correct t_{crit} AWR T Seen or used Allow +1.383 AWR T Allow $p = 0.05605$ Allow 'reject H_0 ' Must be consistent with their conclusion on whether to accept H_0 or not or their t and t_{crit} if not explicitly stated Correct conclusion based upon ft their t Must be in context, must not be definite and all the previous 6 marks must have been awarded
		7	

